Universal Protologics

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Abstract. A system of *protologics* is developed as precursors of a wide range of logical systems by considering the nature of an arrow, $\rightarrow$, linking nodes in a directed pseudograph or *net*. The arrow is the residual sign resulting from *desemantification* of the derivation relation of many logics. The protologics are defined by its incremental resemantification through *inference patterns* specified as schematic subnets that license the addition or deletion of links and nodes in a net. An auxiliary arrow, $\nrightarrow$, is defined to restrict the placement of $\rightarrow$ and represent the notion of *inconsistency*. The graphical symbolization of logical formulae introduces no additional constructs beyond those of conventional linear symbolisms, but avoids the proliferation of variables to link multiple occurrences of the same logical entity. The primary semantics of the protologics are proof-theoretic but notions of *content* and *context* are introduced to provide protosemantics for intension and extension respectively. Other abstract interpretations are investigated to illustrate the general relations between alternative semantics for logical systems, including truth values, intensional, extensional and incompatibility reconstruction, and mereological models. A range of significant generic schemata, or *design patterns*, are analyzed to illustrate the representational capabilities of the kernel protologic including determinables, contrast sets, taxonomies, cluster concepts, family resemblances, graded concepts, conceptual spaces, frames and individuals. The protologics support paraconsistency in that the propagation of inconsistency may be localized. *Definitional connectives* such as conjunction and disjunction are introduced as hypothesized *ideal nodes* that are extremal relative to other nodes having the same links. The addition of a *preference relation* over the nodes of a disjunction allows reasoning about inconsistency to be supported. This enables the representation of the structures developed in recent years to formalize *defeasible reasoning*. The logical system is fully computational through the implementation of an inference engine and an interactive graphic language for entering knowledge structures and viewing inferences, and its applications in a wide range of disciplines are illustrated through computational examples.

1. Introduction

This article extends one on visual languages for knowledge representation which defined semantic network graphic primitives that mapped unambiguously to description logics, and *vice versa* [140]. It provided logical and computational foundations for semantic networks based on the associated description logics having well-defined semantics as fragments of first-order logic with known computational complexity and associated inference algorithms [20]. The examples chosen to illustrate the visual language included the classical knowledge patterns of determinables, contrast sets, genera/differentiae, taxonomies, faceted taxonomies, cluster concepts, family resemblances, graded concepts and frames.

Definitions and rules were also represented but many major knowledge patterns only required the syllogistic primitives of entailment and incompatibility with all concepts being primitive. That is, the concepts involved were *undefined* from a mathematical logic perspective, but strongly constrained through their relations to other concepts, and the logical power of definitions and rules could be recovered through *abductive inference*[1] formalized as a *disjunctive judgement* [141, p.179]. This suggests the need to model inferential primitives and patterns beyond those normally taken as foundational.

Tools supporting the semantic network language have been used to elicit, edit, validate and apply knowledge structures for many practical applications and it has been noted that the translations from the visual language to normal logical symbolism, whilst necessary to export the knowledge to inference engines, are less perspicuous than the visual form. It has been found in training users to express their knowledge in the visual language that the most readily understood expositions are in terms of *design patterns* [145] providing schemata representing common knowledge structures. The logical primitives are also best explained in terms of *factoring* knowledge structures into *modules* and the *virtual copying* [122] of common general patterns to specific ones through arrows representing entailment/subsumption. Exhibiting the processes of inference as a *dynamic animation* of the visual structures has also been found valuable to promoting human understanding of the reasoning process.

All this suggests that the scope of the visual language can, and should be, extended by formalizing the processes of logical inference directly within it. This article retains the emphasis on visualizing knowledge and inference patterns but formalizes these notions within the visual language, presenting the logical primitives and inference patterns that define them in graphical form.

It is minimalist in commencing with a single connective that is common to most systems of logic, and defining other logical operations through structural constraints on the allowed usage of that connective.

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[1] The logical model of *abductive inference* developed in this article treats it as synonymous with inference to the best available templet, stereotytpe, explanation, or similar term [170, 179], that is as *selective* rather than *creative* [324]. The extension of the model to the creative aspects of abduction in Peirce's theory of inquiry [71] is discussed in §???.

It avoids ontological commitments inappropriate to a universal protologic, instead providing a framework for representing various theories of notions such as *singular* and *universal*, *individual* and *concept*, *true* and *false*, rather than treating them as primitive notions of a logical system. The long-standing scholarly debates and massive literatures on these notions [117] suggests that they are extra-logical, derivative rather than foundational in the development of systems of logic.

The motivation and basic approach are similar to those of Lorenzen in his *Protologik* [236] preamble to his *Konsequenzlogik* [235]. Some of the constructions developed may be viewed as a continuation of those of Hertz [**?**], particularly in the use of diagrams, syllogistic constructs and the *ideal elements* of the Hilbert school [176, 237, 355]. Specific links will be noted as they occur.

This approach is also consistent with Béziau's [38] proposal of *universal logic* as the study of *deduction* in any logical system through an integrative conceptual framework that presupposes no particular axioms. He [39, p.23] further proposes that universal logic might be useful to other disciplines if presented in a way that avoids unnecessary symbolism, that, for example, "the definitions philosophers need involve mathematical abstract conceptualization rather than symbolic formalisation." However, most universal logic research has naturally adopted the symbolism of formal logic and this, like mathematical symbolism in general, can be a significant barrier to understanding [173].

The overall objective is to develop a system of *universal protologics* in terms of inference patterns presented graphically that are natural, readily explained and understood, underlie most logical systems, and can be used for wide range of theoretical and practical purposes.

1.1. The nature of logic

What criteria are relevant to assessing a system of protologics? The primary dimension may be characterized as one that ranges from *pure* to *applied*. The protologics should be coherent as abstract systems and applicable as kernels from which may be developed a wide range of logical systems encompassing as many well-established systems as possible. They should also enable logical systems to be built that are targeted on the needs of other disciplines, implementing only the constraints appropriate to those needs.

The pure-applied dimension is not a simple graduated dichotomy but rather a structure branching from a kernel system devoid of sense or reference through an incremental system of interpretations that increasingly enrich its meaning in terms of a wide range of conceptual structures relevant to logic and its applications.

These issues have been studied from many perspectives and those relevant to the protologic are summarized in the following sections.

1.1.1. Pure logic as a Bourbakian mother structure. These considerations are not specific to logic but apply to mathematical formalization in general as, in Bernays'

terms, a *theoretical phenomenology of structures*.[2] They suggest that formal logics be treated as mathematical structures but not what distinguishes them as a subfield of mathematics.

In the context of Bourbaki's [64] *architecture of mathematics*, Béziau [40, p.14] has proposed that logical structures should be considered as "fundamental mother structures in the sense of Bourbaki, having the same status as algebraic, topological and order structures." He proposes that *univeral logic* comprise the study of an arbitrary relation on the product of a set with its power set as the ultimate abstraction of the notion of *deduction*.

This provides a generic criterion of what differentiates logic from other mathematical disciplines. In a special issue on the 'nature of logic,' Benthem [31, p.22] proposes a more specific criterion that logic is a formal system representing "ubiquitous patterns in information representation and reasoning, which arise in any subject matter whatsoever." This is consistent with the notion of mathematics as a *science of patterns* [340, 304].

This article focuses on logic as a *science of inferential patterns* based on order structures but deviating from the standard development of the theory of order relations by introducing additional structural constraints necessary to represent specific logical notions such as *incompatibility*, *inconsistency*, *truth*, *model*, *logical constants*, *definitions*, *rules* and *defeasibility*, and treating the structures as intrinsically *dynamic* to represent additional components resulting from *hypotheses*, *inference* and *structural combination*. These non-standard constraints change the theoretical issues involved sufficiently radically to suggest that logic, whilst having aspects of algebra, topology and order, is a mother system in its own right.

1.1.2. Applied logic as extended cognition. Like other sub-disciplines of mathematics, logic provides foundational capabilities for many other disciplines. Bééziau suggests that a universal logic conceptual framework contributes to these disciplines by providing techniques for tailoring logical systems to address the precise purposes for which they are required. It helps build "the right logic for the right situation" [39, p.19], making it possible to avoid the wholesale import of inappropriate axioms of a logical system that may introduce artifacts by going beyond those deriving from a conceptual analysis of the problématique.

Logical systems may regarded as a tool for expressing and communicating knowledge in a formal, operational language. Krämer [208, 209, 210] has analyzed the evolution of writing from being a linear phonetic transcription of spoken language (*phonographischen Schrift*) to using the two-dimensionality of the page to structure the material presented and to include diagrams and technical material (*operative Schrift*) involving mathematical and logical symbology. She emphasizes the role of such operative writing as "a tool of the mind, a technique of thinking and intelligence amplifier."[3]

[2] "Die Mathematik aber läßt sich auffassen als theoretische Phänomenologisch der Strukturen." [35, p.18]

[3] "ein Werkzeug des Geistes, eine Denktechnik und ein Intelligenzverstärker." [208, p.171]

Benthem [31, p.22] also notes these *cognitive science* aspects of the nature of logic, and they are nicely captured by Brandom, whilst avoiding both *psychologism* [379] and *normativity* [364]. He presents logic as supporting *meta-rationality* enabling us to reason and communicate about our pre-formal rational behaviour, "logic does not define rationality in the most basic sense, but by making it possible for us to express explicitly the already rational relations articulating the contents of all our thoughts, it ushers in a higher level of rationality" [67, p.10].

Béziau [39, p.23] has suggested that *universal logic* might be presented as a formal but comprehensible conceptual framework that avoids unnecessary symbolism, that, for example, "the definitions philosophers need involve mathematical abstract conceptualization rather than symbolic formalisation." However, most universal logic research has naturally adopted the symbolism of formal logic and this, like mathematical symbolism in general, can be a significant barrier to understanding and the development of meta-rationality [173].

Dutilh Novaes enlarges her previous analyses of the nature of formal logic [111] from the pure to the applied in her book *Formal Languages in Logic* [112] by focusing on the role of logic in human rationality. She [112, ch.5] surveys the psychological and neurological literature to determine how formal languages support *extended cognition* [84, 310, 250] and how the acquisition of skills in their uses changes brain processes and structure [103]. She summarizes research on their limitations as models of human reasoning, their promotion of 'system imprisonment' [32], their being deductively incomplete and non-categorical, and their trading tractability for expressiveness, and emphasizes the need to represent defeasible, nonmonotonic reasoning in order to capture human practice [112, pp.128 & 143].

These analyses all avoid presenting logic as a normative criterion for cognition, or *vice versa*, but characterize it as an aspect of human introspection and communication that both describes prior rationality and goes beyond it to support a range of meta-rationalities.

This article attempts to bridge between the rationality of intuition and experience to the meta-rationalities of formal logics in a way that is readily assimilated with a minimum of specialist knowledge and, in this regard, is an extension of one on the relation of universal logic studies to human rationality [141]. In particular it exemplifies the capabilities of the kernel protologic to represent common schemata widely used as templets for structuring and encoding experience, and the inferential processes involved in anticipating it, managing it, understanding it and communicating about it, including defeasible reasoning.

1.1.3. Logic as techical writing, *operative Schrift*. In her analysis of the processes of abstraction involved in technical writing, Krämer argues that sense and meaning making are only possible in conjunction with sense and meaning reduction or deletion,[4] and characterizes the pure end of the dimension as abstraction to

[4] "Sinnbildung und Sinnerzeugung immer nur denkbar sind im Verein mit Sinnreduktion und Sinnlöschung" [208, p.169]

the extreme of complete *desemantification* (*Entsemantisierung*). She views technical material as being essentially algorithmic and computational from its earliest inception, enabling the reader to represent cognitive processes that may be beyond their understanding and use them to solve problems.[5]

Dutilh Novaes [111] has situated the desemantification and computational aspects of logic historically through her analysis of the use of the qualifier *formal* in the logical literature. In addition she identifies the notions of formal as schematic, indifferent to particular objects, topic-neutral, abstracting from intentional content, and pertaining to regulative or constitutive rules. She [112, §6.1.2] introduces the term *resemantification* to describe the process of interpreting a desemantified system in terms of a less abstract one in order to re-introduce more meaningful features.

In this article the term *resemantification* is reserved for the incremental reintroduction of common logical constraints to a desemantified derivation relation (§2, §7, §8). The term *semantification* is used for various extra-logical interpretations that add meaning without changing the underlying logical system, including: those intrinsic to the logical system, such as truth values (§2.7), or intensional (§5.2.1), extensional (§5.2.5), and incompatibility (§5.2.7) reconstruction; pragmatic constructions such as named generic schemata found useful in some applications (§4); and translations into familiar logical systems (§6.3).

There is no conflict with Dutilh Novaes' usage since the boundary between the logical and extra-logical aspects of a derivation relation is to a large extent one of choice and convention.

1.1.4. Universal logic as controlled desemantification/resemantification. Desemantification as discussed by Krämer and Dutilh Novaes may be traced back to Aristotle's [15] exposition of the categorical syllogistic in terms of schemata having meaningless letters as variables representing any terms that might be used as content. However, he retains a strong semantic component by deriving his argument forms from the prior linguistic meanings of the copulae used to join these variables.

In his seminal paper on the structural reasoning underlying universal logic Hertz extends the desemantification process to the copula representing the derivation relation, stating that there is no need to specify "what the symbol → linking the characters a→b or the word 'if' in the corresponding linguistic formulation means."[6]

Bernays further extends Hertz's desemantification when he uses it to exemplify Hilbert's philosophy of mathematics by reducing → to a *sign* rather than a *symbol*: "If the hypothetical relationship 'if A then B' is symbolically represented

[5] "Notationssystem, das eine Doppelfunktion erfüllt: Es ist einerseits Medium zur visuellen Repräsentation einer Domäne kognitiver Gegenstände und andererseits Instrument zum handgreiflichen Operieren mit diesen Gegenständen, um dadurch zu Problemlösungen oder Beweisen zu gelangen, welche sich auf diese kognitive Domäne beziehen." [208, p.161]

[6] "was das Symbol → in der Zeichenverbindung a→b oder das Wort „wenn" in der entsprechenden sprachlichen Formulierung bedeutet, braucht hier nicht angegeben zu werden" [**?**, p.247]

by A→B, then the transition to the formal position is that we abstract from that the meaning of the symbol → and take the linkage by the 'sign' → itself as the primary consideration."[7]

These suggested desemantifications of → may be seen as precursors to Béziau's [38] proposal of *universal logic* as an integrative conceptual framework for all logical systems that presupposes no particular axioms. His conceptual framework has been widely adopted and in a wide range of studies of foundational and historical issues in logic and of the nature and relationship of logical systems, as did Birkhoff's [52] notion of *universal algebra* for foundational studies of algebras.

This article commences with a completely desemantified sign, →. It develops a system of *universal protologics* through the incremental specification of formal properties ascribed to → to resemantify it in order to capture commonly expected features deduction. It expresses these specifications as inference patterns represented by arrows and nodes, including variant forms of → overloaded to represent additional constraints upon those structures that constitute a logic. It further semantifies the protologics by representing within them supervenient logical phenomena as noted at the end of §1.1.3, some of which are not normally regarded as epiphenomenal but are deflated to be so through this approach.

1.2. Protosemantics

1.3. Terminology and examples

At one level this article is about the development of an abstract framework for logics with no connotations other than those of the formal specifications. However, to cast it in this way would undermine much of purpose of this presentation which is intended to provide a minimalist, but powerful and understandable, logical foundation for discussion of fundamental issues in metaphysics and to have practical applications in disciplines dependent on metaphysics such as *expert systems* [191], *computational ontologies* [339] and the *semantic web* [325].

Hence terms have been adopted to name abstract patterns that reflect those in the literature and seem natural to those patterns, but these terms are strictly technical and none of their possible connotations beyond their formal definitions are used to draw inferences. That is, the protologic semantifies the terms and not the terms the protologic.

For example, A→B is read as 'A includes B' which is consistent with Aristotle's [15, 68a23] 'B belongs to every A' (τὸ B παντὶ τῷ A ὑπάρχειν), Leibniz's [225, p.100] 'A includes B' (*A includere B*), Kant's [199, A7/B11] "B is contained in A" (*B in A enthalten ist*) and Wittgenstein's [376, 5.122] "the sense of B is contained in the sense of A" (*der Sinn von B im Sinne von B enhalten*).[8] However, the term

[7] Wird die hypothetische Beziehung „wenn A so B" symbolisch durch A→B dargestellt, so besteht der Übergang zum formalen Standpunkt darin, dass wir von jener Bedeutung des Symbols → abstrahieren und die Der knüpfung durch das „Zeichen'" → selbst als zu Betrachtende nehmen. [34, p.333]

[8] Aristotle's and Wittgenstein's use of letters has been adjusted for consistency with Bernays.

'includes' signifies only the syntax of linking by the arrow and has no connotations as to what, if anything, is included.

Two complementary technical term are also introduced. The term *content* is used to reference that which might be included. That is, A→B may be read as 'the content of A includes the content of B.' This terminology is consistent with that of those who have proposed that logical entailment be explicated as *meaning* [114], *sense* [287] or *content* [65] *containment*.

The term *context*, is used to reference the effect of inclusion, of being within the scope of the meaning. That is, A→B may be read as 'the context of B encloses the context of A.' This terminology is consistent with Aristotle's use of the term *qua* (ᾗ, as) in his *Metaphysics* to introduce a context, for example, to consider physical objects 'qua moving' rather than 'qua bodies',[9] that is in the context of their motion rather than those of their features as objects [25, p.70].

The mereological allusions of these terms are converse to one another. A *includes* the content of B but is *is enclosed in* the context of that content. These allusions have influenced the semantic interpretation of logical systems though the millennia [132] and are analyzed in §5.3 in relation to the inference patterns developed for →.

Any connotations of the terms 'content' and 'context' beyond their initial technical usage derive explicitly from the constraints placed upon the use of →, rather than from *a priori* intuitions. The connotations are relative to the constraints and their acceptability may be used to assess both constraints and intuitions, possibly changing them. In some situations these aspects of the interpretation of → may make little contribution, but in others they provide interesting perspectives on the nature of particular logical structures and inferences, such as those associated with 'meaning,' 'substance' and 'individual.'

Thus, while the formal specification of the protologic is through abstract inference patterns, supervening on that specification is an abstract protosemantics, both aspects being made more perspicuous by the graphical presentation of the inference patterns. The value of the abstractions may be assessed through their applicability to a wide range of logical systems and to the representation of, and inferences about, a variety of significant knowledge structures.

This article does not aim to make a direct contribution to the historical literature, or to philosophical debates on the foundations of logic, ontology, epistemology and psychology. It draws on that literature to provide insights into logical issues from many different perspectives, from syllogistics to the semantic web. Its intended contribution is indirect, that of providing a neutral, well-founded logical framework in which to represent various positions without prejudice.

These considerations have also motivated the examples selected to illustrate the structures that may be represented within the protologic and the inferences that may be made based on them. They are drawn from a wide range of literatures such as ancient logic, philosophical logic, computational logic and its applications,

[9] ᾗ κινούμεα καὶ οὐκ ᾗ ὄντα [17, 1061β30]

anywhere that presents a many-sided critical debate about significant foundational issues. It is intended, for example, that the logical framework be able to represent the diversity of different theories of *things*, *individuals* and *reference* in terms of everyday life, science and fiction, without bias towards any one of them.

If there is any bias it is towards *pluralism*, that there is no unique 'correct' theory of such fundamental notions but there are common foundations to different perspectives. The richness of the debates provides multi-faceted explicata which need to be captured formally and precisely, and represented in such a way as to be readily comprehended.

2. Logic as a science of patterns

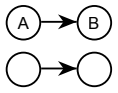
A system of protologics is developed though patterns of inference over a graphical structure constituted by nodes linked by → and additional signs derived from constraints upon its use.

The starting point is Bernays suggestion that the ultimate abstraction of a derivation relation is to treat → as a desemantified *sign* rather than a meaningful *symbol*. The process of resemantification is to let the meaning of the derivation relation *emerge* within a semiotic framework as suggested by Merrell [251, p.x] in his studies of Peirce's notion of signs and their meaning, "Meaning does not emerge through some specifiable contact between people and people and between people and language and language and the world, but rather, it is the very process of emergence... Signs, ultimately, speak for themselves. In a certain sense, they are their own meanings."

The semiotics of arrows stem from their use in archery to project force from place to place, which provides an allegory for any form of directional transfer [133, 213]. The initial resemantification of → is to consider it as a directional connective between two unspecified entities. These entities will be termed *nodes* using a graph-theoretic terminology without drawing on any other notions from graph theory.

2.1. Arrows, links and graphs

Béziau [38, p.85] introduces a minimal resemantification of → as a *deduction* sign by going back to the Latin roots of the term and interpreting it as representing *leading away* from premise to conclusion. That is we may regard the arrow in A→B as a directed connective leading from the symbol 'A' at the tail to the symbol 'B' at the head. The resulting structure may be termed a *link* constituted by the triple (A,→,B). It may also be described as a *link out of* A constituted by the pair (→,B) or a *link in to* B constituted by the pair (A,→).



A link is naturally represented in graphical form as two labelled nodes with an arrow between them or, more generically if the labels are irrelevant, as an arrow between two anonymous nodes. It will be assumed that labels constitute unique identifiers from some family of identifiers

with an equality relation, and that an anonymous node has an implicit label unique to that node.

The only resemantification involved in labelling nodes is the assumption that the entities linked by → may be identified and distinguished. Node labels may be chosen to suggest possible connotations but these make no formal contribution to the logical structure represented by the links. They are logically meaningful only to the extent that the linkage structure represents such connotations.

Nodes with identical labels will be taken to represent the same node shown more than once, and may be merged to form a canonical graph-like structure with no duplication of nodes. This allows structures to be split into substructures, possibly overlapping, that can be merged to reconstitute the original structure. It also allows a structure to be merged with one of several other structures, each representing an alternative component, for example, a different 'ontology' or 'theory.'

Figure 1 shows a number of links merged into a graphical structure, S, with no duplication of nodes.

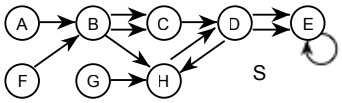


FIGURE 1. Graphical structure specified by multiple links

Note that the graph representation adds no additional information to that of a linear representation as the multiset, S ≡

[A→B, B→C, B→C, C→D, D→E, D→E, E→E, F→B, B→H, G→H, H→D, D→H]

It is only a more perspicuous representation of the structure represented by multiple links involving the same nodes.

Figure 2 shows the same links merged into three overlapping substructures, S1, S2 and S3, representing the same structure in a modular way.

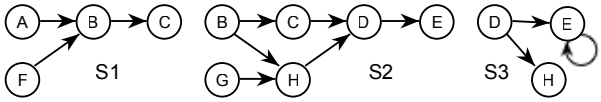


FIGURE 2. Same structure specified modularly as three substructures

The linear representations are shown below and S is their multiset sum:

S1 ≡ [A→B, B→C, F→B]
S2 ≡ [B→C, C→D, D→E, B→H, H→D, G→H]
S3 ≡ [D→E, E→E, D→H]
S ≡ S1 ⊎ S2 ⊎ S3

2.1.1. Terminology for nodes, arrows and graphs. The resemantification of the arrow sign so far is sufficient to support the basic structures of graph theory. However, as evident in the examples, there are no constraints to preclude parallel arrows between the same nodes or loops from a node to itself. Hence, in graph-theoretic terms, the examples given are not strictly *directed graphs* (digraphs) but rather *nets* [169, p.4-7] or *directed pseudographs* [23, p.4].

There is also, as yet, no support for the inferential processes of logic. However, in the continuing resemantification a *repetition inference pattern* may be used to add or delete repeated arrows, and a *loop inference pattern* may be used to add or delete loops, so that the nets may be reduced to digraphs. In addition, cycles such as that between A and D, are shown to generate an equivalence between nodes and taking the quotient of a graph under this equivalence allows a further reduction to a *skeleton* [1, p.51] that is an *acyclic digraph* representing a partial order underlying the inference processes of a logical system [206, 343].

As later sections will define other forms of arrow denoting additional or different constraints, the arrow so far considered, →, will be termed an *inclusion arrow* and a link involving it as *inclusion link*. The tail node will be said to *include* the head node, and the head node to be *included in* the tail node. A series of nodes such that each includes the next will be termed an *inclusion path* or *path* from the initial node to the final node in the series. For example, the series of nodes (B, C, D, E) in the centre of Figure 2 constitutes a path from B to E.

A node is said to *indirectly include* another if there is a path from the first to the second, in which case the second is also said to be *indirectly included* in the first. A node that includes, or indirectly includes, another is said to be *down* a path from it, and that node is said to be *up* a path from the original node. A node with no path to it is said to be a *bottom node*, and with no path from it a *top node*. A *path-independent* or *transitive* interpretation of → is such that when a node is down a path from another it also necessarily includes it, that is, indirect inclusion is also inclusion.

A node that includes a node named 'A' may be termed 'some node included in A,' abbreviated to 'some A.' Issues of *existence assumptions* in interpreting 'some' as a *particular quantifier* are discussed in §3.3.2.

As noted in §1.2 the 'inclusion' terminology will be extended by associating with each node an unspecified *content* and an unspecified *context* that constitute what is included and the effect of that inclusion, respectively. Neither 'content' nor 'context' have any connotations other than providing a convenient terminology for describing inference patterns and other structural constraints in the protologic as they are introduced.

Since directed graphs or pseudographs are common to a range of disciplines there are many other terms used for 'includes', such as 'a predecessor of' , 'a descendant of', 'subordinate to,' 'inferior to', 'lesser than', 'is,' and so on, and for 'included in', 'a successor to', 'superordinate to,' 'an ancestor of', 'superior to', 'greater then', 'belongs to,' 'inheres in,' and so on. Since the direction of the arrow is arbitrary, these terminologies may be reversed and constructions based on → have duals. Similarly, nodes may be referenced through discipline-specific terminology such as 'terms', 'concepts', 'predicates', and so on, and inclusion arrows as 'derivations', 'entailments', 'implications', 'consequences', and so on.

A distinction will be made between nodes and links that are *specified* to be part of a net, and those that are *inferred* to be so through various inference patterns. It is also useful to consider *abstract nets* in which all the labels are

anonymous having form without content and exhibiting structures that may be common to many specific nets, and *concrete nets* where nodes are labeled in a way that is intended to have a meaning additional to that specified in the net and hence extra-logical.

2.2. Inference patterns and invariance under logical interpretations

While it is convenient to use the terminology of graph theory to describe the nets that represent collections of links, logical theory focuses on the *dynamics of change* in such collections. This is not a primary concern of graph theory [168], although it is one of growing interest in many applications [49].

The logical dynamics of nets will be captured in terms of *inference patterns* in which an abstract subnet of a particular form is recognized as licensing the addition or deletion of one or more links while leaving any 'logical interpretation' of the net invariant. These notions are formalized in the following sections.

A *logical interpretation* of a net is defined to be an inference-preserving conservative translation [75] of the net into statements of a logical system supporting some notion of inference. To state that it is *invariant* under a change to the net is to assert that, within that logical system, the translation of the changed net will have the same inferences as that of the original net. Two nets with the same logical interpretations will be termed *logically equivalent*.

This is a constraint upon appropriate translations, that the target logical system must implement within its own framework the inference patterns of the protologic, and this must be verified for each translation. It is also intended that the inference patterns in the protologic can be understood in their own right and that the graphical language can be used to represent the form of knowledge structures and and the dynamics of inference.

An inference pattern may be seen as an *analytic invariant* of a net in that it can dynamically expand a net by adding logical inferences implicit in its links, and hence can also delete them as being superfluous, contracting the net, possibly to a minimal form. Additions or deletions that maintain the net logically invariant will be termed *conservative expansions* or *conservative contractions*, respectively.

This perspective is intended to reserve the term *logical* to denote the dynamic aspects of the formal structure of nets. It leaves open other extra-logical, but logically constrained, perspectives such as those relating to the semantics of nodes, links and nets.

Some aspects of such extra-logical notions can be addressed by further development of notions such as *content* and *context*, and others by specifying *abstract nets* to capture common generic schemata, *concrete nets* to represent more specific ones, and other constructions within the framework of the protologic.

2.3. Repetition inference pattern

A simple example of resemantification through a logical inference pattern is that for adding or deleting parallel arrows between nodes. For many logical interpretations multiple links of the same type in the same direction between two nodes do not

affect the logical interpretation of a net. Their addition or deletion is a conservative expansion or contraction.

This could be implemented by requiring a collection of links to be a *set* rather than a multiset or by requiring the graph to represent a *relation* rather than a net or directed pseudograph. However, in some logical systems, the repetition of a statement can be significant, for example in linear logic [154] or the representation of implicature [181] and resource limitations [366], and the lack of significance is best represented explicitly as an *repetition inference pattern* (Figure 3).

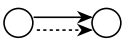


FIGURE 3. Inclusion repetition inference pattern

A metalogical distinction has been made in the abstract net representing the inference pattern by showing the pattern-defining arrow in black and the inferred arrow in light blue (grey in the paper journal). The inference pattern indicates that if the pattern-defining link is found then a parallel link may be added, or any existing parallel link may be deleted, without changing the logical interpretation of the net of which the pattern is part.

If x and y are links, S a multiset of links, and ≡ indicates identical logical interpretations, an equivalent linear representation might be:

$$\text{for any x, y, S, } [S, x{\rightarrow}y] \equiv [S, x{\rightarrow}y, x{\rightarrow}y]$$

One advantage of using the two-dimensional structure of the page to provide a nonlinear graphical presentation is that, as Shönfinkel has noted,[10] the introduction of variable names is distracting because they serve merely to link multiple occurrences of the same logical entity. This may be specified formally in a net without requiring an artificial label for a generic node.

However, the graphic representation has introduced no additional constructs beyond those of a conventional linear representation. It is a formal specification with no dependence on visual intuitions, but avoids the proliferation of symbols.

2.4. Inclusion inference pattern

The *path-independence* or *transitivity* of → is common to its usage in most logical systems and has been taken to be the characteristic feature of 'a logic' [343]. Hertz [174] represented the transitivity of inference in graphical form (Figure 4 top left) in a presentation in September 1921 at the Jena meeting of the German Mathematical Society celebrating Hilbert's 60th birthday.

Hertz used this representation to illustrate various issues that arise in the factoring of systems of axioms but does not discuss its derivation. In a later paper [**?**] using the same figures, he notes its similarity to Zaremba's [380] notation in his article on the independence and consistency of axioms, and to the use of networks to display proofs.

[10] "die Veränderliche in der logischen Aussage ja nichts weiter als ein Abzeichen ist, um gewisse Argumentstellen und Operatoren als zusammengehörig zu kennzeichnen" [319, p.17]

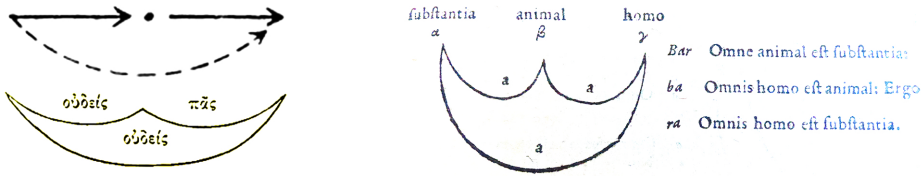


FIGURE 4. The transitivity of inference: Hertz, Ammonius and Pacius

He also notes that his inferences are syllogisms in the mode of Barbara[11], suggesting that the diagrams used in the commonly available Latin translations of Aristotle's organon may have been another inspiration. For example, Pacius' [12, **?**] graphic illustration of Barbara on the right of Figure 4 is similar to that of Hertz except that the arrows are implicit in the left to right direction of inference and the metalogical convention of dotted line for an inference is achieved through a textual label.

He cites Pólya [284] on the explanatory power of diagrams but emphasizes

It is represented by the *inclusion inference pattern* shown in Figure 5 left.[12] The pattern indicates that if a path of two arrows is found an arrow may be added, or deleted, between the end nodes.

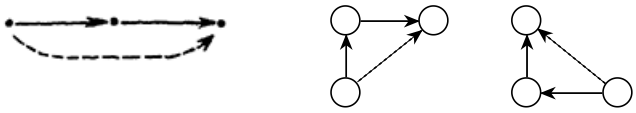


FIGURE 5. Inclusion inference pattern

The same pattern is drawn differently on the right to show that the inclusion inference pattern might be visualized either as the copying, or inheritance, of a horizontal inclusion link downwards from the head to the tail of a vertical inclusion link, or of a reverse horizontal inclusion link upwards from the tail to the head of a vertical inclusion link.

As exemplified in Figure 6 the inclusion inference pattern may be applied iteratively to a longer path to infer that any node that indirectly includes a node also includes it. From the perspective of meaning containment, a node includes the content of, and is enclosed in the context of, all nodes on a path from it.

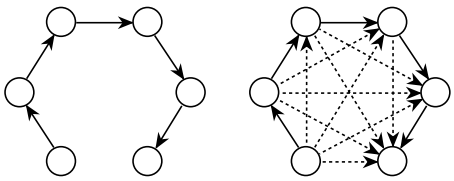


FIGURE 6. Iterative application of inclusion inference pattern

[11] "unsere Schlüsse (sind) Syllogismen nach dem Modus Barbara" [**?**, **?**]

[12] Hertz [**?**, fig.1] uses this diagram to specify the transitivity of →, noting its similarity to Zaremba's [380] figures in his article on the independence and consistency of axioms and the use of networks to display proofs. He also cites Pólya [284] on the explanatory power of diagrams.

2.4.1. Cycles and node equivalence. A cycle of mutual inclusion between two nodes, as shown in Figure 7 left, may be treated as a single symmetric link with the directional arrow heads omitted as shown in the centre.

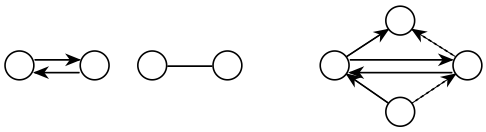


FIGURE 7. Cycles and node equivalence within cycle

Such a cycle induces an equivalence between nodes because the inclusion inference pattern implies that any nodes which includes, or is included in, one node includes, or is included in, the other node as shown on the right.

This generalizes to larger cycles, as shown in Figure 8,[13] because the inclusion inference pattern implies that all nodes in a longer cycle are also linked pairwise in simple cycles, and hence all nodes in a cycle are equivalent in this respect.

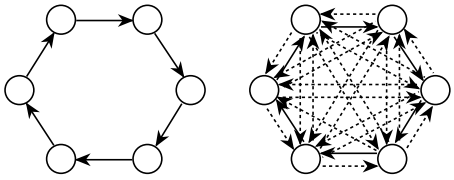


FIGURE 8. Iterative node equivalence within a longer cycle

It will be noted in introducing other arrow types that, as they are defined in terms of →, that the node equivalence induced by a cycle generalizes to commonality of links constituted by any of the arrows that will be defined. Any nodes in a cycle have the same incoming and outgoing links of all types and are equivalent in this respect, and their labels may be regarded as aliases for one another under this equivalence.

2.4.2. Equivalence and loop inference patterns. Another commonly expected logical constraint on → is that should it be *reflexive* so that every node has an associated loop (as illustrated by node E in Figures 1 and 2). Rather than being introduced as an *ad hoc* assumption, reflexivity can be derived from a more specific requirement, that nodes equivalent to any node may be added or deleted without change of logical interpretation. This corresponds to the *equivalence inference pattern* shown on the left of Figure 9, which may be explained in terms of nodes always being able to have more than one label.

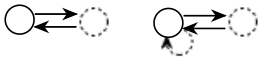


FIGURE 9. Equivalence inference pattern and derived loop

As shown on the right, application of the inclusion inference pattern to the equivalence inference pattern implies that the node has an arrow to itself, a *loop*.

[13]Hertz [**?**, def.10] terms a cycle a *web* (*netz*) for reasons that Figure 8 right makes apparent.

These results may be used to derive the *loop inference pattern* corresponding to reflexivity through the inference sequence shown in Figure 10.



FIGURE 10. Derivation of loop inference pattern

From left to right: the equivalence inference pattern is used to add an equivalent node to an isolated node; the inclusion inference pattern is used to infer that the original node has a loop; and the equivalence inference pattern is used to delete the equivalent node resulting in the loop inference pattern on the right.

Reflexivity is significant in the theoretical development of logics but irrelevant to practical applications where, whatever an inclusion link represents, it is unlikely to be useful to infer that a node includes, or is included in, itself—the epitome of a circular argument.[14]

2.4.3. Protoconjunctions and protodisjunctions. When a node includes, or is included by, two or more other nodes, it is common in interpretations of nets in other logical formalisms to shorten the representation by collecting the labels of the other nodes separated by conjunction or disjunction operators, respectively. For example, a description logic translation of the nets in Figure 11 might be:[15]

$$A \sqsubseteq (B \sqcap C) \qquad (A \sqcup D) \sqsubseteq B$$



FIGURE 11. Protoconjunction and protodisjunction

However, the nets might equally well be translated without the introduction of conjunction and disjunction operators as:

$$A \sqsubseteq B \qquad A \sqsubseteq C \qquad D \sqsubseteq B$$

The use of conjunction and disjunction in the first translation, introduced only to collect terms rather than compose them to define a new ideal node, will be characterized as generating a *protoconjunction* or *protodisjunction*, respectively. Compositional constructions defining ideal nodes satisfying certain extremal conditions will be termed *structural conjunctions* and *structural disjunctions*, and are discussed in §7.1 and §7.2. Similar considerations lead to a distinction between *protonegation* and *structural negation* §2.6.3.

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[14]The circularity of reflexivity can be avoided by explicitly deleting 'premises' from 'conclusions.' It has been argued [141, §8] that the reflexivity of Tarski's [349] consequence operator deviates from the "common usage of the language of every day life" [350] by including premises in conclusions, and shown that Malinowski's [241] *q-consequence* can provide equivalent axioms without this problem.

[15]Description logic notation will be used to exemplify translations of nets because it is concise, well-defined and trnslates readily into other logical notations [20, Appendix].

The significance of these distinctions is that the semantics and inferential complexity of the protoconnectives is substantially simpler than that of the structural connectives, and that the representation of many significant generic knowledge schemata requires only the protoconnectives (§4). However, the linguistic usage of 'and,' 'or' and 'not' does not make a clear distinction between the to aspects of the connectives and neither do most logical symbolism.

2.4.4. The resemantified → as a preorder or partial order. Given any net, if one conservatively contracts it by merging it to canonical form, using the inclusion inference pattern to equivalence the nodes in a cycle, the equivalence inference pattern to delete all but one node in the cycle, and the loop inference pattern to delete any loops, the resultant net satisfies the definition of a *directed graph* [23, §1.2].

It can be further contracted by using the inclusion inference pattern to delete all links that can be inferred from it resulting in the *transitive reduction* [23, p.177] of the cycle-free graph as the *minimal canonical form* of the original net (unique up to the node label chosen to represent all the nodes in a cycle). Figure 12 shows S_{min}, the minimal canonical form of the net specified in Figure 2.

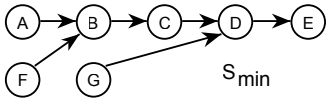


FIGURE 12. Net from Figure 2 reduced to a minimal canonical form

The transitive, reflexive relation defined by the repetition, inclusion and equivalence inference patterns indicates that, if → complies with these patterns, it specifies a *preorder* on the nodes. If the node equivalence of §2.4.1 is taken to define equality of nodes then the preorder becomes a *partial order*.

A preorder has been taken by Straßurger [343] to answer the question "what is a logic?" As the remaining constructions in this article are defined in terms of the preorder it might seem that the preceding sections have already resemantified → adequately to specify a logical system. The sign → has become the symbol for, in De Morgan's terms, an *abstract copula*[16] capturing the essential features of a derivation relation.

However, the preorder alone offers limited representation and reasoning capabilities. The additional metalogical definitions that follow significantly extend the capabilities of the protologic. Because they are always available as constructions within the protologic (as constraints on the usage of →), one could argue that they are inherent in the order relation. One could also argue that their definition is a significant additional resemantification of the sign → to symbolize the derivation relation, leading to a richer notion of what it is to be a logic.

[16]"By an 'abstract copula' is meant a formal mode of joining two terms which carries no meaning, and obeys no law except such as is barely necessary to make the forms of inference follow." [101, p.104]

Resemantification is an incremental process with any combination of inference patterns defining a protologic that can provide the foundation for further resemantification, a branching structure generating different families of logics.

2.5. Definition of an exclusion arrow and link

In representing logical inference, what is most obviously missing in the structures so far defined is the notion that they may be structurally unsound or *inconsistent*. Every net based on the inclusion link, and every net derived from it using the various inference patterns is a legitimate logical structure. Nodes and links may be added to any net without logical constraint. There is support for the notion that derived links are logically necessary but none for the notion that some potential links may be logically impossible.

The *exclusion arrow* $\twoheadrightarrow$ supports the addition of a different type of link to a net that constrains what further links may be added to it. The exclusion link is defined in terms of the inclusion arrow:

Definition: an *exclusion link* may be constituted with $\twoheadrightarrow$ iff any node that includes the tail is excluded from also including the head.[17]

The meta-inference patterns corresponding to this definition are shown in Figure 13. A red cross is used as a metalogical symbol to indicate that an inclusion link is prohibited, either by specification (left) or by inference (right).

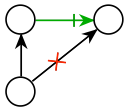


FIGURE 13. Exclusion link definition

The tail node of an exclusion link is said to *exclude* the head node, and the head node to be *excluded by* the tail node. The head node may also be termed *opposite to*, *contrary to* or *incompatible with* the tail node.

2.5.1. Exclusion inference patterns. Three inference patterns may be derived from the metalogical definition of the exclusion link. First, it complies with a repetition pattern, the *exclusion repetition inference pattern* (Figure 14).



FIGURE 14. Exclusion repetition inference pattern

Second, because any node that includes the lower node also includes that to which it is linked (inclusion pattern), it cannot have a link to the node on the right. Hence the lower node satisfies the exclusion link definition and has an exclusion link to the node on the right. The resulting pattern is the *exclusion inference pattern* (Figure 15).

Third, any node that includes the right hand node cannot also include the left hand node since the exclusion link definition would be contravened. Hence there

[17] Leibniz's [225, (2)] explanatory example of his usage of *excludere* follows this pattern: "none who is just is unhappy" (*nullus justus est miser*).

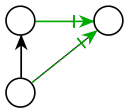


FIGURE 15. Exclusion inference pattern

is a also an exclusion link from right to left, resulting in the *exclusion symmetry inference pattern* (Figure 16).



FIGURE 16. Exclusion symmetry inference pattern

The symmetry inference pattern is what marks a major difference in the inferential dynamics of the two link types, that the exclusion link is inherently symmetric and could be represented by a pair of exclusion links as shown on the left of Figure 17 or as a single undirected link shown on the right[18] (equivalent in interpretation to the pair of exclusion links on the left).



FIGURE 17. Equivalent representations of symmetric exclusion link

The undirected link has repetition and exclusion inference patterns derived from those of the exclusion link as shown in Figure 18 left. The right two nets show a *reverse exclusion inference pattern* that also follows from the exclusion inference and symmetry patterns. The variant on the far right shows how this may also be seen as a *reverse inclusion inference pattern* such that an exclusion link to the tail of an inclusion link may be inferred from one to the head.

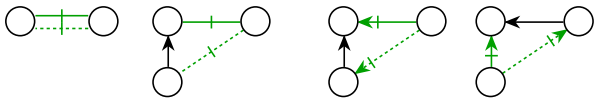


FIGURE 18. Exclusion link and reverse exclusion/inclusion patterns

As illustrated in Figure 19, iteration of the exclusion inference pattern to two paths with each node in the exclusion link as head results in a mixed graph that has some similarities to that for the inclusion inference pattern in Figure 6. However, while the inclusion links travel *up* it to the head (exhibiting *modus ponendo ponens*), the exclusion links travel *down* the chains to the tails (exhibiting *modus tollendo tollens*).

The exclusion inference pattern implies that if two nodes are in a cycle then any node which excludes, or is excluded by one node, excludes, or is excluded by the other. This generalizes the equivalence between nodes in a cycle discussed in §2.4.1 to exclusion as well as inclusion links.

From the perspective of meaning containment, an exclusion link between two nodes signifies that the contents of the two nodes are *incompatible* and their

[18] In graph theory, graphs with both directed and undirected arcs are termed *mixed graphs* [23, §1.7] but interaction between the two types of links as specified in the exclusion inference pattern does not appear to have been studied in that literature

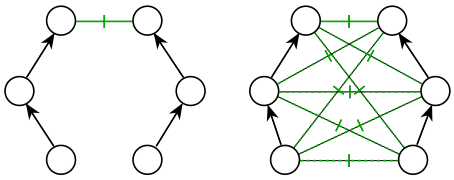


FIGURE 19. Iterated application of exclusion inference pattern

contexts are *disjoint*. That is, some part of the content of one node cannot be included with some part of the content of the other node, and no node may be enclosed in the context of both the nodes. Mereological interpretations of these constraints are discussed in §5.3.

From an inferential perspective the directed exclusion link can always be replaced by the undirected one with no change in logical interpretation; exclusion is always mutual exclusion. However, when considering the semantics of a net it may be appropriate to support the metalogical distinction between the *specification* of the net and the *inferences* ensuing from that specification. There may, for example, be significance to the notion that the specified direction of the arrows depicts how a new node *depends* on existing nodes. Hence the symmetry of the exclusion link is, for some purposes, best represented as an inference rather than an intrinsic feature, even though this may increase the number of steps required in a derivation.

2.5.2. Inconsistency inference pattern. The definition of an exclusion link in §2.5 introduces a metalogical constraint that an inclusion link is not allowed between two nodes. If this constraint is violated in the net, either by specification or by inference, the outcome will be that two nodes are connected by both inclusion and exclusion links as shown in Figure 20 (the exclusion link is shown in symmetric form to encompass all cases of parallel inclusion and exclusion links).

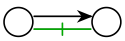


FIGURE 20. Inconsistent link

Such a pair of links will be considered as constituting a single link combining conflicting link types and termed an *inconsistent link* from the tail of the inclusion link to its head. The tail node will be termed an *inconsistent node*, and the net of which it is part an *inconsistent net*.

The inconsistency inference pattern shown in Figure 21 derives from inclusion and exclusion inference patterns. If one node has an inconsistent link to another then any node that includes it also has an inconsistent link to it and to that node, as will any down a path to it. Since any node is down a path from itself, any node with an inconsistent link also has an inconsistent link to itself.

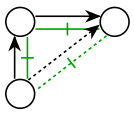


FIGURE 21. Inconsistency inference pattern

An inconsistent link in a net, whether specified or inferred, implies that the exclusion link definition has been violated by the specification of a node that includes both the tail and the head of an exclusion link. It derives from the specification of net and indicates a structural inconsistency in that specification.

2.5.3. Paraconsistency. In translations of the protologic to some logical systems inconsistency cannot be represented and is treated simply as an error in the specified logical structure. In a *paraconsistent* system [290, 44] meaningful inference will generally still be possible, and the protologic supports both possibilities. Whether the inconsistency corresponds to an intrinsically inconsistent entity or to an erroneous specification is an extra-logical issue that does not affect the inference patterns involved in representing inconsistency.

An inconsistent link in a net is *localized* in that it can only propagate through the inclusion and exclusion patterns to a node which includes the tail of its inclusion link. Inconsistency of some links in a net does not 'explode' to inconsistency of all links in the net.[19]

Thus, the logical inference schemata that have been defined are paraconsistent in their containment of inconsistency. It is possible to reason normally both about the parts of the net which are not inconsistent and about the propagation of the inconsistencies themselves. The principle of non-contradiction continues to apply, but, if the specification of a net violates it, the adverse impact of the violation is localized to a sub-net.

For example, the net shown in Figure 22 illustrates the inferences when the definition of the exclusion link specified between nodes A and B is violated by node C which is specified to include both. C is inferred to have inconsistent links to A and B. These propagate to node D which is specified to include C. However, the exclusion link between A and E propagates without inconsistency to C and D, as does the inclusion link from B to F.

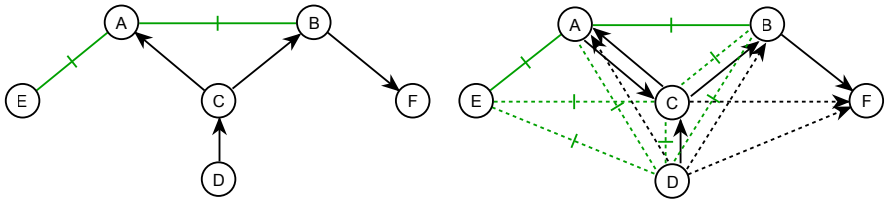


FIGURE 22. Localization of inconsistency

With an appropriate interpretation of the nodes and arrows, this can represent the Meinongian distinction between *sein* and *sosein* [193]. An inconsistent

[19]The origins of the notion of *explosion*, or *ex contradictione quodlibet*, are interesting [257]. In early times it might have arisen from confusion with *ex falso quodlibet* on the assumption that a contradiction in a bivalent logic is logically false, rather than metalogically not representable. Later symbolic, or truth-table, 'proofs' of the principle seem to neglect the same metalogical issue [356, II5b-g]. Theorem provers for bivalent logics do not attempt prove anything from a contradiction but simply halt and report that it cannot be represented.

entity can still exhibit normal patterns of inference for aspects unrelated to its inconsistency, and inconsistency itself propagates in a meaningful way..

2.5.4. Contraction to resolve inconsistency. It is also possible to eliminate inconsistency in a net by deleting links. A *maximal contraction* [3] of an inconsistent net is a consistent subnet formed by deleting one or more links such that none of the deleted links may be reinstated without the net becoming inconsistent. Such contractions are not conservative and result in a loss of links that changes any logical interpretation.

Maximal contractions are generally not unique and any preference between them derives from additional considerations about what specified or derived links it is most desirable to preserve. For example, Figure 23 shows the three maximal contractions of the net of Figure 22.

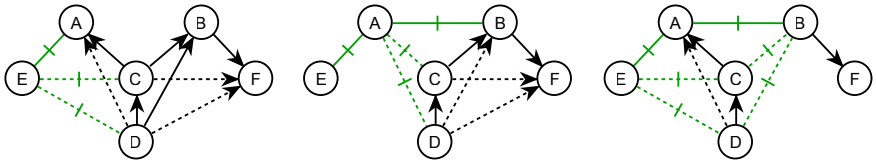


FIGURE 23. Minimal contractions to resolve inconsistency

On the left the exclusion link between A and B has been deleted, and the inferred exclusion links from C to to A, B and D, and from D to A, B and C have been lost. In the centre the inclusion link from C to A has been deleted, and the inferred exclusion links from C to to B, D and E, and from D to B, C and E have been lost. On the right the inclusion link from C to B has been deleted, and the inferred exclusion links from C to to A and D, from D to A and C, and inclusion links from C and D to F, have been lost.

Thus, the inconsistency may be eliminated in a number of ways, each leading to different consistent nets with a minimal loss of inferences, or allowed to remain, preserving all the inconsistent and consistent inferences that may be made.

2.5.5. Generic inference patterns for the kernel protologic. The protologic based on the resemantification of $\rightarrow$ with the inference patterns defined so far will be termed the *kernel protologic*. It is constituted through inference patterns constraining $\rightarrow$ to represent the partial order expected of derivation relations together with a further constraint $\nrightarrow$ restricting the placement of $\rightarrow$ to represent inconsistency in logical specifications.

The kernel protologic is extremely simple yet logically powerful enough to: illustrate the dynamics of logical systems that differentiate them from other mathematical structures (§3); provide protosemantics underlying a wide range of semantic interpretations of many logics (§5); and to support the representation of a wide range of common knowledge representation schemata (§4).

Structural connectives may be defined through constraints on nodes defined in terms of the basic inference patterns, but only very restricted forms of them

are needed for much practical reasoning (§7). Nonmonotonic inference may be represented through a preference relation between nodes (§8).

The inference patterns for the kernel protologic have similarities that allow them to be condensed to a generic form. For example, the exclusion pattern (Figure 15) mimics the inclusion pattern (Figure 5). If one shows them together (Figure 24), Fahlman's [122] process of *virtual copying* is apparent. Both inclusion and exclusion links out of the top centre node are 'copied' by inference to the lower node so that the links do not need to be shown explicitly but can be treated as 'implicit,' 'virtual' or 'inherited.'

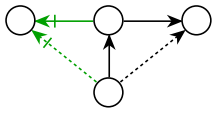


FIGURE 24. 'Virtual copying' of links

The inference patterns for $\rightarrow$ and $\nrightarrow$ are common to six link types, inclusion, equivalence, exclusion (in both directions and symmetric form), and inconsistent, It is useful to treat these links as parametrized instances of a generic link, $\dashrightarrow$, allowing the inference patterns to be represented generically as shown in Figure 25. This enables a number of inference patterns in other logics to be reduced to a single pattern (§6.3.3).

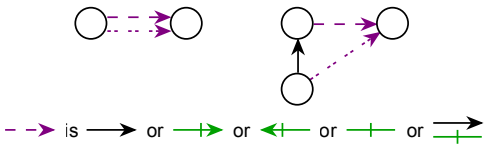


FIGURE 25. Generic inference patterns

From an algebraic perspective, the inference patterns for $\dashrightarrow$ signify: left, that the instances represent relations; right, that the relations are idempotent when residuating $\rightarrow$ on the right. That is, in the Boolean algebra of binary relations, $\rightarrow \backslash \dashrightarrow \; = \; \dashrightarrow$ for the five substitution instances of $\dashrightarrow$ that are shown. This is the algebraic basis of notions of 'copying' and 'inheritance' and foundational to studies of logics as residuated lattices [143].

2.6. Coexclusion links

One may interpret the inference pattern for an inclusion link (Figure 5) as requiring that any node with an inclusion link to the tail also has an inclusion link to the head, and for an exclusion link (Figure 15) that it has an exclusion link to the head. There two obvious complementary link types where an exclusion link to the tail node requires either an inclusion or exclusion link to the head node. The latter is already available as the converse of an inclusion link (Figure 18, with the notions

of 'head' and 'tail' reversed) but the former is a new link type that will be termed a *coexclusion link* and represented by an arrow with two bars ↠.[20]

The formal definition of a coexclusion link is:

Definition: a *coexclusion link* may be constituted with ↠ iff any node that excludes the head must include the tail.

This definition is illustrated in graphical form in Figure 26:

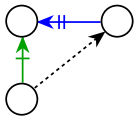


FIGURE 26. Coexclusion link definition

From the perspective of meaning containment, a coexclusion link specifies that if the content of the head is excluded then that of the tail is included, and if a node is not within the context of the head then it is within the context of the tail.

One might also interpret this definition as specifying that the content of the tail of a coexclusion link is included in the content of the tail of any exclusion link with the same head, and that the tail of a coexclusion link provides a context for the tail of such an exclusion link.

2.6.1. Coexclusion inference patterns. Figure 27 shows derived inference patterns for coexclusion links. From left to right, coexclusion links: have a repetition pattern; are symmetric allowing the arrow heads to be dropped (subject to semantic considerations) to provide an equivalent line form; and have a downward inheritance pattern similar to that of the reverse inclusion link (Figure 5).

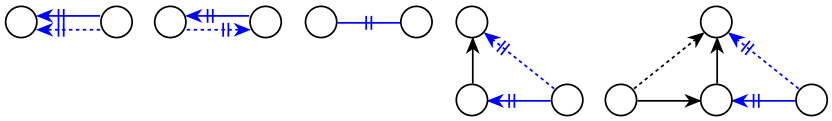


FIGURE 27. Coexclusion inference patterns

Figure 28 shows a generic representation of the coexclusion inference patterns.

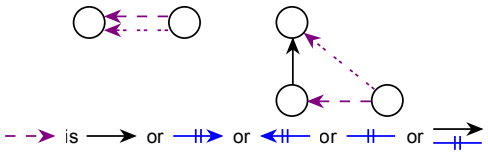


FIGURE 28. Generic inference patterns for inclusion and coexclusion

From an algebraic perspective, the inference patterns for -↠ signify: left, that the instances represent relations; right, that the relations are idempotent when

[20] *Coexclusion* is a neologism reflecting the duality between exclusion and coexclusion links. In the ancient logical literature the terms *subpares* and *subcontarias* have been used for a coexclusion relation in the metalogic of the syllogistic (§6.3.2).

residuating → on the left. That is, in the Boolean algebra of binary relations, -→/→ = -→ for the five substitution instances of -→.

2.6.2. Coexclusion inconsistency inference patterns. A node that violates the definition of a coexclusion link by having an exclusion link to both nodes may be inferred to be inconsistent (Figure 29 left).

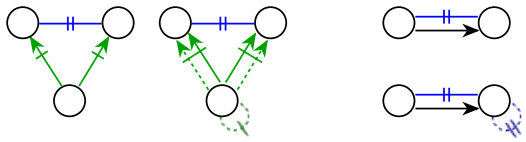


FIGURE 29. Coexclusion inconsistency patterns

A coexclusion and an inclusion link between the same nodes are not compatible and lead to the head node of the inclusion link being inconsistent in having a coexclusion link to itself (Figure 29 right). This is a different form of inconsistency from that of a node having an exclusion link to itself, and, where necessary, the two forms will be distinguished as coexclusion inconsistency and exclusion inconsistency, respectively.

Coexclusion inconsistency occurs in any node that is included by both nodes of a coexclusion link (Figure 30 left).

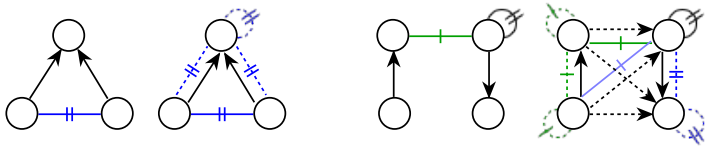


FIGURE 30. Further coexclusion inconsistency patterns

Any node on a path to an exclusion inconsistent node is exclusion inconsistent as is one with an exclusion link to a coexclusion inconsistent node. Any node on a path from a coexclusion inconsistent node is coexclusion inconsistent.

Figure 30 right shows how the two forms of inconsistency interact. The top right node is specified to be coexclusion inconsistent and the lower right node on a path from it may be inferred to be so. The top left node excludes the coexclusion inconsistent node and may be inferred to be exclusion inconsistent as may the lower left node on a path to it. In addition the lower left node may be inferred to both include and exclude the top right node, and the two left nodes may be inferred to include the lower right node.

2.6.3. Negation inference patterns. A coexclusion link is compatible with an exclusion link between the same nodes. In particular, the combination of exclusion and coexclusion links acts as a structural negation in the protologic. Figure 31 shows some of the possibilities, all of which are logically equivalent but may be semantically distinct in indicating different origins of the negation.

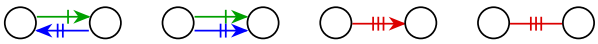


FIGURE 31. Negation as combined exclusion and coexclusion links

The negation aspects of the combination are apparent in the inference patterns shown in Figure 32. On the left it can be seen that an inclusion link to one node results in an exclusion link to the other, and *vice versa*.

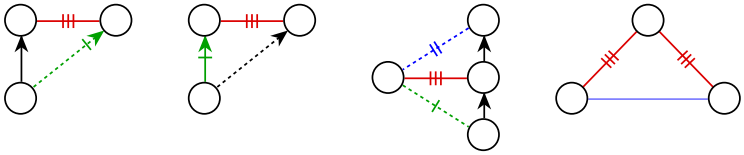


FIGURE 32. Negation inference patterns

In the centre it can be seen that a node which includes one node excludes the other, and one that is included coexcludes the other. The lower exclusion inference captures Brandon's [68, p.126] definition that a contradiction is that which is entailed by any contrary, and Dunn [110, p.10] and Brady's [65, p.20] that it is a disjunction of all possible contraries.

The upper coexclusion inference provides dual definitions: that a negation entails any coexclusion and is the conjunction of all possible coexclusions. The inference pattern may also be seen as splitting a negation into its exclusion and coexclusion components.

On the right is shown the inference pattern for double negation, that if two nodes have negation links to the same node then they are equivalent. The inferred inclusion links derive directly from the coexclusion definition of Figure 26. This pattern also shows that the negation of a node is unique modulo equivalence.

In nets where the links between nodes derive from some relation between their contents, coexclusion and exclusion are closely associated such that coexclusion is generally also negation. This arises because if the exclusion of node A requires the inclusion of node B because of some relation between their contents then the inclusion of node B will also lead to the exclusion of node A, an exclusion relation between A and B.

However, if coexclusion arises, or is specified, in a way that does not derive from the nodes' contents then the nodes need not necessarily also exclude one another. A typical situation where this occurs is through a binary, non-exclusive structural disjunction (§**??**) such that either A or B or both must be included. In this case if A is excluded then B must be included, and *vice versa*, a bare coexclusion.

A deflationary account of truth values through the propagation of marks (§2.7) is consistent with the interpretation of combined exclusion and coexclusion links as representing negation. Structural definitions of negation as a node with a maximal exclusion link (§7.1.1), or one node of a binary disjunctive judgment (§**??**), also allow the combination of exclusion and coexclusion links to be inferred.

The description logic translations of the exclusion, coexclusion and negation links in Figure 33 are:

$$A \sqsubseteq \neg B \qquad \neg C \sqsubseteq D \qquad (E \sqsubseteq \neg F) \sqcap (\neg E \sqsubseteq F)$$



FIGURE 33. Protonegation and structural negation

The translations illustrate that exclusion involves only *protonegation* that does not introduce an ideal node defined to be the negation of another, whereas coexclusion alone or in combination with exclusion involves structural negation in which an ideal node is defined.

2.6.4. Negation and semantic fragmentation. The nature and role of negation in language and logic were recognized as being problematic by Parmenides, Plato and Aristotle [180], and their concerns were echoed in the development of modern logic as Russell, Demos and Wittgenstein debated the nature of 'negative facts' [309, 268, 272]. Ferejohn [123] provides a detailed analysis of the issues with negation that Plato discusses in *Sophist* and *Statesman* in terms of the *semantic fragmentation* that can occur through the unrestricted negation of a concept, and the solution proposed by Aristotle that negation should always be relative to a context. These considerations are also relevant to the modern debate on the representation of negative facts [272, 24], and it is interesting to determine whether they can be usefully represented in the kernel protologic.

Aristotle criticized an unrestricted negation such as 'not man' as an indefinite (ἀόριστος, aorist) term not designating anything as does a name.[21] Plato exemplified the problem in *Statesman* by considering the tendency in Greece to divide the human race into 'Greek' and 'not Greek' and name the 'not Greek' as 'barbarian' as if that were a single kind despite the disparity of origins, cultures and languages (Figure 140).[22]

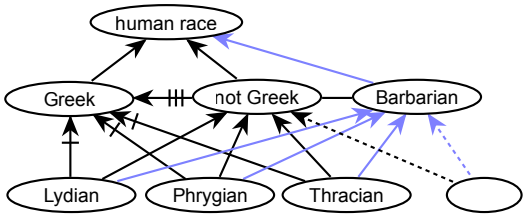


FIGURE 34. Plato's example of semantic fragmentation

The semantic fragmentation that they criticize may be seen as arising through the coexclusion component of a negation link, that every node that excludes its

[21] τὸ δ' οὐκ ἄνθρωπος οὐκ ὄνομα ... ἀλλ' ἔστω ὄνομα ἀόριστον [14, 16a30-32]

[22] τὸ μὲν Ἑλληνικὸν ὡς ἓν ἀπὸ πάντων ἀφαιροῦντες χωρίς σύμπασι δὲ τοῖς ἄλλοις γένεσιν ἀπείροις οὖσι καὶ ἀμείκτοις καὶ ἀσυμφώνοις πρὸς ἄλληλα βάρβαρον μιᾷ κλήσει προσειπόντες αὐτὸ διὰ ταύτην τὴν μίαν κλῆσιν καὶ γένος ἓν αὐτὸ εἶναι προσδοκῶσιν [281, 262d]

head must also include its tail (Figure 26). From a meaning containment perspective, this implies that every node that is incompatible with a given node contains a common meaning, but it is not apparent that members of the races that include 'not Greek' have anything else in common.

The issues managed through a common context specifying the *universe of discourse* (§4.1) are apparent if one neglects the context provided by 'human race' in Figure 140, and considers 'not Greek' as applied to literature, cuisine, mountains, and so on. What is the common component of meaning to this diversity of kinds that accounts for their being 'not Greek'? Within the context of 'human race' it is reasonable to envision that there might be a common content to 'being Greek,' such as having a parent who is Greek, that leads to an equally simple notion of not 'being Greek.' However, the broader the context the more problematic it becomes to envision such commonality.

Many significant notions in a wide range of disciplines are characterized by semantic heterogeneity through their inclusion of diverse terms having no commonality of meaning, with their only connection being their joint involvement in certain states of affairs. They present problems of semantic fragmentation that make a coherent account of their natures problematic. A good example is the controversy in the literature on the philosophy of art about the definition of an 'art object' [98, 76] (§4.3.3).

This section has shown that the kernel protologic supplemented by coexclusion can be used to analyze the basic issues involved in semantic fragmentation. This analysis is extended in later sections to: the role of a common context in managing fragmentation in §**??**; the use of structural disjunction to avoid it in §**??**; the relevance of inductive reasoning in §**??**; and of enthymematic reasoning in §**??**.

2.6.5. Pseudo inference patterns between inconsistent nodes. Nodes that are exclusion and/or coexclusion inconsistent have a number of pseudo inference patterns linking them. These patterns are termed *pseudo* because they do not derive directly from the inference patterns already defined but rather supervene on them in that the links involved may be added or removed without any change in the logical interpretation of the net. If specified, they are logically meaningless.

Figure 35 shows the pseudo inferences for the six possible combinations of inconsistent nodes. On the left, the logical interpretation is invariant to: an exclusion link between exclusion inconsistent nodes; a coexclusion link between coexclusion inconsistent nodes; or an inclusion link from an exclusion inconsistent node to a coexclusion inconsistent node. On the right are shown the patterns that derive from these if one or both nodes is both exclusion and coexclusion inconsistent.

The inconsistent nodes are shown to be specified as inconsistent but will usually have been inferred to be so from other links in the net. The pseudo inferences link nodes that otherwise have no connection and the main reason for defining them is to allow links to be deleted that have been specified between inconsistent nodes but have no logical effect. Such links lead to ambiguity in the definition of the

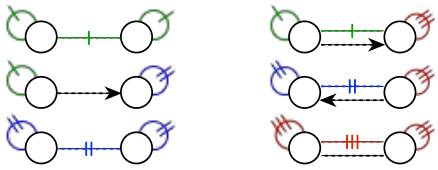


FIGURE 35. Inconsistency inference patterns

minimal canonical form of a net (§3.1) and are also significant in the extensional reconstruction of inconsistent nets (§5.2.6).

2.6.6. Coexclusion squares of oppositions. Figure 36 left shows a square net with coexclusion links on the diagonals and an exclusion link on the top side such that inclusion links may be inferred on the left and right sides and a coexclusion link on the bottom side. Similar structures have been studied in the logical literature, both ancient [232] and modern [46, 45], as *squares of oppositions*.

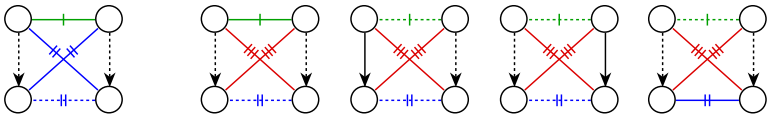


FIGURE 36. Coexclusion squares of oppositions

The inferred implications on the left and right of the square are derived directly from the coexclusion definition. The inferred coexclusion link is derived because any node with an exclusion link to one of its nodes will also have an exclusion link to the node above (exclusion inference pattern) and hence an inclusion link to the other node (coexclusion inference pattern) and hence satisfies the coexclusion definition.

In the net on the left, having only coexclusion links as diagonals, the additional inclusion and coexclusion links may be derived from these and the exclusion link at the top, but, if this link is omitted, it cannot be derived by specifying one of the links previously derived.

However, if exclusion links are added to the coexclusion links on the diagonals to make them negations then specifying the link on *any* side allows those on the other three sides to be derived (Figure 36 right). These are the classic *squares of oppositions* that are common to many logical systems [45].

The net on the left demonstrates that the main phenomena of interest in squares of oppositions derive from the co-exclusion component of the negations on the diagonals, that a node that excludes a node at one end of the diagonal must include that at the other end.

2.6.7. Hexagon of oppositions. Blanché [53] extended the square of oppositions to a hexagon of oppositions by adding the disjunction of the top nodes and the conjunction of the bottom ones. His construction has proved very fruitful in a universal logic framework in representing the relations between major constructs in a wide range of logical systems [41, 43], and is interesting to see how it may be represented in the kernel protologic with coexclusion.

Figure 37 shows an inference pattern in a partial hexagon whereby an inclusion link from the node on the left to that at the top leads to the inference of one from the node at the bottom to the one on the right. In the Blanché hexagon the other links are negations, but exclusion between the top and bottom nodes and coexclusion between left and right is sufficient for this inference.

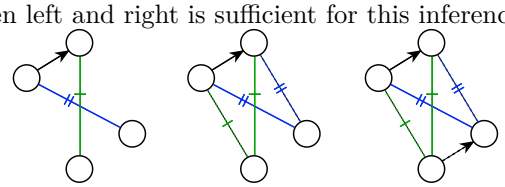


FIGURE 37. Downward inference of inclusion in a partial hexagon

Figure 38 shows an inference pattern in a partial hexagon whereby an inclusion link from the node on the bottom to that at the right leads to the inference of one from the node on the left to the one at the top. In the Blanché hexagon the other links are negations, but coexclusion between the top and bottom nodes and exclusion between left and right is sufficient for this inference.

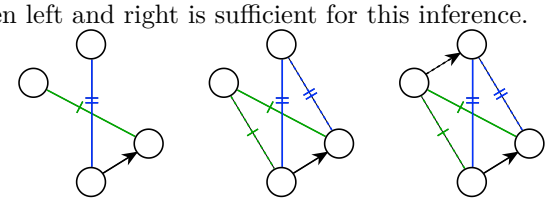


FIGURE 38. Upward inference of inclusion in a partial hexagon

Figure 39 combines Figures 37 and 38 to form a hexagon with negation links on its three diagonals and uses the inference patterns already derived to show that: from a pair of inclusions forming a protodisjunction of the top two nodes of the square may be inferred a pair of inclusions forming a protoconjunction of the bottom two nodes of the square, and *vice versa*; and a pair of inclusion nodes on the left may be used to infer a pair on the right, and *vice versa*.

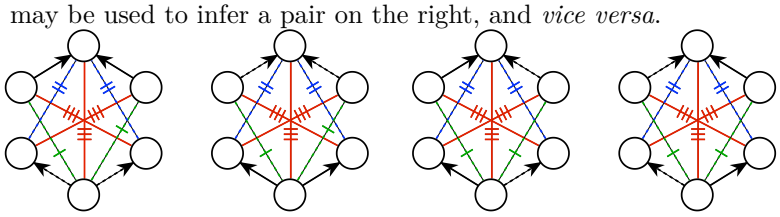


FIGURE 39. Inference of inclusions in a square extended to hexagon

Figure 40 extends Figure 39 to a full hexagon of oppositions by adding an exclusion link between the top two nodes of the square and inferring the other three links on the sides of the square. As shown in Figure 36 any of the four links on the sides of the square could be added to infer the others.

Thus the logical derivations involved in the Blanché hexagon may be factored through the inclusion, exclusion and coexclusion links involved in their derivations, and only protodisjunctions and protoconjunctions are required for the top an bottom nodes that he added, not full structural ones.

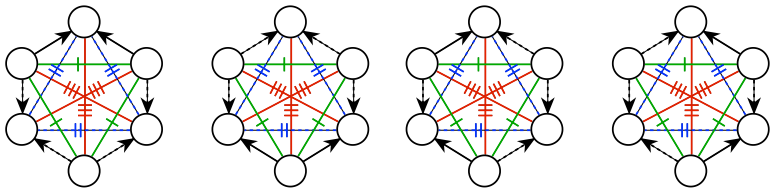


FIGURE 40. Inference of inclusions in a hexagon of oppositions

2.6.8. Role of coexclusion in the protologics. It has been appropriate to define coexclusion links at this juncture for formal completeness. As noted in the introductory paragraph, they are defined by an inference pattern similar to those for inclusion and exclusion links that completes the set of possible patterns of that type. However, whereas the other patterns correspond to named logical notions such as derivation or entailment, incompatibility or opposition, the coexclusion pattern is unusual and somewhat unnatural playing, for example, no role in the significant generic knowledge representation schemata of §2.5.5.

One is tempted to echo the phrase that Lenat [227, p.26] has AM use to describe its discovery of Goldbach's conjecture, that the coexclusion link is 'cute but useless.' However, exclusion and coexclusion links together define structural negation which plays a foundational role in the exposition of many logical systems. In particular, while the inclusion and exclusion links of the kernel protologic alone are adequate for all the inference patterns of the syllogistic (§6.3), Aristotle has a preference for proof by *reductio ad absurdum* or *reductio per impossibile* arguments [61, §5.2] that require an inference pattern based on contradiction, that is, metalogical negation [192].

The similarity of the definition and inference patterns of coexclusion to those of inclusion and exclusion together with its semantic problems suggest that it exemplifies the boundary between the pure and applied, between mathematical logics where elegance and completeness of functionality are important, and practical reasoning where naturalness and ease of understanding are more significant. In particular, the issues involved may be clarified by studying the impact of omitting coexclusion from protologics, the techniques that enable effective natural and computational ontologies to be built without the use of, or highly constrained us of, negation, and the role of semantic fragmentation in the tractability and understandability of reasoning with such ontologies.

2.7. Visualizing inferential dynamics, propagating marks, truth values

The graphical representation of inference used so far is to show new links and nodes in a lighter shade. However, as Figure 44 right illustrates, in larger nets the inferred links may become difficult to discriminate. The graphic language provides an alternative way of displaying the same information by allowing one or more nodes to be marked and then marking the other nodes to show what kind of specified or inferred link the marked nodes have to them.

A vertical mark placed in a node results in all those it includes being marked with a vertical bar, and those it excludes, or exclude it, by a horizontal bar. A

horizontal mark placed in a node results in all those nodes that include it being marked by a horizontal bar, and all those that it coexcludes, or coexclude by it, being marked with a vertical bar.

Figure 41 shows a net in which the centre node labelled 'A' has different links to five other nodes, and the effect of marking that node with a vertical mark, a horizontal mark, or both. The marks are coloured to indicate the nature of the source: green for an asserted mark; blue for a vertical bar inferred from an inclusion link; gold for a vertical bar inferred from a coexclusion link; grey for a horizontal bar from an inclusion link; and black for a horizontal bar from an exclusion link (with black having priority).

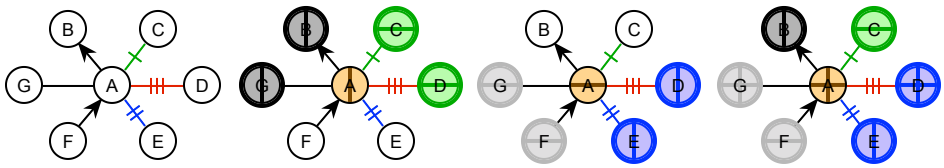


FIGURE 41. Propagating marks from one node to linked nodes

One can recover a familiar logical vocabulary by terming a vertical mark logically *true* and a horizontal one logically *false*. The inference patterns shown can then be interpreted as: 'if A is true then B and G are true, C and D are false;' 'if A is false then D and E are true, F and G are false;' 'if A is incoherent (both true and false) then B and E true, C and F are false, D and G incoherent.'

The propagation pattern between A and B in the second net exhibits *modus ponendo ponens*, and between A and D in the third net *modus tollendo tollens*, providing truth-theoretic accounts of these argument forms corresponding to the graphical accounts in §2.5.1.

When a node is marked false all those nodes included in its context are also inferred to be marked false, and this may be viewed as the context determining their *relevance*. One might prefer to say that the node has become *irrelevant* because its context is inapplicable rather than that it is simply false. The gray colour of a horizontal bar for falsity distinguishes this type of inference from the other two possibilities of the node being asserted to be false (green), or being inferred to be so from a node with an exclusion link being inferred false (black).

These propagation patterns for 'true' and 'false' are those used in theorem prover algorithms [99], and propagation algorithms have been increasingly optimized for large networks over the years to provide the high-speed *SAT-engines* of today [47]. They are also the basis of *spreading activation* models of human *semantic memory* [87] and its emulation in *semantic networks* [221] and *connectionist computation* in *neural nets* [311], where the inclusion link represents activation and the exclusion link inhibition.

The notion of logical truth in the protologic is simply an alternative way of representing the inference patterns in a net. The associated semantic issues are discussed in §5.1.1.

2.7.1. Paraconsistent truth values. The paraconsistency of the protologic corresponds to it not being bivalent and supporting inference from nodes that are incoherent in being marked both 'true' and 'false.' The inference patterns of Figure 41 are those of Belnap's [30] *useful four-valued logic* based on 'told true' and 'told false' as independent assertions [29]. This interpretation is shown to be consistent with further structural connectives as they are added (§7.3.2).

Figure 42 exemplifies the propagation of marks applied to various nodes in the inconsistent net of Figure 22. The nets are labelled by the node which is marked true, and the marking of the other nodes is derived from the inclusion and exclusion inference patterns. Marking nodes C or D false leads to them and other nodes being marked incoherent but, as discussed in §2.5.3, nodes E and F being marked consistently.

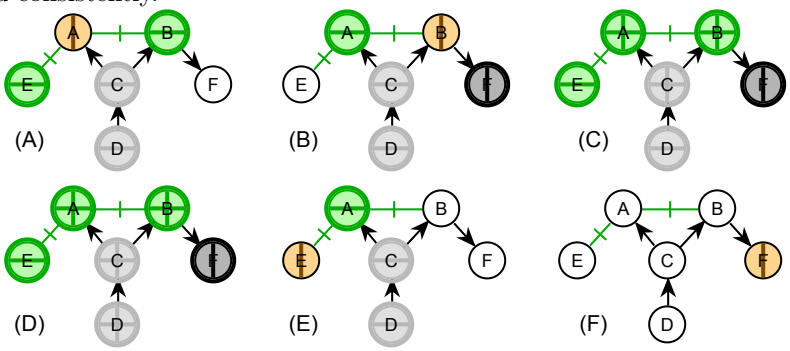


FIGURE 42. Propagating marks in the net from Figure 22

Reasoning about inconsistent concepts is important in many applications. In taxonomic articulations a node that is always false indicates that a proposed species concept has no application [354]. Aristotle considers inconsistent terms such as "goat-stag" (τραγέλαφος) in several of his works and these give rise to significant logical and philosophical issues [370, 372, 22, 306].

Figure 43 shows how a notion such as goat-stag that is comprehensible but has no being can be represented in the protologic as a node including two incompatible nodes. The first three nets show the possible consistent markings, and the fourth the inconsistency when goat-stag is marked true.

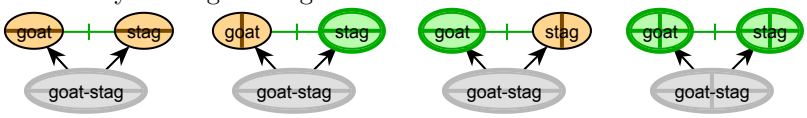


FIGURE 43. Representing an inconsistent concept

The representation in Figure 43 is appropriate if the incompatibility between goat and stag is part of the meaning of the terms, and the violation on the right is one of the inappropriate use of those terms. However, if it an empirical relation based on evidence that goats and stags cannot interbreed then a material conditional (§**??**??) or defeasible material conditional (§**??**??) would be a more appropriate representation.

3. Dynamics of logical structures

As noted in §2.2, although logical structures may be represented in graph-theoretic form, logic goes beyond graph theory in its emphasis on the *dynamics of change* those structures. This encompasses not only the addition and deletion of links in nets through inference patterns but also various ways of restructuring nets, splitting and merging of nets, the specification of additional nodes and links, and the consequences for the logical interpretations of the nets involved. It also involves considerations of effective means of communicating the dynamics to those developing and using knowledge structures represented in nets.

3.1. Minimal and maximal canonical forms

In has been noted (§2.4.4) that a net with inclusion links can be reduced to a minimal canonical form having the same logical interpretation. The exclusion and coexclusion inference patterns support a similar deletion of exclusion and coexclusion links that may be regarded as superfluous because they can be inferred from others. However, the symmetry of these links means that the direction in which they are specified is inferentially irrelevant and it is appropriate to treat the symmetric form as canonical in deriving unique canonical forms. Similarly the repetition inference pattern for all links means that it is appropriate to treat reduction to a single link as canonical for the sake of unicity.

These considerations lead to a minimal canonical form for a net with that is unique up to the choice of node label used to represent all the nodes in a cycle, and has the same logical interpretation as the net from which it was derived. The inclusion, exclusion and coexclusion inference patterns may be used to expand this minimal canonical form to a unique maximal one. Figure 44 illustrates the minimal and maximal canonical forms of a specified net of inclusion and exclusion links.

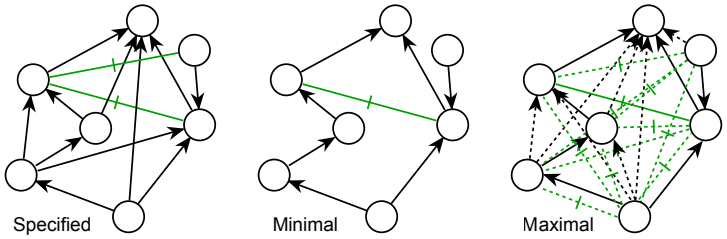


FIGURE 44. Example of minimal and maximal canonical forms

Figure 45 illustrates the minimal and maximal canonical forms of a specified net of inclusion, exclusion and coexclusion links.

The restriction of the canonical form to having no repeated links and only symmetric exclusion and coexclusion links may be too restrictive from a semantic perspective. A net that would be in minimal or maximal canonical form if the repetition inference patterns were used to delete superfluous links will be termed super-minimal or super-maximal, and one that would be in minimal or maximal canonical form if the symmetry inference patterns were used to replace exclusion

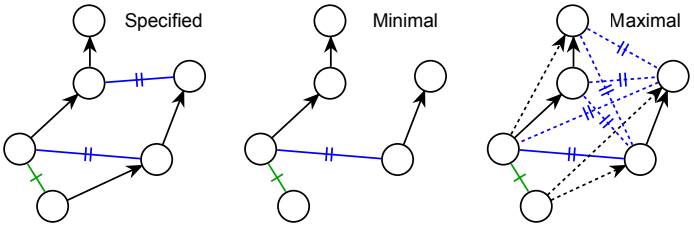


FIGURE 45. Canonical forms with coexclusion links

and coexclusion links with their symmetric forms will be said to be sub-minimal or sub-maximal.

The inference patterns of §2 and their use to expand or contract the net as discussed in this section exemplify net dynamics through the addition or deletion of links. The following sections extend this to expansion and contraction of nets through the addition or deletion of other nets (including single nodes). Since some expansions do not change the logical interpretation of the net, this allows for further expansion beyond the maximal canonical form to a net that is logically equivalent modulo added nodes.

3.2. Consistent and conservative imports and merges

Nets are dynamic, not only through inference patterns that do not change their logical interpretation, but also though the addition of links that are not implicit in the inference patterns and hence change the interpretation, and through the addition of nodes and associated links that may have a variety of effects.

When nodes are added they may already be named and/or linked, and the general case may be considered as one where one net is *imported* by another, either through merging nodes with the same names, or by linking nodes in one net with nodes in the other, or both.

One major consideration is whether the resultant net has additional inconsistent links. An import is termed *consistent* if it does not. Usually the focus is on the import of a consistent net by a consistent net to produce a consistent net, but it can be appropriate to consider situations where the importing or imported net is inconsistent.

Another major consideration is whether the import is *conservative* in that no additional links may be inferred between the nodes in the imported net. This is of concern, for example, in many computational applications where the imported net is intended to be a generic module representing a 'library,' 'ontology' or 'theory.' The intent is to generate additional inferences in the importing net but not in that which is imported [164].

A conservative import may impact the dynamics of the importing net by making inconsistent potential links that could have been added to that net, that is, constraining it more strongly. An import that imposes no such constraints is termed *ultra-conservative*.

An import of one net by another may also be regarded conversely as an import by the second net of the first. If both imports are conservative, or ultra-conservative, it may be termed a conservative, or ultra-conservative, *merger* of the nets.

Since the inference patterns are such that only inclusion or coexclusion links from nodes in one net to nodes in another can generate new links within the first net, importing a net such that there are no inclusion or coexclusion links from the imported net to the importing one is always a conservative import, but generally not ultra-conservative.

Conservative imports are reversible through deletion of the imported net provided this deletion is from the maximal canonical form. This restriction is necessary because the purpose of some imports is to simplify the net by reducing the number of links in the minimal canonical form and, hence, the links removed from the original net must be restored before the imported net is removed, (for example, in the *interpolation* (§3.3.1) and *factoring* (§3.3.3) expansion patterns).

3.3. Single node and link expansion patterns

Adding an additional anonymous node as an *ideal element* with up to one link to a node in a net is an ultra-conservative merger, and the single node and link expansion patterns of Figure 46 are always available. However, the issues of *semantic fragmentation* (§2.6.4) indicate that the ideal elements introduced through coexclusion or negation links may have rather more problematic semantics than those introduced through inclusion or exclusion links.



FIGURE 46. Single node and link expansion patterns

Thus, nets are extensible by an anonymous node with a single link without logical constraint. However, extra-logical or metalogical constraints may be applied such as a node being specified to be a bottom or top node. More complex expansions of nets, and associated issues, are discussed below.

Expansion patterns based on ultra-conservative mergers provide a symmetric contraction pattern, similar to those of the inference patterns, in that the anonymous node may be removed without affecting the logical interpretation of the original net.

The addition of a labeled node is conservative if there are no nodes with that label in the net, but generally not conservative if this is not so unless the link is one that may already be inferred. It is generally not ultra-conservative because it may affect the consistency of potential imports of other nets containing a node with that label.

3.3.1. Interpolation expansion pattern. As shown in Figure 47 adding a single node interpolated between the tail and head nodes of a link is a also an ultra-conservative merger. The sequences of three nets on the left and right illustrate the principles involved. The node may be freely added to an existing link without

creating any new inferences, and then that link may be treated as inferred. This type of pattern is useful in *factoring* more complex nets to simplify their structures (§3.3.3).

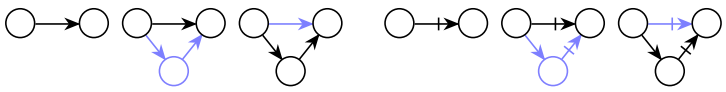


FIGURE 47. Node interpolation patterns

The interpolation pattern is analogous to introducing the middle term of a syllogism, and the Kantian scholar Vaihinger [358, p.212-3] discusses it as an illustration of the introduction in logic of *fictions*, abstract notions that are treated 'as if' (*als ob*) they were real.[23] Hertz [**?**, p.248-9] uses this pattern to simplify systems of axioms and notes that the interpolated node may be regarded as an *ideal element* (*idealen Elementes*) in the sense of the Hilbert school [176, 237]. He cites Vailhinger's analysis, noting that the introduction of such abstract notions is common in the sciences, for example, the concept of force (*der Kraftbegriff*).

3.3.2. Existential status of added nodes. Vaihinger's analysis raises the issue of the existential status of added nodes. One may distinguish at least five aspects of the notion of *existence* in the protologic: being a node (*conceptual existence, daseinfrei* [248, p. 51]); being a consistent node (*logical existence*, [131, p.12]); being specified as an essentially top node representing a maximally generic entity (*categorical existence*); being specified as an essentially bottom node representing a single entity (*singular existence*); and being a bottom node representing an entity within a specified universe of discourse (*situated existence*), such as a state of affairs, phenomenon, event, experience or individual, in some situation, world or time interval specified in logical terms.

A universe of discourse may be specified in the protologic as a contextual node representing the situation (§4.1). A node's situated existence within that context is then simply a combination of logical and singular existence (§8.4).

There has been controversy about what kind of existential assumption, if any, is being made in adding a particular node designated 'some A' in the categorical syllogistic [83], in its interpretation in modern logic [261] and in logic in general [289]. In the protologic, 'some A' is defined to be a technical term abbreviating 'some node included in node A,' with the only existential assumption being that a node may always be added that includes another. From the perspective of meaning containment, an additional node may always be specified that includes the content of another and is enclosed by its context.

The additional node will be consistent if the node it includes is consistent. It may then be said to be *logically existent* under Hilbert's [131, p.12] definition in an 1899 letter to Frege that the consistency of axioms is sufficient to ensure the

[23] "Was ist dieses 'überhaupt'? Es ist eine reine Einbildung, eine Fiktion, ein fiktives Vorstellungsgebilde."

existence of entities that comply with them.[24] Such *logical existence* captures Leibniz's [226, p.101] conception of the nature of the particular quantifier as ranging over *possibila*, the realm of ideas.[25]

The addition of nodes can also raise more subtle issues of the changing *nature* of the nodes that were already in the net, particularly if nodes have been added as *ideal elements* to simplify a formal model. Hilbert's [176] standard examples are the supplementation of the reals by imagineries and assumption of points at infinity in projective geometry. Russell [314, ch.7] emphasized that one should be careful in treating the extended system as if it included the unextended system as a special case. The extended system may have emergent properties that the unextended system did not but which are now attributed to the latter when it is regarded as embedded in the extended system.

The semantification involved in naming ideal elements is not a resemantification of the underlying logical system. It is the extra-logical semantification provided by attaching domain-specific meanings to abstract patterns within that system that represent concepts in the domain.

3.3.3. Factoring expansion patterns. What has been minimized in the minimal canonical form (§3.1) is the number of links whilst keeping the number of nodes constant (treating nodes in a cycle as a single node). There are other significant forms of restructuring that can further reduce the number of links at the expense of adding additional nodes. For example, one can use a similar technique to the interpolation pattern (§3.3.1) to factor out several links with a common tail or head, and add an additional node to group them.

Figure 48 left shows a net having 8 nodes and 12 links, with the links at the top having common tails and those at the bottom having common heads. In the centre is shown a net with 10 nodes and 10 links where two anonymous nodes have been interpolated to factor out the commonality. The two nets have the same logical interpretation with respect to the nodes in the original net. The import of the anonymous nodes is ultra-conservative, and the original links are implicit, or virtual, in that they may be inferred from the inference patterns.[26]

On the right the factored net has been split into modules and the added nodes have been named. The *interface* nodes between the modules have been duplicated to separate the net into two simpler components.

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[24] "Wenn sich die willkürlich gesetzten Axiome nicht einander widersprechen mit sämtlichen Folgen, so sind sie wahr, so existieren die durch die Axiome definierten Dinge. Das ist fur mich das Criterium der Wahrheit und der Existenz."

[25] "quidam homo est ridens, esse veram in regione Idearum"

[26] Hertz [**?**, fig.4] uses a similar diagram (of inclusion links only) in which an ideal element is interpolated to reduce the number of links representing axioms in order to provide logical support for Hilbert [175] and Nelson's [263] notions of deconstructing the conceptual frameworks of the exact sciences to minimal sets of independent axioms from which their theories may be derived.

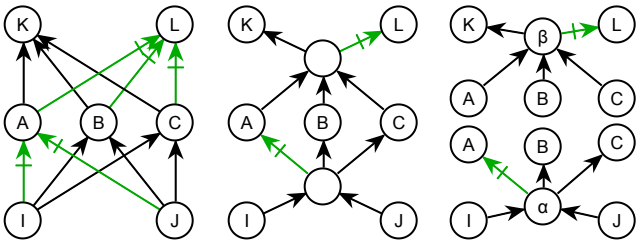


FIGURE 48. Example of factoring a net

Factoring clusters some collection of links in a way that reduces the arrow crossings making common structures more apparent. In an application the resulting sub-structures may well suggest labels for the anonymous nodes that have been created as significant entities in the domain of the application.

Factoring, in general, does not lead to unique canonical forms as there will usually be several possible factorings leading to different structures having the same logical interpretation with respect to the nodes in the original net.

The increased perspicuity that can result from factoring is not an artifact of the graphical presentation of the net. If one represents the structures in Figure 48 as multisets of links, then the corresponding link multisets are:

$$[I{\twoheadrightarrow}A,\ I{\to}B,\ I{\to}C,\ J{\twoheadrightarrow}A,\ J{\to}B,\ J{\to}C,\ A{\to}K,\ B{\to}K,\ C{\to}K,\ A{\twoheadrightarrow}L,\ B{\twoheadrightarrow}L,\ C{\twoheadrightarrow}L]$$
$$[[I{\to}\alpha,\ J{\to}\alpha],\ [\alpha{\twoheadrightarrow}A,\ \alpha{\to}B,\ \alpha{\to}C],\ [A{\to}\beta,\ B{\to}\beta,\ C{\to}\beta],\ [\beta{\to}K,\ \beta{\twoheadrightarrow}L]]$$
$$[[[I{\to}\alpha,\ J{\to}\alpha],\ [\alpha{\twoheadrightarrow}A,\ \alpha{\to}B,\ \alpha{\to}C]],\ [[A{\to}\beta,\ B{\to}\beta,\ C{\to}\beta],\ [\beta{\to}K,\ \beta{\twoheadrightarrow}L]]]$$

where the more modular representation of the net after factoring is also apparent.

The description logic translation (§6.1) makes the *protodisjunctions* and *protoconjunctions* represented graphically by the interpolated nodes more apparent:

$$(I \sqcup J) \sqsubseteq \alpha \sqsubseteq (\neg A \sqcap B \sqcap C) \qquad (A \sqcup B \sqcup C) \sqsubseteq \beta \sqsubseteq (K \sqcap \neg L)$$

From a meaning containment perspective the factoring is innocuous because the logical interpretation of the the original net is unchanged; the links between the original nodes are unchanged. The ideal nodes which have been introduced derive content only through their links. Their lack of intrinsic content may be represented logically by making the protoconjunction or protodisjunction structural (§7), *defining* the ideal nodes in terms of their links. If this is not done and they are named to introduce additional content then extra-logical criteria are involved.

3.3.4. Factoring exclusions. There is another common factoring pattern in which a number of exclusion links are represented as deriving from a common exclusion link. Figure 49 left provides an example.

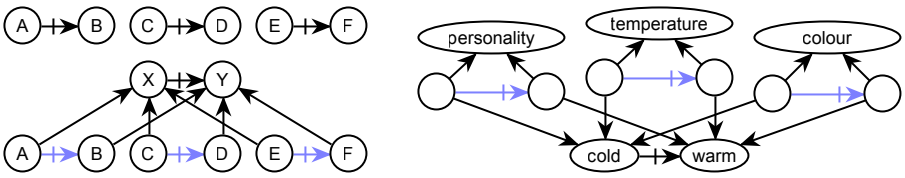


FIGURE 49. Factoring exclusions

This factoring is problematic because it changes the logical interpretation of the original net, introducing additional links, for example, an inferred exclusion between A and D. From a purely logical perspective it is not a successful factoring. However, such factoring is common in human discourse where, as exemplified on the right, the distinction between 'cold' and 'warm' in the context of temperature is extended metaphorically to an identically named distinction in the contexts of personality and colour, even though the three contexts have no apparent meaning connection.

In human discourse the *context* of usage generally disambiguates the confusion of terminology. This is a situation where common psycho-social processes in communication and reasoning challenge logic to provide an appropriate representation and interpretation. §4.1.1 addresses this issue through a simple mechanism that pushes the metalogical notion of context down to the logical level within the framework of the kernel protologic.

3.3.5. Adding a top node. Adding an anonymous top node that is included by, or provides a context for, some or all nodes in a net is ultra-conservative because the inference patterns can add no additional links and does not constrain the further addition of links to the original net. Adding a named top node is conservative but not ultra-conservative. Since more top nodes may always be added, specifying any node as being essentially a top node, for example as representing a *category*, is an extra-logical constraint, ontological in nature [8].

The node added is not unique and there is no way of specifying it to be so within the protologic so far defined. Unicity can be specified in the metalogic by defining it relative to its links as being the *least node* in the preorder that has those links. The general form of this additional constraint is analyzed in §7.2 and, if pushed down to the protologic, is shown to provide a *structural disjunction* [206, ch.14], or, if the nodes are mutually exclusive, a Kantian *disjunctive judgement* [233].

If the nodes included by the top node are those of an exclusion link (Figure 50), the node added has been argued to be essential to the interpretation of that link, representing a *context*, *genus*, *kind*, *sort*, *range* or *universe*, common to, and constraining, the included nodes. In the constructivist psychology literature this three-node schema has been proposed as the basic building block underlying all conceptual schemata and termed a *construct* [204]. This common schema is discussed further in (§4.1).

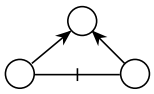


FIGURE 50. An exclusion link included in a common node

3.3.6. Adding a node that includes one in a net, particular, singular. Adding a node that includes one in a net (Figure 46 left) represents the construction used in syllogistic proofs by *ecthesis* (ἔκθεσις, setting out, exposition) [335]. The node

added is often a bottom, anonymous node but the anonymity is not significant; Aristotle [15, 28a24-26] and Galen [144, p.41] label their ecthetic nodes for convenience of exposition. As Smith [336, p.228] notes, what is significant is that the node added is a new one, a construction, extra to the premises.

As discussed in §?? and illustrated in Figure 51, the node added may be designated by the particular quantifier, *some*, applied to the label of the node that it includes.



FIGURE 51. Designating a node as particular relative to one it includes

This corresponds to Mignucci's [252, p.12] analysis of Aristotle's usage of the quantifier 'particular' (ἐν μέρει, κατὰ μέρος) as denoting a term that is less general than another, possibly but not necessarily singular. It is consistent with his statement [15, 24a19] that he means 'by particular that it belongs to some or not to some or not to all'.[27]

The usage of 'some' specified in Figure 51 allows for the degenerate case that *some B* is the same as *all* B, that is B itself (Figure 52 left). To avoid the degenerate case and specify *properly particular* one can add a second particular node with an exclusion link to the first (Figure 52 centre). The exclusion link need not be specified but can be inferred from another (Figure 52 right).

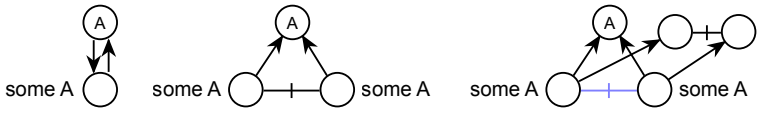


FIGURE 52. Degenerate and non-degenerate (properly particular) examples

The bottom nodes in the *properly particular*, non-degenerate examples may be described as *some A*, or *not some A* or *not all A*, as noted by Aristotle.

A bottom node is significant because not being included by another node, *impredicability*, may be used as a criterion of the node being *singular*, intended to represent an individual [160]. It represents Aristotle's distinction between a universal (καθόλου) being that which is predicated of a number of things, and a singular (καθ᾽ ἕκαστον) being that which is not.[28]

In the protologic a bottom node is included only in itself, satisfying Aristotle's definition of singular, and Leibniz's criterion that for singulars the universal and particular quantifiers coincide.[29]

Being singular is usually taken as a necessary feature of being *individual* [160] but it is not a sufficient one since the notion of an individual is normally taken to have some essential aspect of *reference* to some extra-logical entity. The representation of individuals is addressed in §8.4.

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[27] ἐν μέρει δὲ τὸ τινὶ ἢ μὴ τινὶ ἢ μὴ παντὶ ὑπάρξειν

[28] λέγω δὲ καθόλου μὲν ὃ ἐπὶ πλειόνων πέφυκε κατηγορεῖσθαι, καθ᾽ ἕκαστον δὲ ὃ μή, οἷον ἄνθρωπος μὲν. [15, 17a37-40]

[29] "quidam Petrus Apostolus et omnis Petrus Apostolus coincidunt, quia terminus est singularis" [226, p.101]

3.3.7. Adding a bottom node, probes. A simple but important structure is a *probe*, a bottom node that is added to a net but not regarded as part of it and has an inclusion or exclusion link some of the nodes constituting the net. Two probes will be termed *distinct* if they have different sets of links.

A probe is *compatible* if it is in maximal canonical form, having all the links that may be inferred from the net. A probe is *admissible* if it is compatible and has no inconsistent links other than those resulting from a link to an inconsistent node; that is, it has no unnecessary inconsistent links. If the net itself is consistent then an admissible probe is also consistent.

Figure 53 illustrates these distinctions for a simple net with a single link between two nodes: P1 is consistent, admissible and compatible; P2 is incompatible because the inferable inconsistent links are not shown; P3 is inconsistent but still incompatible because only one inconsistent link is shown; P4 is compatible but not admissible since the inconsistent links do not derive from an inconsistent node; P5 is admissible because its inconsistent links derive from one in the net.

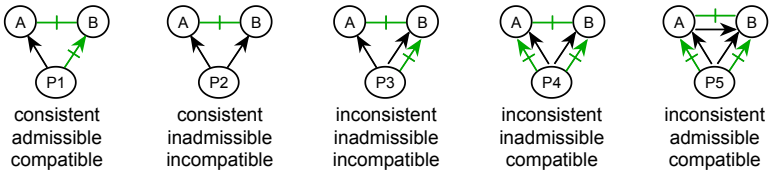


FIGURE 53. Examples of different forms of probe

A *full probe* is one with a link to every node in the original net, otherwise it is a *partial probe*. If there are n nodes with different names in the net then there will be 2^n distinct full probes having different combinations of link types, but not all of them may be consistent. A set of all possible distinct admissible probes is said to *saturate* a net.

A probe is not unique and there is no way of specifying it to be so within the protologic so far defined. Unicity can be specified in the metalogic by defining it relative to its links as being the *greatest node* in the preorder that has those links. The general form of this additional constraint is analyzed in §7.1 and, if pushed down to the protologic, is shown to provide a *structural conjunction* [206, ch.13].

3.3.8. Admissible probes characterizing possible links between two nodes. The four possible probes for the two nodes A and B are:

P0: A↞ ↠B
P1: A← →B
P2: A↞ →B
P3: A← ↠B

There are sixteen possible combinations of consistent probes and these may be used to distinguish between the sixteen nets having two nodes linked by all possible types of link or link combination. Figure 54 shows the admissible probes (including the inconsistent probe Pi) for all possible links between A and B. It can

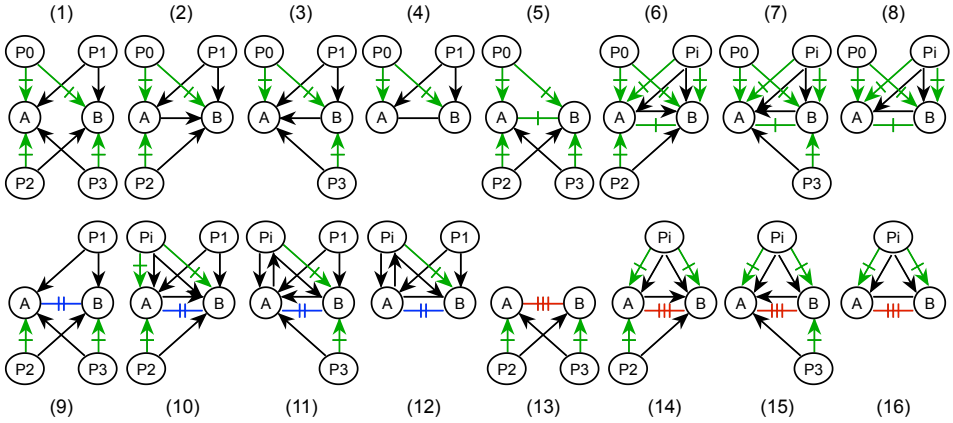


FIGURE 54. Admissible probes for sixteen link types

be seen that each net has a distinct characteristic set of consistent probes, and hence that such a set may be used to identify the link between the two nodes.

One relation between probes and links that may be derived from Figure 54 is that from the inadmissibility of one of the four probes may be inferred the presence of one of the four basic link types:

if inadmissible	then infer
P0: A↞ ↠B	A↠B
P1: A← →B	A↠B
P2: A↞ →B	A←B
P3: A← ↠B	A→B

which provides a useful derivation of metalinks in the metalogic of nets (§6.2).

More generally, one can reconstruct the nets through the functions, π and $\underline{\pi}$, associating with each node the set of admissible probes that include or exclude that node, respectively. There is an inclusion link from A to B if the set of probes with inclusion links to A, πA, is a subset of that with inclusion links with B, πB, and the set of probes with exclusion links to B, $\underline{\pi}$B, is a subset of that with exclusion links to A, $\underline{\pi}$A, that is $\pi A \subseteq \pi B$ and $\underline{\pi}B \subseteq \underline{\pi}A$. There is an exclusion link from A to B if $\pi A \subseteq \underline{\pi}B$ and $\pi B \subseteq \underline{\pi}A$, and a coexclusion link if $\underline{\pi}A \subseteq \pi B$ and $\underline{\pi}B \subseteq \pi A$.

The conjunction of the two tests is necessary to allow for inconsistent nodes that would otherwise lead to spurious links because a probe has an exclusion link to every exclusion inconsistent node and an inclusion link to every coexclusion inconsistent node. Unless both nodes are inconsistent the conjunction of the two tests ensures a correct reconstruction. If both nodes are inconsistent then pseudo links will arise but can be identified through the inference patterns discussed in §2.6.5.

As shown in Table 1, for the sixteen nets of Figure 54 the links are correctly reconstructed from the relations between the sets of consistent probes. The final example for net (16) is a pseudo inference pattern; all completely inconsistent

nodes are both equivalent to and contradictory to one another, playing no useful role in inference processes.

TABLE 1. Reconstructing links from probe sets

	πA	πB	$\underline{\pi}A$	$\underline{\pi}B$	$\pi A \subseteq \pi B$ $\underline{\pi}B \subseteq \underline{\pi}A$ $A \rightarrow B$	$\pi B \subseteq \pi A$ $\underline{\pi}A \subseteq \underline{\pi}B$ $B \rightarrow A$	$\pi A \subseteq \underline{\pi}B$ $\pi B \subseteq \underline{\pi}A$ $A + B$	$\underline{\pi}A \subseteq \pi B$ $\underline{\pi}B \subseteq \pi A$ $A \# B$
(1)	{1,3}	{1,2}	{0,2}	{0,3}	false	false	false	false
(2)	{1}	{1,2}	{0,2}	{0}	true	false	false	false
(3)	{1,3}	{1}	{0}	{0,3}	false	true	false	false
(4)	{1}	{1}	{0}	{0}	true	true	false	false
(5)	{3}	{2}	{0,2}	{0,3}	false	false	true	false
(6)	{i}	{i,2}	{0,i,2}	{0,i}	true	false	true	false
(7)	{i,3}	{i}	{0,i}	{0,i,3}	false	true	true	false
(8)	{i}	{i}	{0,i}	{0,i}	true	true	true	false
(9)	{1,3}	{1,2}	{2}	{3}	false	false	false	true
(10)	{i,1}	{i,1,2}	{i,2}	{i}	true	false	false	true
(11)	{i,1,3}	{i,1}	{i}	{i,3}	false	true	false	true
(12)	{i,1}	{i,1}	{i}	{i}	true	true	false	true
(13)	{3}	{2}	{2}	{3}	false	false	true	true
(14)	{i}	{i,2}	{i,2}	{i}	true	false	true	true
(15)	{i,3}	{i}	{i}	{i,3}	false	true	true	true
(16)	{i}	{i}	{i}	{i}	true	true	true	true

In §5.2.5 it is shown that this reconstruction technique scales up to consistent nets with any number of nodes, and in 5.2.6 that a slight variation does so for inconsistent nets. The set of admissible probes may be used to reconstruct the structure of any net, and provides extensional semantics for the paraconsistent protologic.

4. Significant generic schemata, design patterns

The kernel protologic developed in §2 is economical in its primitive constructs, being based on a few properties ascribed to a single sign → and some simple constraints upon its use. However, the system is adequate to represent a number of significant knowledge schemata and associated inference patterns and, before extending it further, it is appropriate to examine some of these structures.

If one regards the nodes as 'atoms' and the arrows as 'bonds' then the following generic schemata may be seen as analogous to 'molecules' common to many knowledge 'substances.' They are widely studied and used in many literatures and, nowadays, in computational applications are termed *design patterns* [145]. They semantify the protologic by naming schemata represented within it that are well-known and have many significant applications.

4.1. Constructs

The structure discussed in §3.3.5 and shown in Figure 50, a *construct* constituted by an exclusion link whose nodes are included in another node, is a generic building

block of many other structures that has been continually reinvented in diverse literatures.

For example, Aristotle [14, 14a19-20] notes in *Categories* that "contraries should in all cases either belong to the same genus or belong to contrary genera, or be themselves genera." In *Metaphysics* [17, 1054b23-26] he explains that a contrary, in order to express a *difference* (not just 'other than'), needs to be relative to a *likeness*: "But difference (διαφορὰ) and otherness (ἑτερότης) are distinct (ἄλλο). For, while the other and that which it is other than are not necessarily other in something (for everything that is some being is either other or the same), the different differs from something in something, so that it's necessary for there to be something the same in which they differ."

In developing theoretical foundations for *personal construct psychology*. Kelly [204, p.8-9] makes a similar argument for the basic structure of cognition being a difference relative to a likeness. He terms the structure of Figure 50 a *construct* [142, p.193], describing it as a *templet* that we create and fit to experience in order to give it meaning and attempt to anticipate further experience. He termed the top node the *range of convenience* and the bottom nodes *dichotomous poles*.

De Morgan [100, p.379-380], in his reconstruction of syllogistic reasoning, addresses the issue that Aristotle [14, 16a30-32] raised when noting that a general negation, such as *not-man* was an *aorist* (ἀόριστος) or indeterminate term, by arguing that in everyday language such negations are not aorist but understood as a contrary within an implicit context which De Morgan terms a *universe*. He discussed his notion of a universe divided by a contrary with his protégéé Boole, who reconceptualized it in psychological terms as a *universe of discourse* [334, p.102] determined by the "state of mind of the speaker."

These analyses are all intensional, that the meaning of an exclusion link is relative to the context provided by the node which includes it. The terms, 'universe of discourse' or 'domain of discourse,' are sometimes used to denote the *set* of entities under consideration. This is not problematic if that set is constituted as the consistent entities that are logically possible in a context that has been logically specified.

An extensional specification of the universe is possible if the logic includes constructs for set inclusion and cardinality constraints such as the description logic KRS [138] or description logics with the power of $\mathcal{SHOIQ}$ [182]. However, the complexity of reasoning with such constructs is very high and impractical except for small sets.

If the universe is ill-defined then a logical specification is not possible. For example, Nedzynski [262, p.132] remarks that "In the modern theory of existential import, the domain of discourse is the real world." However, the 'real world' is unlikely to be a logically sound domain of discourse because there is not currently, and probably never will be, a logical specification of the possible entities within that world. As Peirce [274, p.337] noted, "The real world cannot be distinguished from a fictitious world by any description."

Figure 55 provides some concrete examples of constructs.

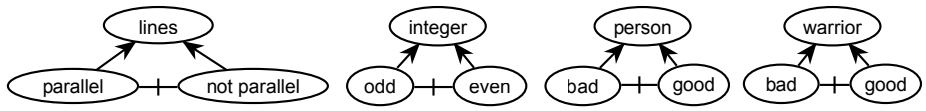


FIGURE 55. Some concrete examples of constructs

At the left is De Morgan's example of 'not parallel' having meaning only in the context of lines in geometry. He states that there is no natural opposite to 'parallels, such as 'intersectors,' but the negative form, 'not parallel' only makes sense relative to the context of 'lines.'

Next is an example where, in the context of arithmetic, the contrast between 'odd' and 'even' implicitly assume that the universe of numbers under consideration is that of integers.

The two constructs on the right illustrate how the meanings of the evaluative terms 'bad' and 'good' are dependent of the context in which they are used.

The logical structures in Figures 50 and 55 exhibit the two aspects of a construct that underly Aristotle, De Morgan and Kelly's intuitions. It is inconsistent for a node to include both nodes of the exclusion link—they are incompatible. Any node that includes one of them also includes the including nodeÑit is of the same kind, or genus, or within the context, universe of discourse, or range of convenience.

These are weak constraints and there will be many others deriving from the way in which the construct is embedded in a larger net, for example, an inclusion or equivalence link from 'even' to 'being a multiple of 2', but a triple of nodes linked as shown is a component of many significant structures.

4.1.1. Context, *qua*. Constructs provide an example of the way in which the interpretation of the labels of nodes is dependent on the context in which they are used. Lear [217, p.168] characterizes Aristotle's *qua* as introducing a "veil of ignorance" that *filters* out those *aspects* of a topic that are not relevant to the context specified by the *qua operator*.

Thus, in terms of Figure 55, we are able to represent that someone is 'bad' in the context of being a 'person' but 'good' in the context of being a 'warrior.' However, this meaning variance is problematic if the different 'bad' versus 'good' constructs occur in the same net. The conventions that node labels are unique identifiers and that nodes with the same identifier may be merged as instances of the same node (§2.1) override the intended contextual differentiation.

This is not an issue of logic but rather one of abbreviating labels. The graphical language makes provision for managing this by allowing a node to be represented as a *context box* rather an an oval, with the convention that a node contained within the box includes it and has a unique identifier constituted by the tuple of its own label and those of each surrounding context box.

Figure 56 shows the constructs of Figure 55 represented by context boxes. The only difference is that the unique identifiers of the nodes within the boxes are tuples including the label of the surrounding node, for example (person, good)

and (warrior, good) rather than just (good). Thus the two nodes no longer have the same identifier and will not be merged.

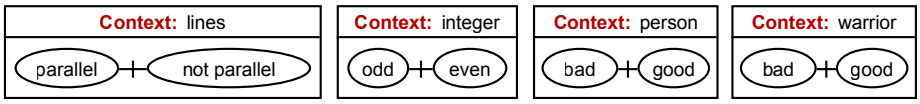


FIGURE 56. Construct labels localized through context boxes

How the tuple will be represented lexically when interpreted as a variable name in a logical system depends on what forms of name that system will support, for example, 'person.good' for the tuple (person, good). All that is required is that the unique identifier and equality constraints on labels are preserved.

This technique also provides a solution to the problem discussed in §3.3.4 of representing the common practice in human discourse of using the same terms for an exclusion link in several disparate contexts. Figure 57 shows context boxes being used to used to provide a logically sound representation of the factoring of several exclusion links as exemplified in that section.

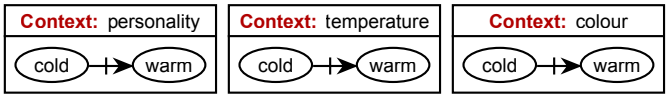


FIGURE 57. A logically sound representation of exclusion factoring

One could bypass the context boxes by expanding the labels in Figures 56 and 57 to incorporate the relevant context, emphasizing again that the problem is one of the lexical structure of labels as unique identifiers, not one of logic. The issue is significant in practice if modules representing different conceptual structures that are unambiguous in their own right are combined but have unintended interactions [266]. Context boxes implement what are termed *name spaces* in information engineering [82, §4.1], and the automatic inclusion links ensure that they represent a *context* for the nodes they enclose in the technical sense of §1.2.

4.1.2. The commonality of abstract generic terms. Providing appropriate contexts to isolate knowledge structures is an important technique to modularize them without introducing excessively verbose terminology. People use generic terms in a variety of situations, and resolve any resulting conflicts through the context in which they are used, and it is appropriate to support this in a system intended to be as perspicuous as possible. However, there is usually some abstract commonality of content across contexts which also needs to be represented.

The commonality of all uses of 'bad' versus 'good' as generic terms is a metalogical construct, that the terms represent a generic dimension of evaluation. This can be pushed down to the protologic by adding an inclusion link from a specific use to a more general one outside the specific context boxes. This may be done automatically in the graphical language by assuming that there is an implicit inclusion link from a node to one whose identifying tuple is the same with

contextual components omitted, for example from a node with the identifying tuple (good, warrior) to one with the tuple (good).

This is exemplified in Figure 58 where, for example, the node 'good' within the context of the node 'person' has an implicit inclusion link to the node 'good' within the context of the node 'evaluable' but none to the node 'good' within the context of the node 'warrior.'



FIGURE 58. Specific nodes automatically linked to more general nodes

There is no exclusion link between the nodes 'bad' and 'good' in the outer context because one node does not necessarily exclude the other except, possibly, within a specific context. What the outer structure represents is that the generic context of these terms is evaluative, that they are generally used *qua* being evaluable in some context.

This is the basis of Aristotle's criticism of Socrates' requiring Meno to provide a general definition of 'virtue' rather than provide instances of its meaning in particular contexts (§5.2.4). The essence of notions such as 'excellence' and 'virtue' is that they are evaluative in the generic contexts of skilled and moral behaviour, respectively. The terms only acquire richer meanings in the specific contexts in which they are used, and defining them fully involves exemplifying their usage over the range of possible contexts.

4.1.3. Contrast sets, taxonomies, determinables. In order to represent folk taxonomies, Conklin [90, p.48-49] generalized Kelly's notion of a bipolar construct to a structure of many mutually exclusive nodes included in one common node that he termed it a *contrast set*. Figure 59 shows three alternative representations of a contrast set with three nodes: left as a simple extension of a binary construct; centre as a cascade of a pair of constructs; right as a more complex structure. In all three representations the three leaf nodes form a mutually exclusive contrast set.

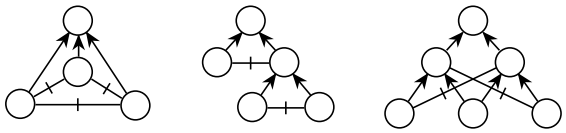


FIGURE 59. Three representations of a contrast set with three nodes

The centre and right representations interpolate intermediate nodes between the top node and the leaf nodes. In computational applications the availability of these protodisjunctions of the concepts represented by the contrast set has been found to simplify knowledge representation and speed inference, and their introduction has been termed the *vivification* of the knowledge base [228, 119].

Figure 60 shows concrete versions of the representations of Figure 59 taken from an expert system for contact lens prescription [80] where the concept 'presbyopia' had three specific possibilities, 'young,' 'pre-presbyopic' and 'presbyopic.' The introduction of the interpolated protodisjunctions, 'old' or 'older' and 'younger' reduces the number of rules or cases in the knowledge base and supports more meaningful explanations of the inferences made.

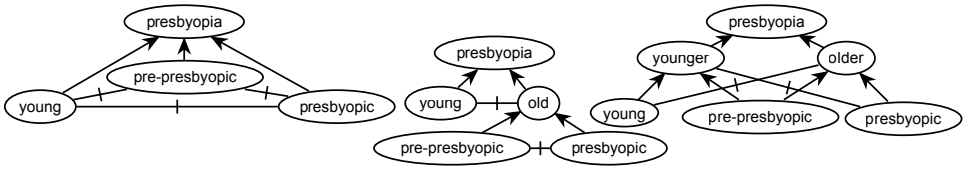


FIGURE 60. Examples of a contrast set with three nodes

Figure 61 shows a contrast set with four nodes, as a basic net on the left and using a context box on the right. The three-node contrast set can also be considered as three closely connected dichotomous constructs dividing a common universe of discourse, and the four-node set as six. In general an n-node contrast set corresponds to n(n-1)/2 dichotomous constructs.

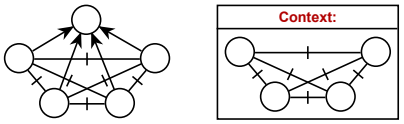


FIGURE 61. Two representation of a contrast set with four nodes

4.1.4. Determinables and determinable taxonomies. The top node of a contrast set in which the exclusion links are specified directly is called a *determinable* and the nodes its *determinates* [291, 292]. Johnson [194, ch.11] used this terminology to characterize structures such as colour names where there is no logical differentiation of the terms except that those at the same level in the hierarchy exclude one another.

Johnson notes that determinates often have a hierarchical and graded structure. The centre net in Figure 60 illustrates a hierarchical structure for the determinable 'presbyopia' having determinates 'young' and 'old' with the latter also being a determinable with determinates 'pre-presbyopic' and 'presbyopic.' §4.2 analyzes how graded scales emerge from such structures.

While Johnson names the determinable/determinate structure and provides the first logical analysis of it *per se*, it was a common logical structure in antiquity and is instantiated many times in Aristotle's writings [162, 316], including examples of graded contrast sets [13, 12b11-25].

Contrast sets can have an indefinite number of nodes and their graphical representation can have such a large number of exclusion link crossings as to be confusing. Hence it is often appropriate to factor larger contrast sets by interpolating intermediate nodes to create a hierarchical structure in which the bottom

nodes constitute the contrast set. Figure 62 shows a hierarchical representation of an eight-node contrast set. It may be considered to be a hierarchy of seven bipolar constructs.

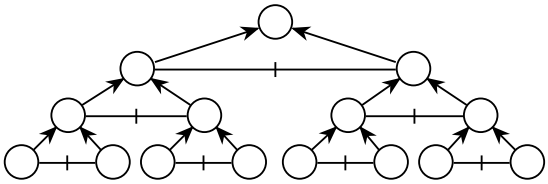


FIGURE 62. Hierarchical representation of a contrast set

Figure 62 looks like the common representation of a *taxonomy*, although in informal diagrams the horizontal contrasts are often implicit with the lines omitted, as are the vertical inclusions with the arrows shown as undirected lines. The *determinable/determinate taxonomy* of Figure 62 represents a determinable six of whose determinates are themselves determinables.

4.1.5. Genera/differentiae taxonomies. The exclusion links in a contrast set need not necessarily be specified *per se* but may be represented indirectly as virtual links inferred from the exclusion inference pattern applied to links from the nodes in the set to others outside it, the *differentiae*.

Figure 63 shows a representation of Porphyry's [285] exposition of Aristotle's analysis of substance as a *genus/differentia taxonomy* where the exclusion links within the taxonomy are all virtual. In particular those between the five-node contrast set of bottom nodes, body, spirit, mineral, plant, human, beast, are all inferred through those of the differentiae.

At the top each of the *differentiae* is shown as a determinant of a determinable having as its universe the genus that is divided by the differentiae. However, as noted in [140, §3.2] which represents the same structure with their universes outside the context of the genera, there are many other possible choices that have no impact on the logical structure of the taxonomy.

Such variations may be significant in considering issues of the *meaning* of the structure, and may also have different logical inferences if it is merged with other structures. As Dutilh Novaes [111] has noted in the context of the various semantic models of the syllogistic, variations having no consequence for a particular logical system may have significant differences when that system is extended.

In an analysis of scholarly issues in the interpretation of Aristotle's ontology of substance, Morrison [259, §3] has discussed the extensive debate on the correct representation of the differentiae and made recommendations which were followed in [140]. The issues *per se* cannot be resolved by a logical system, but the graphical protologic discussed in this article provides a formal tool for representing them visually and exhibiting, for each option, the assumptions made, their basis in terms of other structures, and their logical consequences—a contribution to the presentation and comparison of issues and arguments.

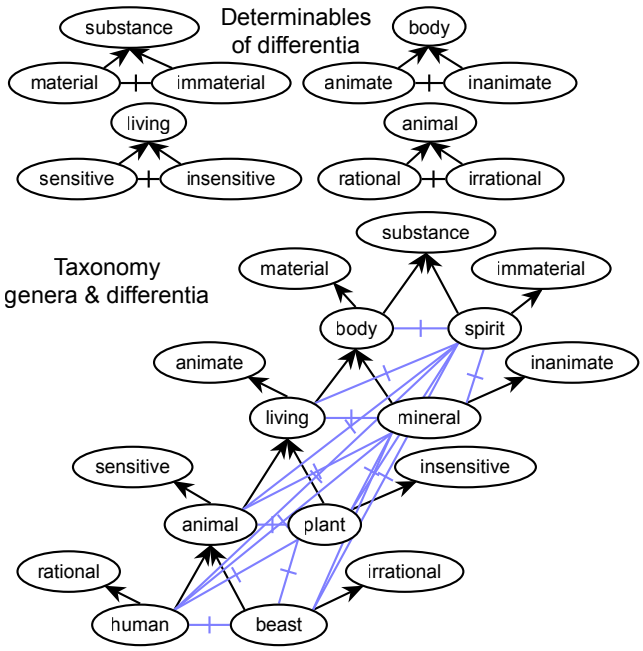


FIGURE 63. A representation of the tree of Porphyry

4.1.6. Merging biological taxonomies. Reasoning about taxonomies is not only a matter of philosophical scholarship but also important to biological science, for example in analyses of the consequences of merging taxonomies of the same species from different sources [354], or doing so across species to develop an overall *tree of life* [50].

Figure 64 presents an example from the literature [354, p.198] of two simple taxonomies, their proposed merging through the articulations shown, and queries that may arise about the outcome.

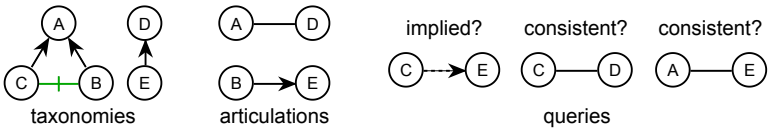


FIGURE 64. Some taxonomy articulation queries [354, p.198]

Figure 65 shows responses to the queries based on the inclusion and exclusion inference patterns: there is no inferred link from C to E; equivalencing C and D makes the net inconsistent; but doing so for A and E does not.

The study cited represents the taxonomies and their articulation with similar node and arrow diagrams, omitting the implicit exclusion links, but regards the inferences one may make from them as *informal* and uses a powerful first-order logic theorem prover to answer the queries. What Figure 65 demonstrates is that formal reasoning in terms of the diagrams themselves can be used to provide

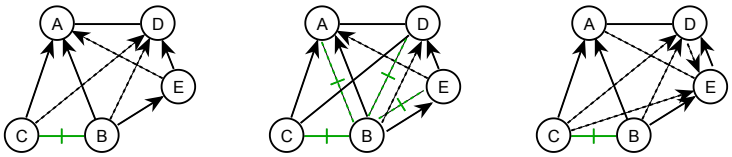


FIGURE 65. Inferred responses to the taxonomy articulation queries

logically well-founded responses to the queries. §7 provides additional techniques for such analyses.

4.2. Mereology of determinables, graded contexts, scales

Johnson [194, ch.11 §5] notes that a characteristic of many determinables is that their determinates are comparable, having a serial order such that one can say that one determinate is qualitatively between two others. The contexts provided by such determinables may be seen as representing graded *dimensions* rather than simply contrast sets. In linguistics this phenomenon has been studied as one of *dimensional adjectives* [48] and in cognitive psychology Rosch [308] has conducted empirical studies showing that far more concepts are used as if they were graded than one might expect.

4.2.1. Representing graded contexts in the protologic. It is often assumed that the representation of an order relation over determinates requires a logic with graded truth values [128]. However, this aspect of determinates may be modelled in the metalogic of nets of inclusion and exclusion links by considering structures from which order relations naturally emerge [140, §7].

For example, Figure 66 shows the evaluable, 'good' versus 'bad' structure of Figure 58 extended as in Figure 59 right with additional nodes intended to extrapolate and interpolate the original structure as an ordered dimension.

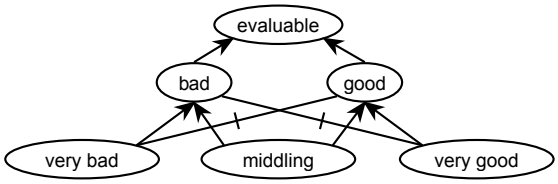


FIGURE 66. A graded evaluation dimension

This net exemplifies a hierarchical representation of a determinable that is not a taxonomy. The three bottom nodes form a mutually exclusive contrast set but the nodes above them appear to interpolate between them. This order structure emerges formally if we characterize each node by its content, the set of its inclusion and exclusion links to the nodes labeled 'bad' and 'good.' That is:

The cardinality of the symmetric difference between these sets is a distance measure, δ, show in in Table 2, that is, a linear ordering of the five nodes included by 'evaluable.' It results from a metalogical characterization of the nodes that has no direct representation in the protologic but is latent in the structure of the

very bad: {(→, bad), (↠, good)}
bad: {(→, bad}}
middling: {(→, bad), (→, good)}
good: {(→, good)}
very good: {(↠, bad), (→, good)}

net. Similar structures representing order relations over more than five nodes are exemplified in [140, §7].

TABLE 2. Distance measure between sets of links

δ	very bad	bad	middling	good	very good
very bad	0	1	2	3	4
bad	1	0	1	2	3
middling	2	1	0	1	2
good	3	2	1	0	1
very good	4	3	2	1	0

The calculation of δ is most readily apparent if the characterization of the nodes is presented as the matrix of Figure 67. Each column is a *feature vector* for a node having as values the link arrows to the two designated nodes.

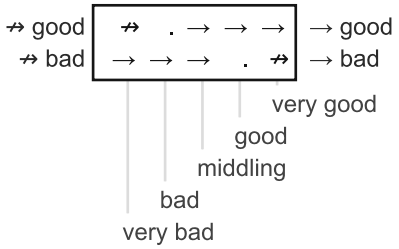


FIGURE 67. Feature vectors characterizing the nodes in Figure 66

4.2.2. Representing graded contexts with incompatibility semantics. Order relations may also be represented by incompatibility structures having exclusion links only (except for the loops around individual nodes). For example, Figure 68 top shows an Euler diagram proposed by Fourali [128, p.152] as a graded concept to be represented in a fuzzy logic. At the bottom is shown an equivalent net that represents the mereological structure of the diagram through exclusion links between the disjoint nodes.

The order structure emerges if we characterize each node by the feature vectors of its links to the nodes labeled 'very unhappy' and 'very happy' (Figure 69) where, again, the δ measure leads to a five-point linear scale.

In this example, the net represents whether the ovals overlap in the upper Euler diagram. Fourali [128, p.152] relies on such "areas of ambiguity" to represent a graded scale. Without them one would have five disconnected ovals with the intended scale suggested only informally by their spatial relations.

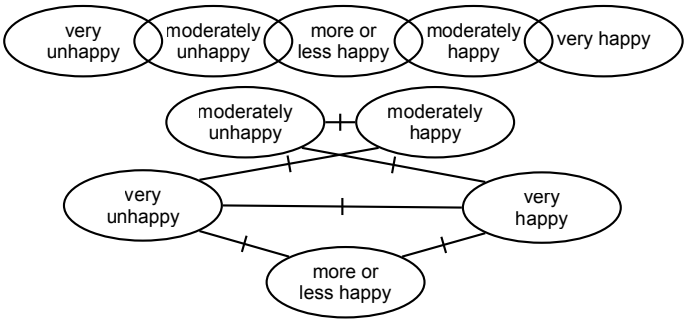


FIGURE 68. Euler diagram for a graded concept and equivalent net

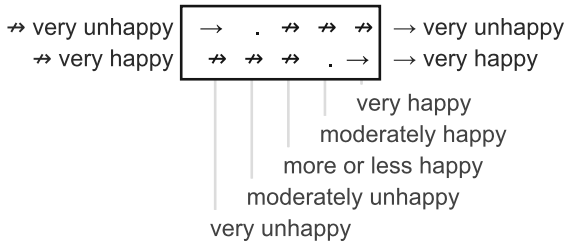


FIGURE 69. Feature vector for the net at the bottom of Figure 68

Figure 70 top shows an alternative Euler diagram for the same graded concept. The mereological part-of and disjoint relations are represented by inclusion and exclusion links in the net below which has the same abstract structure as that in Figure 66. It generates exactly the same characterization in terms of its links to the nodes labeled 'very unhappy' and 'very happy' as the net of Figure 68 even though it appears very different in its structure.

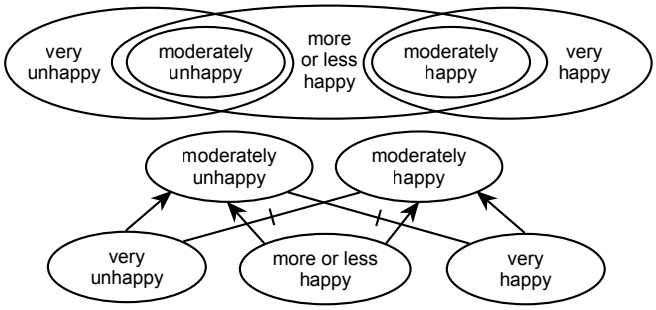


FIGURE 70. Alternative representation of the same graded concept

4.2.3. Spatial modelling of graded contexts by multi-dimensional scaling. Deriving latent order relations for a larger net can be more complex, and and the distance matrix derived from the net's feature vectors in a may best be represented in a lower dimensional space through *multi-dimensional scaling* [63]. Scaling based

on those in Figures 67 and 69 generates one-dimensional spaces corresponding to linear scales as shown in Figure 71.

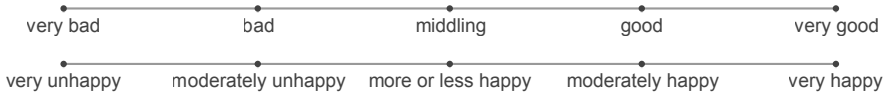


FIGURE 71. Multidimensional scaling of data from Figures 67 and 69

A more complex example is shown in Figure 72 that combines a net representing the standard nine-point hedonic rating scale [279] with two forms of five-point scale representing the constructs used in a whisky flavour map [374, Fig. 6].

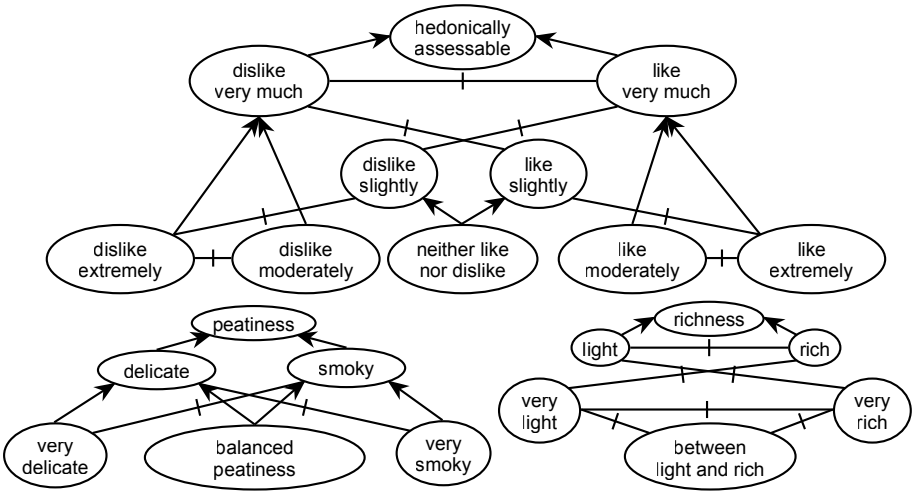


FIGURE 72. A net representing three distinct scales

When the feature vectors for the entire net are extracted and the distance matrix derived from them is analyzed through multidimensional scaling the resulting plot [159] is that shown in Figure 73. The latent three-dimensional *conceptual space* [146] represented by the three determinables in Figure 72 is apparent.

The three disconnected subnets in Figure 72 may each be represented by Euler diagrams. However, it is not possible to combine these diagrams to represent the entire net since the lack of connection has to be represented by each closed curve representing a node in a subnet overlapping, but not being contained in or contained by, every closed curve in the diagrams for the other subnets. Euler diagrams are useful visual/formal representations of highly linked nets but do not scale up to larger structures, or even to smaller ones having largely disconnected subnets.

The conversion of an Euler diagram to a net followed by the spatial modelling of the net illustrates an interesting interplay between various representations of conceptual space. The mapping from Euler diagrams to nets is well-defined: each closed curve becomes a node and its label, if it has one, the node label; the nodes

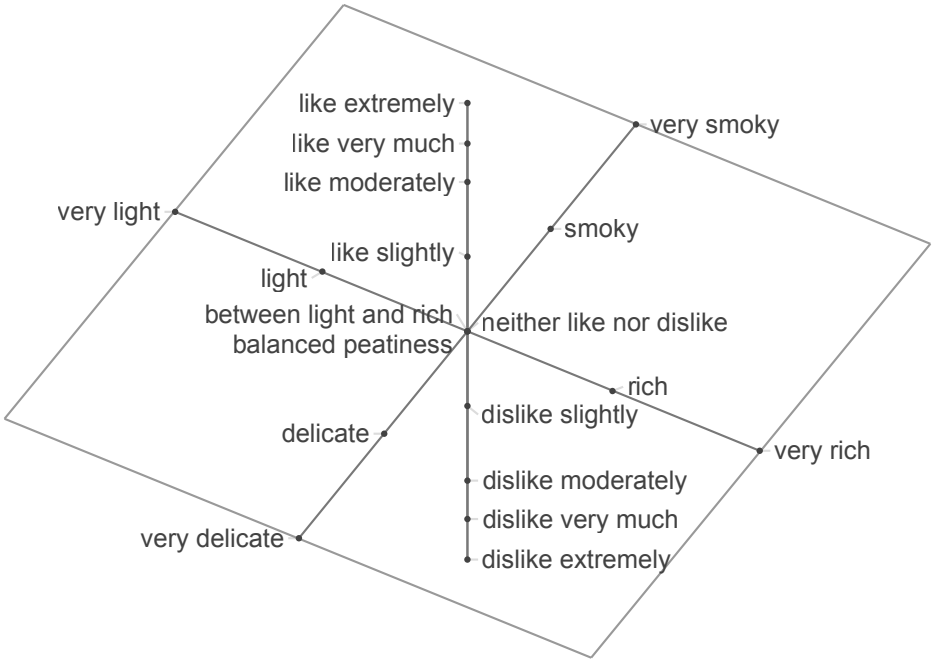


FIGURE 73. Multidimensional scaling based on the links in Figure 72

corresponding to spatially disjoint closed curves are connected by exclusion links; the node corresponding to a closed curve contained in another has an inclusion link to the node corresponding to the other (the logical inclusion is converse to the spatial inclusion).

Characterizing a node by a feature vector of its links to other nodes is intensional, a reflexive characterization of the net in terms of its own structure. The spatial representation that emerges is a topological model based on the logical connections constituting the net. It appears to have metric aspects because the cardinality of the set differences imposes an implicit metric, equivalent to assuming that the ordered triple $(\twoheadrightarrow, ., \rightarrow)$ is one of equidistant values, and the results are plotted on linear orthogonal axes. As Kelly [203, p.105] emphasized when presenting constructs as axes of a *psychological space,* the conceptual space of Figure 73 reflects a topology of connectivity rather than that of a metric space.

The scalar properties of the nets in Figure 72 are metalogical. They could be pushed down to the protologic by extending it to provide some form of multivalued logic such as uncertainty [135], or fuzzy, logic [134].

The labels may be factored as bipolar oppositions modified by generic qualifiers, and this genericity could be represented by partially specified nets providing metalogical templets. These could pushed down to the protologic to extending it to provide graded quantifiers similar to those proposed by Åqvist [11].

Templets at the metalogical level seem the most appropriate way to capture the wide variety of generic qualifiers since scales and their terminology seem indefinitely extensible [230], and the intermediate levels of the hierarchical structure are valuable in making available coarser gradations supporting what has been termed the *vivification* of knowledge [119].

4.3. Frames

An important structure that is discussed in many literatures under several different names is a *frame*, a collection of determinables whose overlap provides a context in which to represent a family of *concepts*, *situations*, *problems*, *states of affairs*, and so on. Particular types of situations are represented as sub-contexts specified by the overlap of determinants of the determinables, and may be termed in various literatures *templets*, *prototypes*, *stereotypes*, *schemata*, *epistemic objects*, *epistemic states*, *hypotheses*, *explanations*, *theories*, and so on.

The term *frame* originates from the role played by a picture frame in separating the picture from its surroundings, an analogy that was first developed by Simmel [331] and used to compare the unity of a work of art with that of social structures. It was developed in detail in Goffman's [156] *frame analysis* to provide a theory of how human behaviour was influenced by the social ethos in which it was framed. Minsky [256] introduced the notion in artificial intelligence studies, initially in studies of computer vision. Fillmore [126] introduced it in linguistics in studies of *interpretive frames* for text understanding. Ranganathan [297] introduced the notion in library science terming it *facted classification*. Gardenförs [146] extends the notion in his studies of *conceptual spaces* by emphasizing the scalar aspects of determinables and using these to develop metrics of similarity between sub-contexts within a frame.

In the protologic a frame may be represented by a *frame node* constituted by the overlap of the contexts of a collection of determinables, and itself providing the context for some structure of sub-contexts constituted by determinates of some or all of the determinables.

4.3.1. Frames and abduction to the best, or only, prototype. Figure 74 illustrates a frame for malt whisky tasting experiences constituted by the three determinables of Figure 72.

The top net specifies the frame node 'malt whisky' as being within the three contexts of 'richness,' 'peatiness' and ' hedonically assessable, and shows three sub-contexts of 'thin whisky,' 'pleasant whisky' and 'great whisky,' each of which includes three determinates. These were derived by abstracting from the experience of whisky tasting. At the bottom are current choices being offered specified only in terms of the two determinables of a whisky flavour map.

Figure 75 illustrates how the frame and prototypes might be used to anticipate the assessment of a possible choice of whisky not previously tasted but whose features are known from the flavour chart.

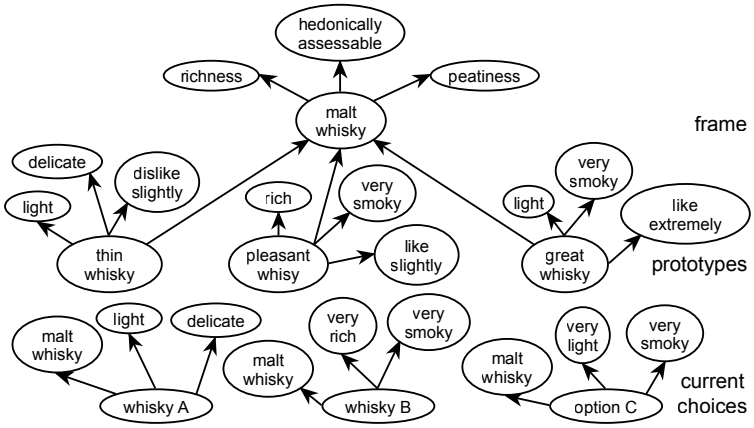


FIGURE 74. A frame for whisky tasting experiences

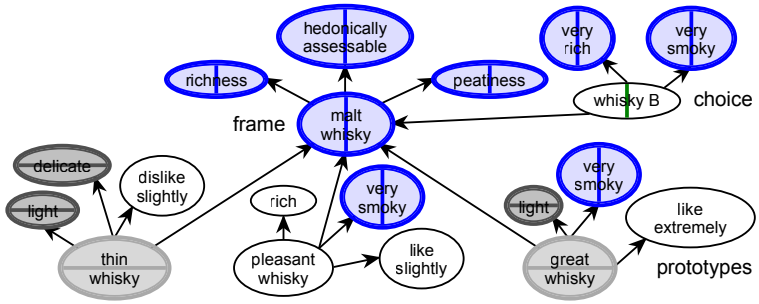


FIGURE 75. A frame used to anticipate whisky tasting experiences

At the top right a possible choice has been brought into the frame or context by asserting it to be true. This leads to two of the prototypes being inferred to be irrelevant. No further inference can be made unless an abductive assumption is made that one of the three prototypes must apply and, since two do not, the third may be assumed to apply. In which case the possible choice is anticipated by abductive inference to the best, or only, prototype [51], to be a 'pleasant whisky' that the taster will 'like slightly.'

The abductive assumption that one of the prototypes must apply is a tacit convention in biological taxonomy, the "latent taxonomic assumption" of *species coverage* [354, §4.3.3]. It is equivalent to assuming that the relation between the frame node and its sub-contexts is a structural disjunction, and it is defeasible unless the sub-contexts provide full coverage of every logical possibility. §7 introduces structural disjunctions in the protologic, and §8 defeasibility.

4.3.2. An abduction-based solution to an expert systems problem. Figure 76 is a more substantial example of frame-based reasoning to solve an expert systems problem that will be used to illustrate the impact of extending the protologic to encompass structural connectives and default reasoning.

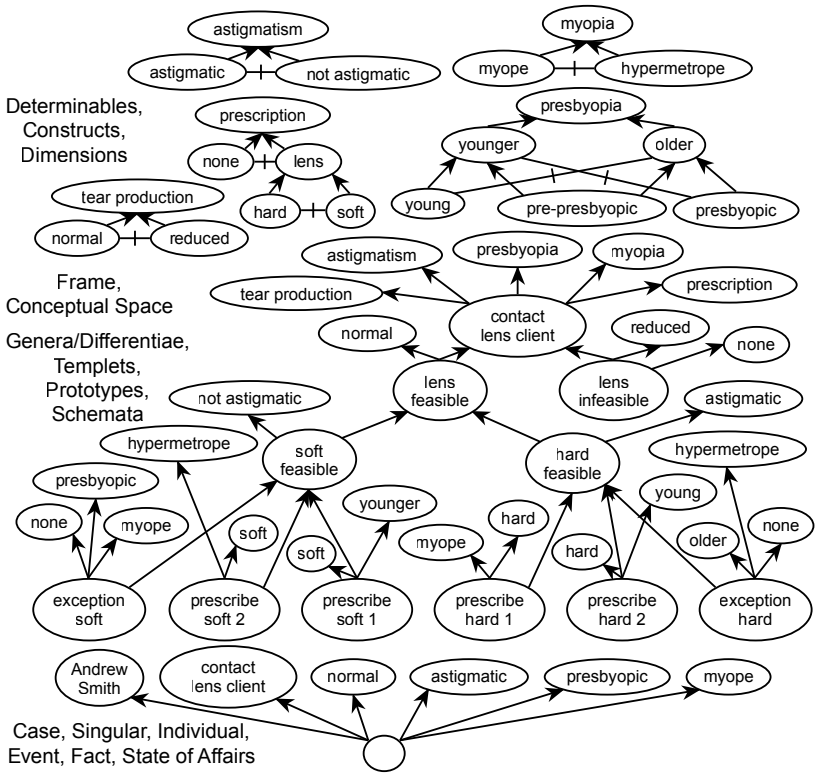


FIGURE 76. Frame for solving a contact lens prescription problem

The problem was originally contrived to illustrate problems of machine learning algorithms based on information gain when attributes interact [80]. Current algorithms have no difficulty but the problem is also useful for illustrating how erroneous data and irrelevant attributes can interact to make inductive modelling very much more problematic [136].

The task is one of prescribing hard or soft contact lenses for a client having the attributes the four determinables shown at the top left leading to one of three prescriptions shown at the top right. The frame shown in the centre characterizes seven possible prototypes represented by its bottom nodes. The representation of a specific case is shown at the bottom.

The solution can be stated simply in terms of rules and exceptions: a patient whose tear production is normal should be prescribed a hard lens if astigmatic and a soft lens if not; however, there is an exception to the soft prescription if the patient is presbyopic and myopic, and to the hard if hypermetropic and old.

Figure 77 shows the inferences when the case at the bottom of Figure 76 is activated by marking it true. The only relevant prototype is 'prescribe hard 1,' and abductive inference, assuming that at least one prototype applies, sets this true leading to the 'hard' lens prescription being inferred.

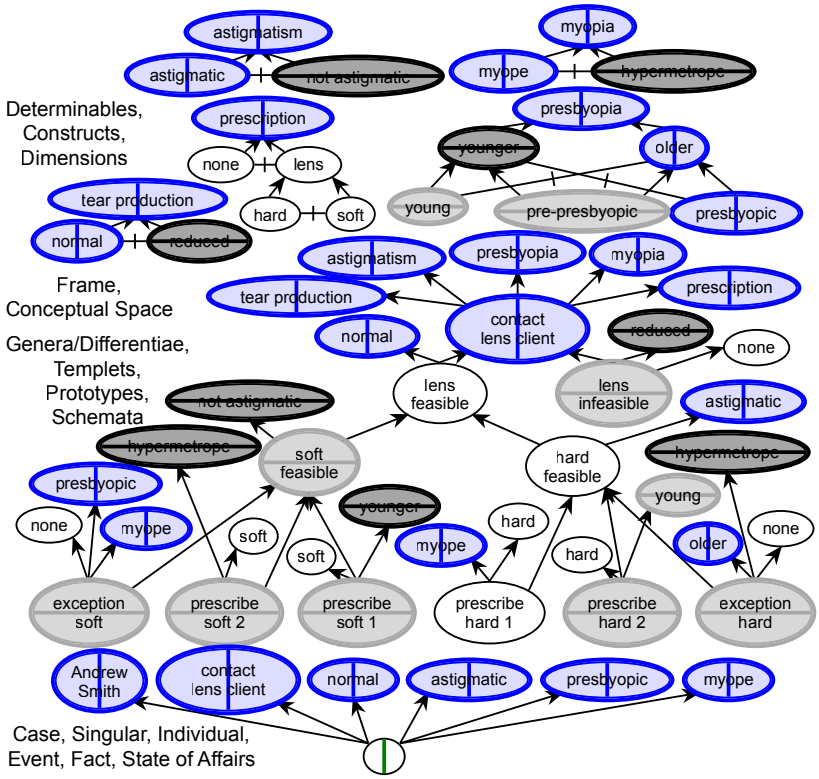


FIGURE 77. Abductive inference used to recommend lens prescription

Thus a logic using only the weak syllogistic connectives can represent the solution to a decision problem that is usually solved through definitions and rules based on structural conjunction [80, 139]. As already noted, the abductive assumption is equivalent to a structural disjunction in the metalogic, but its use is so specific that is reasonable to suppose that it may be a tacit assumption of everyday reasoning.

This example is used to illustrate the effect of introducing structural connectives in the protologic; definitions and rules in §7.1.3, and disjunctive judgements supporting defeasible abduction in §8.2.2.

4.3.3. Family resemblances, cluster concepts. Two sub-contexts of a frame may be disparate in that they have no common determinates. For example, in Figure 74, 'thin whisky' and 'pleasant whisky' have no determinates in common, and their only commonality is that they are both specified through determinates of the same determinables.

This feature of frames has been seen as addressing the issues of Wittgenstein's [377, §65-77] *family resemblances*, that we cluster under terms such as 'games' a

variety of notions having nothing in common, and that there are no necessary and sufficient conditions defining a game [74, 5].

Frames provide formal foundations for another approach to modeling family resemblances, that of *cluster concepts* where the presence of a sufficient number of a collection of characteristic properties is deemed sufficient to define a concept, although no particular property is necessary [147, 91]. Cluster concepts have been presented as modelling Waismann's [365, p.121] notion of the essentially *open texture* of all concepts that Hart [172] saw as foundational to the operation of the legal system and Weitz [371] used to argue that the notion of 'art object' is intrinsically undefinable.

In the philosophy of art, Gaut [148] has defined the concept of an *art object* as a cluster concept. Gaines [140] has argued that the definitional aspects of his model lie in the frame (Figure 78) constituted by the eleven determinables that Gaut uses to characterize art objects. Each determinable has a preferred determinate, for example that the aesthetics are positive, and it is the sub-context constituted by the preferred determinates that is the 'cluster concept' for art object, but it is the frame constituted by the determinables which defines the universe of what we construe as art objects.

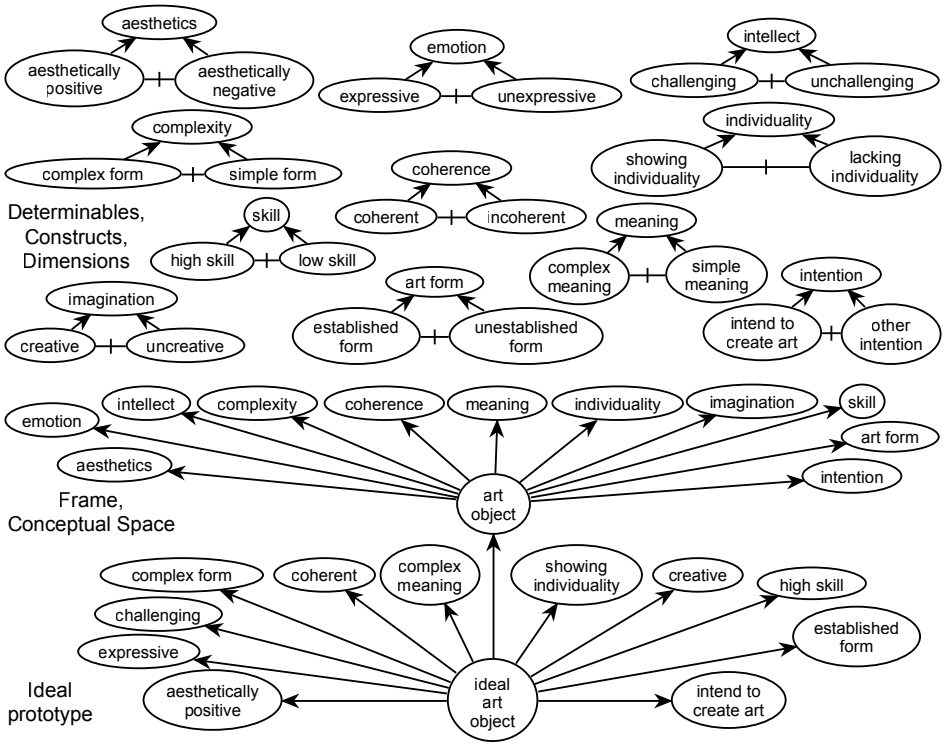


FIGURE 78. A frame used to anticipate art experiences

A necessary and sufficient definition of an art object might be constituted by the structural conjunction of the determinables but this would need to be tracked diachronically since the determinables of art objects have changed historically.

In Figure 74 if the frame node, 'malt whisky,' was defined by the structural disjunction of its outgoing inclusion links that would constitute a formal definition of malt whisky within the context of flavour assessment.

If it was defined by the structural disjunction of its incoming inclusion links that would constitute a formal definition of malt whisky for a particular individual in terms of his or her experiences of tasting it.

4.3.4. Mereology of frames. As discussed in §5.3.1, contexts have a spatial logic, and the notion of a context being within the overlap of other contexts may be formalized merelogically. However, the two-dimensional images of spatial overlaps often used in illustrations can be misleading. They are projections of what is often a high-dimensional space of into a much smaller one, and may provide a distorted view of situations such as Gaut's eleven-dimensional space of art objects.

The determinables are usually best conceptualized as *dimensions* (Figure 73), and the overlap of their contexts as a multi-dimensional space. When the number of dimensions is three or less this is readily visualizable. For example, whisky flavour maps provide a useful two-dimensional representation of malt whiskies in terms of richness and peatiness, and the hedonic dimension is typically represented by a numeric rating on the map indicating how much a person liked the taste. However, empirical studies [374] show that the two dimensions shown on a flavour map account for less than fifty per cent of the variance in consumer preferences, and that there are several other significant determinables. Presenting all these spatially is problematic.

Gardenförs [146, §4.4.2] has argued that the extension of the notion of a frame to one of a *conceptual space* with metric properties that support similarity measures has significant advantages over logic-based approaches. The results of §4.2 show that the spatial logics underlying a frame's determinables correspond to a latent conceptual space underlying that frame. The spatial and logical approaches complement, rather than rival, one another.

Mereological research focuses on three or four dimensional spaces because the target domains are usually spatial or spatio-temporal reasoning. However, mereological logics and constructions are largely independent of the number of dimensions of the space they represent. In abstract mereology the category-theoretic links between topology and logic are well-established [363]. The subtle interplay between locales, toposes and Lindenbaum algebras provides profound insights into fundamental issues of universal logic.

The readily comprehended and visualizable mereological structure of the universal protologic provides a bridge to these deeper studies and their significance in applications.

5. Protosemantics

Sher [326] has noted that there are problems in "explaining logic" because of its *unbounded generality*, *ultimate basicness*, *supreme normative force* and *absolute certainty*. Logic is intrinsically foundational and any specific interpretation in another system is only a test of the applicability of logic to that other system through that interpretation. It not a validation of logic *per se*, in the same way that finding a system that exemplifies '2 + 2 = 4' does not validate arithmetic.

However, a wide range of significant and effective interpretations can provide insights into the relations between those other systems, and into the role that logic plays in facilitating these insights. That is, they offer a variety of semantifications that provide insights into what is otherwise an abstract mathematical system.

The notion of *semantics* in the logical literature is based on *translations* between a logical system and some other systems, either formal or informal. If one side of the translation is regarded as better founded than the other then it may be used to provide 'foundations' for the other. If not, then, through a process of *reflective equilibrium* [303] both sides may provides insights into aspects of the other.

If one system is naturalistic and informal then the more formal system may be seen as providing an *explicatum* of the less formal. Conversely the naturalistic system may be seen as providing a test of the *adequacy* of its representation by the formal one.

For a universal protologic one would expect there to be a very wide variety of significant translations between it and many other systems, providing protosemantics underlying each of the wide range of approaches to semantics for various logical systems. Some of these have already been mentioned such as the algebraic interpretations in terms of graph theory, residuated relations and preorders. Others have been suggested in terms of potential formalizations of notions of *content* and *context*. The spatial allusions of these terms in the literature have been noted suggesting a possible link to mereology. The possibility of the reconstruction of nets from probes, bottom nodes that may be regarded as *protoindividuals*, suggests truth-functional and extensional semantics.

The patterns of inference for the protologic discussed so far are minimal and others will be defined that add substantial power. However, the capability to represent a range of significant and widely used schemata (§4) illustrates that rather more can be achieved with the kernel protologic than might be expected.

This section addresses basic semantic interpretations for the kernel protologic that are common to many logical systems, and later sections examine how they extrapolate to further patterns of inference as they are introduced. §6 presents some specific translations to other logical systems.

5.1. Proof-theoretic semantics, inversion, harmony

Sher's issues of providing foundations for logic are long-standing ones. Lorenzen [236, p.351] argued that most semantic interpretations of basic logical notions use

those same logical constructs in their metalogical explanations and, hence, cannot constitute foundations for logic.[30]

Gentzen [150, §II 5.1.3] provided proof-theoretic foundations for logical systems when he extended Hertz's [**?**] approach to the definition of logical systems by defining natural deduction inference schema (*Schlussfiguren*) comprising paired rules for the introduction and elimination of logical constants. He remarks that the introduction rules could be seen as definitions of the constants and the elimination rules as consequences of those definitions.[31]

Von Kutschera [214] argues that this approach provides *Gentzen semantics* (*Gentzensemantik*) for logical systems, and Schroeder-Heister [320] introduced the phrase *proof-theoretic semantics* for the technique of attributing meaning to the logical constants through their introduction rules. Lorenzen [234, p.176] termed the pairing of introduction and elimination rules a "principle of inversion" (*Inversionprinzip*) and Prawitz [286, ch.2] applied this term to Gentzen's natural deduction system.

Belnap [28], in countering Prior's [293] introduction of *tonk* as a logical constant with inappropriately paired introduction and elimination rules, argued that there should be *harmony* between the rules. However, while the notions of proof-theoretic semantics [367, 295, 323], inversion [322, 258] and harmony [351, 108, 255] have become core foundations of universal logic, their precise general formalization has proved to be extraordinarily difficult and remains a significant area of research [321, 352, 341].

The protologic has a foundational proof-theoretic semantics though its specification in terms of *inference patterns*, each of which licenses the addition and deletion of a link (including some to an additional node). The patterns form complementary pairs where the addition of a link defines a feature of a logical constant, such as transitivity, and deletion is in harmony with addition, because the link deleted remains 'virtually' present, always available to be reintroduced.

This leads to two levels of *meaning* for the protologic connectives: at the logical level it has operational semantics in terms of the inference patterns; at the metalogical level it has conceptual semantics in terms of phenomena induced by the operations such as the *idempotence*, *transitivity*, *reflexivity*, and *symmetry*.

The partial order (§2.4.4) corresponding to inclusion, $\rightarrow$, underlies the proof-theoretic semantics for the protologic. All other logical constants are defined in terms of this: exclusion, $\nrightarrow$, (§2.5) as a constraint on the placement of $\rightarrow$; coexclusion, $\nRightarrow$, as an inference pattern complementary to, and completing, those for $\rightarrow$ and $\nrightarrow$; structural conjunction (§7.1) and disjunction (§7.2) as ideal elements that are extremal in the order defined by$\rightarrow$; negation as a combination of exclusion and coexclusion (§2.6.3), or a degenerate case of conjunction (§7.1.1) or disjunction

[30] "die Semantik für die Anwendung ihrer Definition der logischen Gültigkeit die Gültigkeit der logischen Regeln selber schon voraussetzen muß".

[31] "Die Einführungen stellen sozusagen die 'Definitionen' der betreffenden Zeichen dar, und die Beseitigungen sind letzten Endes nur Konsequenzen hiervon."

(§**??**); and the material conditional (§7.2.6) as a schema based on disjunction. Inference schema based on rules (§7.1.2), abduction (§7.2.3) and defeasible reasoning (§8) are developed in terms of these.

Other interpretations deriving from these proof-theoretic foundations which provide protosemantics for the semantics of many logical systems are developed in the following sub-sections.

5.1.1. Truth-theoretic semantics. The terms 'true' and 'false' were introduced in §2.7 as appropriate terminology for the marking of nodes in such a way as to show the dynamics of inference patterns, that is, as a way of representing and visualizing the proof-theoretic semantics of the protologic. This is consistent with, but converse to, presentations of logical inference that take truth and falsity as primitive notions and present inference as a derived notion.

The analysis of §2.7 is *deflationary* according to Gupta's definition [167, p.57], and a form of *inferential deflationism* in Horsten's terminology [183]. The complexity of more profound notions of truth [212, 327] beyond those of logic is epistemological, the consideration of issues of the extra-logical justifications for making assertions.

Logic has an important role to play in modelling various approaches to these issues by aiding assessment of the basis of making assertions though analysis of their consequences relative to networks of logical constraints, emanating from a variety of sources. It is essential that a universal protologic makes no commitments with regard to the issues so as not to bias the analysis.

5.2. Reconstructing nets from full or partial partitions

Nets may be partitioned in a wide variety of ways, some of which have been discussed in §3.3.3. Since nets are constituted by their multisets of links (§2.1), any full partition that *covers* the original net (in that it includes all its links at least once) may be used to reconstruct a net logically equivalent to it.

The reconstruction will not be isomorphic to the original net unless the partitions are such that each link in the original net occurs only once in the partitions. In general partitions may overlap and/or neglect the distinction between specified and inferred links, so that what is reconstructed will be a logically equivalent net having the same minimal canonical form (§3.1).

In some circumstances, nets may also be reconstructed from partial partitions sufficient to enable the full set of links to be inferred (§5.2.5, §5.2.7), including inconsistent nets §5.2.6).

5.2.1. Intensional partitions of the kernel protologic. Partitions with one node common to all the links may be seen as characterizing a node in terms of all or some of its links with other nodes. If the node characterized is the tail node of all the links, the partition is often termed *intensional* and conceptualized as representing the content of the common node in terms of those of the nodes whose content it includes or excludes.

A node includes itself, and this link is part of its intension. This corresponds to nodes in the protologic so far developed being *primitive*, constrained by their links to other nodes but not defined by them. A primitive node has unspecified internal content beyond that of the nodes to which it links. In §7 further constraints are introduced that *define* the content of a node through some of its links such that its link to itself makes no additional contribution to its content.

Glashoff [155] has developed an intensional semantics for the categorical syllogistic in which a term, A, is characterized by an *intensional interpretation* comprising a pair of disjoint subsets of a set $\mathcal{O}$, (s(A), σ(A)), such that the inclusion and exclusion links may be recovered as:

$$A{\rightarrow}B \text{ iff } s(A) \supseteq s(B) \text{ and } \sigma(A) \supseteq \sigma(B)$$
$$A{+\!\!-}B \text{ iff } s(A) \cap \sigma(B) \neq \varnothing \text{ or } \sigma(A) \cap s(B) \neq \varnothing$$

He does not exemplify particular interpretations of members of $\mathcal{O}$ but it is reasonable to suppose each member is a 'merkmal,' 'mark,' 'feature,' 'property,' 'quality,' 'characteristic' or a similar term used technically or colloquially for a component of content or meaning, or *atom of intensionality*.

The pair of sets may be reduced to a single set by using a syntactic marker, such as an overbar, to distinguish each member of σ(A) from those of s(A), and merging the two sets to represent the *content* of A, c(A). If content is defined to be *inconsistent* if one pair of members has the same label both marked and unmarked, then one can recover the links as:

$$A{\rightarrow}B \text{ iff } c(A) \supseteq c(B)$$
$$A{+\!\!-}B \text{ iff } c(A) \cap c(B) \text{ is inconsistent}$$

which nicely characterizes content inclusion as meaning containment, and content exclusion as meaning incompatibility.

Glashoff's construction provides the basis for the mereological explication of the inclusion of content in the kernel protologic (§5.3.4), complementary to Euler diagrams as a mereological explication of context (§5.3.1).

An intensional partition of a net in the kernel protologic (without coexclusion links) is the set of subgraphs each constituted by a single node together with the two sets of nodes at the head of the inclusion and exclusion links of which it is the tail (corresponding to Glassoff's [155, §1.2.2] sets of positive and negative terms). If X is a node in a net:

$$s(X) = \{Y\colon X{\rightarrow}Y\}$$
$$\sigma(X) = \{Y\colon X{\twoheadrightarrow}Y\}$$

defines an intensional partition where all links are included, and the recovery of the net is a simple merger of the node sub-graphs equivalent to:

$$A{\rightarrow}B \text{ iff } B \in s(A)$$
$$A{+\!\!-}B \text{ iff } B \in \sigma(A)$$

When the net characterized is in maximal canonical form, Glashoff's recovery of the connectives may be seen as a consequence of the inclusion inference pattern and the exclusion link definition. More complex intensional partitions satisfying

these definitions may be developed by following Glashoff and treating node labels as designating non-empty sets which conform with the inclusion and exclusion inference patterns and whose members represent atoms of intensionality.

Such sets may be subsumed within the protologic by representing their members as additional nodes with links that have not been explicitly specified as part of the net. The acceptance that node labels may designate implicit links provides an intensional semantics in terms of the explicit and implicit structure of a net that explicates notions of meaning and content within the protologic without introducing additional constructions. In particular, it recognizes that nodes may have unspecified common content.

5.2.2. Intensional partitions with coexclusion. Coexclusion adds a third link type to the kernel protologic and the intensional partition of a net may be extended to encompass this by defining a third set, $\omega(X)$, characterizing the coexclusion links of node X such that:

$$\omega(X) = \{Y\colon X \twoheadrightarrow Y\}$$

The net can be recovered by the merger of the node sub-graphs, equivalent to adding the recovery of coexclusion links through:

$$A \twoheadrightarrow B \text{ iff } B \in \omega(A)$$

For a net in maximal canonical form, the set relations deriving from the inference pattern for each connective are:

$$\text{if } A \rightarrow B \text{ then } s(A) \supseteq s(B),\ \sigma(A) \supseteq \sigma(B),\ \omega(B) \supseteq \omega(A)$$
$$\text{if } A \dashv\!\vdash B \text{ then } s(A) \supseteq \omega(B),\ s(B) \supseteq \omega(A),\ s(A) \cap \sigma(B) \neq \varnothing,\ \sigma(A) \cap s(B) \neq \varnothing$$
$$\text{if } A \dashv\!\dashv B \text{ then } \omega(A) \supseteq s(B),\ \omega(B) \supseteq s(A),\ s(A) \cap \omega(B) \neq \varnothing,\ \omega(A) \cap s(B) \neq \varnothing$$

The reversal of the direction of inclusion for ω in the first set means that, with the addition of coexclusion, there is no longer a simple content containment interpretation of $\rightarrow$ as there was for the kernel protologic. The duality of exclusion and coexclusion is apparent in the last two sets of conditions.

The reverse inclusion of ω raises some interesting issues for the notion of content containment and the nature of negation. The reversal could be interpreted as a failure of meaning containment because B may have coexclusion links that A does not. However, if we define the set $\overline{w}$ as the complement of the set ω relative to the set of all nodes, then we have:

$$\text{if } A \rightarrow B \text{ then } s(A) \supseteq s(B),\ \sigma(A) \supseteq \sigma(B),\ \overline{w}(A) \supseteq \overline{w}(B)$$

That is, the set of coexclusion links *absent* in the content of A includes the set of those *absent* in the content of B.

The triple of sets may be reduced to a single set by using a bar to distinguish members of $\sigma(A)$ from those of $s(A)$, a double bar to distinguish members of $\overline{w}(A)$, and merging the three sets to represent the *content* of A, $c(A)$. One can recover the links as:

$$A \rightarrow B \text{ iff } B \in c(A)$$
$$A \dashv\!\vdash B \text{ iff } \overline{A} \in c(B) \text{ or } \overline{B} \in c(A) \text{ (exclusion inconsistency)}$$

$$A \dashv\!\vdash B \text{ iff } \overline{\overline{A}} \notin c(B) \text{ or } \overline{\overline{B}} \notin c(A) \text{ (coexclusion inconsistency)}$$

The use of absence in order to preserve the content containment model of derivation raises significant philosophical issues [77, 337] but these are not the topic of this article. The relevant logical issue is that this analysis could not even be contemplated unless negation were factored into its exclusion and coexclusion components. This suggests that the known issues of incorporating negation in logics based on the notion of content containment [110, 65] would bear reanalysis in terms of this factoring rather than treating it as a unitary notion.

The exclusion/incompatibility component of negation is not problematic. It is the coexclusion component from which issues arise. As already discussed (§2.6.4) hypothesizing an ideal element that is the common content of every node incompatible with a given node can be so problematic as to suggest the hypothesis itself should be avoided, or managed to be innocuous. From this perspective a focus on the *absence* of coexclusion links in the notion of content appears natural.

5.2.3. Contextual partitions. If the node characterized is the head node of all the links, the partition might be best termed *contextual*. It can be conceptualized as representing the content of the common node in terms of those of the nodes enclosed by its context, each of which includes that content.

The nodes in the partition with inclusion links to the node being characterized were at one time termed its *extension* [132], the particular nodes included in it. In modern logic, the term 'extension' has become reserved for the 'individuals' enclosed by a node's context, and §5.2.5 discusses the reconstruction of a net from that part of the context of each node that constitutes its 'extension' in the modern sense of the word.

Thus, a net from the kernel protologic can be reconstructed from its nodes characterized either intensionally by the nodes whose content it includes or excludes, or contextually by the nodes included or excluded by its context. Such reconstructions are theoretically simple, depending only on the reconstruction of a multiset from a partition that covers it, but they play important roles in theories of logical structures.

5.2.4. Other full partitions, generic design patterns, semantic fields. Nets can also be reconstructed from partitions into larger fragments that may be viewed as having semantic coherence, for example, *constructs* constituted by a triple of nodes with an exclusion link included in a common node as discussed in §3.3.5. Such partitions into generic semantic components are also important in envisioning nets in terms of *design patterns* [145, 54]. Examples of significant generic schemata commonly used to partition a net are provided in §4.

While, from a logical perspective, content and context partitions are each sufficient to reconstruct a net, they provide different constraints on the elusive notion of the 'meaning' of a particular node. Their different roles are a matter of long-standing debate. For example, Plato [282, 71d3-72d1] has Socrates chastise Meno for providing only examples of 'virtue' when he is looking for one notion

of the essence of virtue, not a swarm (σμῆνός) of them. Aristotle [18, 1259b21-1260a31] comments that the enumeration of examples can be better than abstract definitions because the abstractions are instantiated in different ways according to the nature of the substances that instantiate them.

Linguistic field theory [222, 265] combines the context and context perspectives to characterize a node in terms of *all* its links to other nodes. It derives from the structuralist analyses of word meaning of Humboldt, Saussure, Trier and others, who propose that a single word can be understood, and has meaning, only within the *semantic field* of its relations to other words.

Field theory corresponds to the meaning of a node being constrained by, but not defined by, its links with other nodes. How extensive the surrounding net appropriate to characterize a node in this way is an empirical question. Too local a neighbourhood may not capture significant connotations but one that is too wide may include some that are not normally relevant.

Trier's [357] analysis of semantic fields has been evaluated in studies of the correspondence between such links between words having similar meanings in different languages [305]; that is, by determining the extent of the logical invariance of a net when the node names are translated into different natural languages.

Structuralist linguistics has been criticized for its neglect of reference [106, ch.13], that it focuses only on the relations between words rather than on how those words are used to refer to entities. Many issues of reference are clearly extra-logical, psychological, social and pragmatic. The modeling of how we come to be able to use a word in ways that others would regard as meaningful involves not only the logical constraints representable in a net but also experiential issues that are extra-logical.

Fiction writers dramatize the phenomena associated with coming to link a word with relevant experience, such as having a variety of emotions and behaviours associated with being with another person and having a *expert speaker* [294] explain that you are 'in love.' This corresponds to an anonymous node whose links represent a significant experience becoming classified and labeled, with further links associated with the associated 'substance templet' [254] being inferred.

In a scientific situation, Shubin [329, 63-66] provides an account of trekking for months alongside an expert in a fossil-rich desert attempting to link the word 'fossil' to his visual perception, to 'see bones.' Verbal explanations of what he should try to see were of little help; only prolonged experience provided the referential skill. This corresponds to a named node becoming linked to a pattern of non-verbalizable experience.

Semantic analysis in the protologic, and in modern logic in general [277], is inherently structuralist in nature, capturing significant constraints upon the meanings of abstract entities that derive from their locations in a network of logical relations with other such entities, but leaving open major issues that are essentially extra-logical.

5.2.5. Standard extensional semantics. Extensional semantics in which a logical structure is characterized through the set-theoretic relations between the sets of individuals providing models that are consistent with that structure play a central role in modern logic. They can be studied in the protologic without introducing an additional construct to represent an 'individual' by noting that the only feature required in a suitable entity is that it be impredicable or non-instantiable [160], that is, specified to be a bottom node in the protologic.

Thus, the question becomes one of whether a sufficient collection of bottom nodes may be used to reconstruct the inclusion and exclusion links between the non-bottom nodes in a net. Probes were introduced in §3.3.7 as bottom nodes added to investigate a net from outside in order to infer its internal structure as constituted by the links between its nodes.

Extensional semantics for syllogistic systems with essentially the same connectives as inclusion and exclusion links have been developed in depth by several authors [92, 332, 86, 246] and the techniques and results are applicable to the protologic. The focus in this section is on characterizing sets of probes sufficient to reconstruct a consistent net, and in the following on the need to take into account paraconsistency, that the net may be inconsistent.

The standard result is that, for a consistent net in the kernel protologic, a set of consistent full probes that *saturates* the net by including all possible distinct examples of such probes, may be used to reconstruct the inclusion and exclusion relations in the net [246]. If πA is the set of probes that include node A, then:

$$A \rightarrow B \text{ iff } \pi A \subseteq \pi B$$
$$A \dashv B \text{ iff } \pi A \cap \pi B = \varnothing$$

The algorithm is basically that of §3.3.7 illustrated in Table 1 since $\pi A \cap \pi B = \varnothing$ is equivalent to $\pi A \subseteq \underline{\pi} B$ if the net is consistent (since $\underline{\pi}$ is then the complement of π relative to the set of admissible probes).

Thus providing extensional semantics for the kernel protologic does not require the addition of *individuals* as primitive entities to the protologic. Probes satisfy Gracia's logical criterion for individuality and could be termed *proto-individuals*. They need not satisfy Strawson's [345, p.214] criteria that they be distinct and reference some other entity.

Abstract extensional semantics derive from the capability to reconstruct part of a net from the constraints it induces in a subnet. They are intrinsic to the protologic and involve no additional connotations or denotations.

Coexclusion relations may be reconstructed in terms dual to those for exclusion relations: $A \dashv\!\dashv B$ iff $\underline{\pi} A \cap \underline{\pi} B = \varnothing$, which is equivalent to $A \dashv\!\dashv B$ iff $\underline{\pi} A \subseteq \pi B$ if the net is consistent.

The definitions of structural connectives in §7 are such that they also have extensional semantics in terms of probes but the algorithms for reconstructing them have greater complexity as they require combinatorial search rather than computation of simple inclusion relations between sets.

Standard extensional semantics require the net to be consistent but can be extended to take into account the paraconsistency of the protologic and reconstruct inconsistent nets (§5.2.6).

5.2.6. Extensional semantics with coexclusion and paraconsistency. If the net is inconsistent, there will be one or nodes where the associated set of consistent nodes is empty and the basic algorithm would infer that such nodes include and exclude all others, that is, the structure of the links leading to inconsistency would be lost. The algorithm can be extended to recover the full structure from the set of admissible probes some of which may be inconsistent (§3.3.7).

The inference of A→B from the subset relation carries over, but that of A⊣B and A⫤B from the disjoint relations does not; inconsistency arises because a node has been specified that violates the exclusion or coexclusion link constraints. However, these relations can be inferred through a slight extension to the definitions (noting that $\underline{\pi}$, the set of probes that exclude a node, is no longer the complement of π in inconsistent nets). The extended reconstructions are:

$$A{\to}B \text{ iff } \pi A \subseteq \pi B \text{ and } \underline{\pi} B \subseteq \underline{\pi} A$$
$$A{\dashv}B \text{ iff } \pi A \subseteq \underline{\pi} B \text{ and } \pi B \subseteq \underline{\pi} A$$
$$A{\dashv\!\!\dashv}B \text{ iff } \underline{\pi} A \subseteq \pi B \text{ and } \underline{\pi} B \subseteq \pi A$$

For consistent nets, where $\underline{\pi}$ is the complement of π, these definitions reduces to those of §5.2.5 that A→B iff $\pi A \subseteq \pi B$, A⊣B iff $\pi A \cap \pi B = \varnothing$ and A⫤B iff $\underline{\pi} A \cap \underline{\pi} B = \varnothing$.

It is necessary to use the conjunction of two conditions because, for example, if a node X is exclusion inconsistent there is an exclusion link from every probe and $\underline{\pi}$X would include πY for any Y. In addition, a pseudolink between inconsistent nodes (§2.6.5) has no logical effect and needs to be highlighted since it would, for example, join two inconsistent nets that were otherwise disjoint.

As an illustration of the reconstruction algorithm, consider the set of admissible probes (Table 3) for the net of Figure 22, the last two probes of which are inconsistent, one having a positive link to C and another to D. Table 4 shows the π and $\underline{\pi}$ sets of probes associated with each node.

TABLE 3. Admissible probes for the net of Figure 22

	A	B	C	D	E	F
1	↠	↠	↠	↠	↠	↠
2	↠	↠	↠	↠	→	↠
3	↠	↠	↠	↠	↠	→
4	↠	↠	↠	↠	→	→
5	↠	→	↠	↠	↠	→
6	↠	→	↠	↠	→	→
7	→	↠	↠	↠	↠	↠
8	→	↠	↠	↠	↠	→
9	↠→	↠→	↠→	↠	↠	→
10	↠→	↠→	↠→	↠→	↠	→

TABLE 4. Admissible probes associated with each node

	π	$\underline{\pi}$
A	{7,8,9,10}	{1,2,3,4,5,6,9,10}
B	{5,6,9,10}	{1,2,3,4,7,8,9,10}
C	{9,10}	{1,2,3,4,5,6,7,8,9,10}
D	{10}	{1,2,3,4,5,6,7,8,9,10}
E	{2,4,6}	{1,3,5,7,8,9,10}
F	{3,4,5,6,8,9,10}	{1,2,7}

The subset relations for π and $\underline{\pi}$ reconstruct the maximal canonical form of the net of Figure 22 as shown in Table 5.[32]

TABLE 5. Extensional reconstruction of links for the net of Figure 22

A	(+,B), (← +,C), (← +,D), (+,E)
B	(+,A), (← +,C), (← +,D), (→,F)
C	(→ +,A), (→ +,B), (+,C), (← +,D), (+,E), (→,F)
D	(→ +,A), (→ +,B), (→ +,C), (+,D), (+,E), (→,F)
E	(+,A), (+,C), (+,D)
F	(←,B), (←,C), (←,D)

To illustrate the reconstruction of a more complex net having several unrelated inconsistent nodes, the net shown in Figure 79 extends that of Figure 22 with an additional inconsistent net and specifies an exclusion link between the inconsistent nodes D and S.

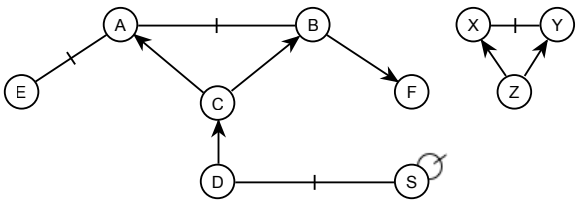


FIGURE 79. Reconstruction of a net with unrelated inconsistent nodes

Table 6 shows the extensional reconstruction of the links in the net of Figure 79. All the specified and inferred links are correctly reconstructed except that the redundant exclusion link specified between D and S cannot be discriminated from a pseudolink.

The reconstruction algorithm is also effective with nets that include coexclusion links. For example, Figure 80 is a labeled version of the net of Figure 45. The admissible probes are shown Table 7, the sets of probes associated with each node in Table 8, and the reconstructed links in Table 9.

[32]The computer implementation of the algorithm does not show the inclusion link from a node to itself and appends an asterisk to a possible pseudolink.

TABLE 6. Extensional reconstruction of links for the net of Figure 79

A	(+,B), (← +,C), (← +,D), (+,E)
B	(+,A), (← +,C), (← +,D), (→,F)
C	(→ +,A), (→ +,B), (+,C), (← +,D), (+,E), (→,F), (+*,S), (+*,Z)
D	(→ +,A), (→ +,B), (→ +,C), (+,D), (+,E), (→,F), (+*,S), (+*,Z)
E	(+,A), (+,C), (+,D)
F	(←,B), (←,C), (←,D)
S	(+*,C), (+*,D), (+,S), (+*,Z)
X	(+,Y), (← +,Z)
Y	(+,X), (← +,Z)
Z	(+*,C), (+*,D), (+*,S), (→ +,X), (→ +,Y), (+,Z)

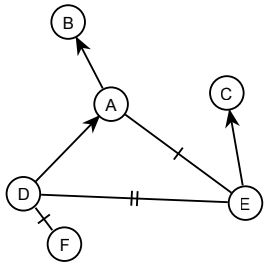


FIGURE 80. Reconstruction of a net having exclusion links

TABLE 7. Admissible probes for the net of Figure 80

	A	B	C	D	E	F
1	↠	↠	→	↠	→	↠
2	↠	↠	→	↠	→	→
3	↠	→	→	↠	→	↠
4	↠	→	→	↠	→	→
5	→	→	↠	→	↠	↠
6	→	→	→	→	↠	↠

TABLE 8. Admissible probes associated with each node

A	{5,6}	{1,2,3,4}
B	{3,4,5,6}	{1,2}
C	{1,2,3,4,6}	{5}
D	{5,6}	{1,2,3,4}
E	{1,2,3,4}	{5,6}
F	{2,4}	{1,3,5,6}

The examples of reconstruction in this section have all been based on the set of all distinct admissible full probes but the set used can be any set of compatible full probes that includes all those admissible. It is also possible to generate sets of partial probes that are adequate for reconstruction but these sets are generally larger because each full probe carries more information.

TABLE 9. Extensional reconstruction of links for the net of Figure 80

A	(→,B), (⊣,C), (—,D), (⊣⊣,E), (⊢,F)
B	(←,A), (⊣,C), (←,D), (⊣,E)
C	(⊣,A), (⊣,B), (⊣,D), (←,E), (←,F)
D	(—,A), (→,B), (⊣,C), (⊣⊣,E), (⊢,F)
E	(⊣⊣,A), (⊣,B), (→,C), (⊣⊣,D), (←,F)
F	(⊢,A), (→,C), (⊢,D), (→,E)

5.2.7. Incompatibility semantics. Brandom [68] has proposed a general *incompatibility semantics* for logical systems in which the entailment relation is reconstructed from that of incompatibility. Brandom and Aker [69] have developed this formally, and Peregrin [277] has compared their framework with Koslow's [206] structuralist one, showing that the two approaches are closely related and provide similar foundations for a wide range of logics.

For the protologic this raises the question of whether the inclusion links may be reconstructed from the exclusion links and, since this is not feasible in general, what features of a net might make it possible.

The reconstruction of the inclusion links from the exclusion links in a consistent net is simple provided an *incompatibility saturation condition* is satisfied, that every node has at least one exclusion link specified that cannot be inferred from other links. An inclusion link can then be inferred from node A to node B if the set of nodes to which A has exclusion links, σA, contains that to which B has exclusion links, σB. That is, A→B iff σA ⊇ σB.

Figure 81 will be used to illustrate this process for the simple net of Figure 50. An additional node. D, with an exclusion link to the top node has been added to satisfy the incompatibility saturation condition.

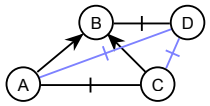


FIGURE 81. Reconstruction of inclusion links from exclusion links

The exclusion sets are:

$$\begin{aligned} \sigma A &= \{C, D\}, \\ \sigma B &= \{D\}, \\ \sigma C &= \{A, D\}, \\ \sigma D &= \{A, B, C\}, \end{aligned}$$

allowing the two specified inclusion links, and no spurious ones, to be reconstructed.

In an inconsistent net, even though the definition of the exclusion link has been violated, the exclusion inference pattern still allows the inclusion links to be reconstructed. Figure 82 will be used to illustrate the reconstruction of the inconsistent net of Figure 22. Additional nodes, G and H, have been added with

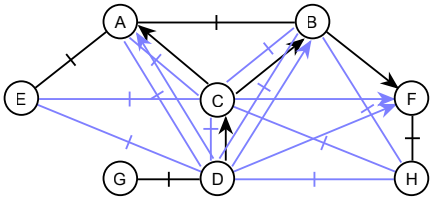


FIGURE 82. Reconstruction from exclusion links when net is inconsistent

exclusion links to F and D, respectively, in order to satisfy the incompatibility saturation condition.

Taking into account the exclusion link from an inconsistent node to itself, the exclusion sets are:

$$\begin{aligned}
\sigma A &= \{B, C, D, E\},\\
\sigma B &= \{A, C, D, G\},\\
\sigma C &= \{A, B, C, D, E, G\},\\
\sigma D &= \{A, B, C, D, E, G, H\},\\
\sigma E &= \{A, C, D\},\\
\sigma F &= \{G\},\\
\sigma G &= \{B, C, D, F\},
\end{aligned}$$

allowing the eight specified/inferred inclusion links, and no spurious ones, to be reconstructed.

The role of the saturation condition is apparent since: without the added node G, σF would be empty and it would be inferred that there is an inclusion link from every node to F; without the added node, H, σC and σD would be equal and a mutual inclusion link would be inferred.

5.3. The mereology of content and context

It was noted in §1.2 that describing A→B in terms of content and context as 'the content of A includes the content of B' and 'A is enclosed by the context of B' had converse mereological allusions. The first suggests that some aspect of B is part of A and the second that some aspect of A is part of B.

Both notions are to be found in the logical literature going back to Aristotle [17, 1023b23-25] who, for example, in his analysis of the notion of 'part' in his *Metaphysics*, notes that "the genus is called a part of the species, though in another sense the species is part of the genus."[33]

Aristotle provides rationales for the use of 'part' in these converse senses: that the elements constituting a definition may be regarded as parts of it; and that the elements into which a kind may be divided may be regarded as parts of it. That is, in the terminology defined in §1.2, the contents of the nodes that a node includes are parts of its content, and the contexts of the nodes that are included in it are parts of its context.

Logical models of mereology [78, 184] are founded on an order relation such as that specified by the inclusion link → (§2.4.4). The direction of the *part of* relation

[33] τὸ γένος τοῦ εἴδους καὶ μέρος λέγεται, ἄλλως δὲ τὸ εἶδος γένους μέρος

is well-defined for material objects but not for immaterial ones. If A→B is taken to specify that the *content* of B is part of the *content* of A, then the mereological order is content(B) ≤ content(A). If A→B is taken to specify that the *context* of A is part of the *context* of B, then it is context(A) ≤ context(B).

The exclusion link + may be used together with → to define a mereological model for each order relation. The model for context is simplest, requiring one only spatial region for each node and having 5 distinct configurations of two nodes [152]. That for content requires two regions (§5.2.1) and has 200 distinct configurations of two nodes.

5.3.1. Merelogy of context enclosure in the kernel protologic. Figure 83 explicates context enclosure for the inclusion and exclusion links of the kernel protologic in terms of the constructs of Casati and Varzi's [78] *minimal mereology*. At the top is Hovda's [184] mereological notation for the patterns shown in the protologic, in the middle a description of the spatial relations between the contexts of the nodes, and at the bottom a two-dimensional representation in terms of regions of space.

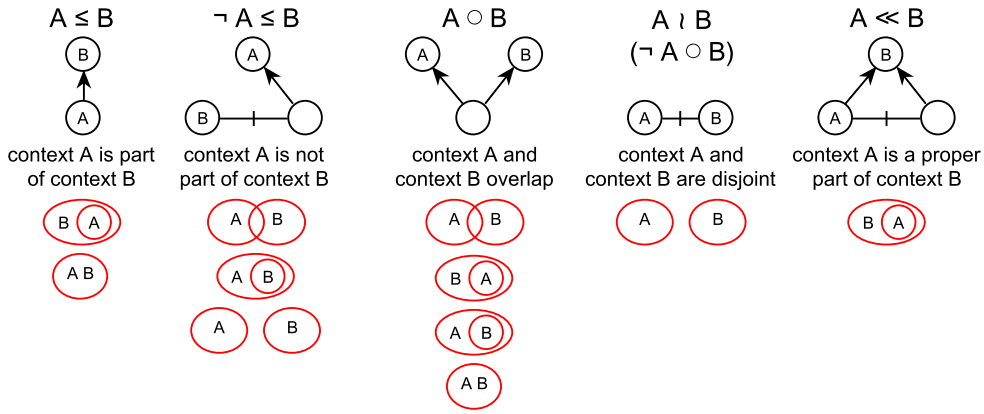


FIGURE 83. Mereological relations of contexts in the kernel protologic

From left to right:-

- the foundational preorder or partial order in ground mereology of being *part of*, A ≤ B, is represented by the inclusion link of the protologic, A→B, that A is enclosed by the context of B;
- the negation of this, that A is *not part of* B, is represented by A having a node enclosed by its context that is incompatible with B;
- the derived mereological notion that A *overlaps* B, A ∘ B, is represented by the contexts of A and B being compatible in having a node enclosed by both contexts;
- the negation of this, that A is *disjoint* from B, A ≀ B, is represented by an exclusion link between A and B, A+B;
- the mereological notion of A being a *proper part* of B, A ≪ B, is captured by *weak supplementation*, that A is enclosed by the context of B but there is

also a node enclosed by that context that is incompatible with A (the same structure as that for *properly particular* in Figure 52 centre).

Other mereological constructions, such as *product*, *sum/fusion* and *complement* also correspond to significant operations in the protologic (§7.3.4).

The spatial representations at the bottom of Figure 83 were used by Euler to explain syllogistic reasoning diagrammatically in his tutorial correspondence with a German princess [**?**, 14-24-Feb-1761]. They were developed formally by Gergonne [152] who added an equivalence relation corresponding to A—B in the protologic (§2.4.1).

The spatial regions shown are *pointless* [198], representing the general topology of regions delimited by closed curves being identical to, enclosed in, overlapping or not overlapping another, rather than specific set-theoretic inclusion relations. Quasi-points may be introduced as minimal regions that cannot have another region inscribed within them, but the geometric points of the diagrams have no status in the logical interpretation.

One problem with drawing such diagrams is that it requires over-commitment to specific spatial relations so that several distinct figures are necessary to represent a single spatial constraint. Venn [362] criticized this and developed his own similar diagrams with all the ovals in general positions where they overlapped and shading of their intersections specified the logical relations.

Neither form of diagram scales up to logical structures with more than a few terms, but they are helpful in presenting simple logical schemata. Bernhard [37] has compared different diagrams of the basic syllogistic relations, including those of Euler and Venn, Frege's [129] *Begriffsschrift* and Peirce's [307] *existential graphs*. There is substantial ongoing research on modern extensions of Euler diagrams, such as *spider diagrams* [104].

5.3.2. Mereology of relevance. Context boxes (§4.1.1), as nodes that can enclose other nodes, provide a graphic representation of the mereology of context analyzed in §5.3.1. This spatial analogy also applies to the notions of *relevance* as being 'within context' and *irrelevance* as being 'out of context.'

It was noted in §2.7 that, when a node is marked false, all nodes within its context are also marked false but that one would normally express this by saying that they have become *irrelevant* rather than that they are simply false. Because nodes within a context box have implicit inclusion links to the context box they will exhibit this behaviour. If the context box is inferred to be false it will be marked black but the nodes within it will be marked grey; they will be 'greyed out' as irrelevant.

Figure 84 illustrates this. The construct 'ill' versus 'well' is specified within the context 'alive,' and the construct 'dead' versus 'alive' within the context 'animal.'

At the top right, 'alive' has been asserted true and it is inferred that 'animal' is true and 'dead is false, leaving open the question of whether the animal is 'ill' or 'well.'

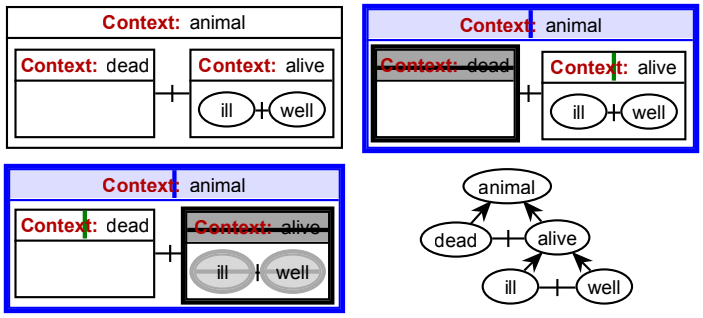


FIGURE 84. Distinguishing irrelevance from simple falsity

At the bottom left, 'dead' has been asserted true and it is inferred that the 'animal' is true and 'live is false, and that 'ill' and 'well' are not just false but irrelevant.

The final net at the bottom right shows the same structure without context boxes to emphasize that these results are logical aspects of the protologic rather than its graphical syntax.

This distinction being made corresponds to the question 'is he well?' becoming irrelevant when we are told someone is dead. It provides a gloss to Aristotle's [13, 13b17-19] remark that if Socrates did not exist then 'Socrates is ill' and 'Socrates is well' would both be false. The assertions are not just false but irrelevant if they are assumed to apply *qua* existent living beings (leaving open the question of whether the 'existence' is physical, fictional or just logically possible).[34]

5.3.3. Mereology of context with coexclusion. In mereological terms a coexclusion link corresponds to partial complementation as shown in Figure 85 left. If a node is not enclosed in the context of node A then it must be enclosed in that of node B.

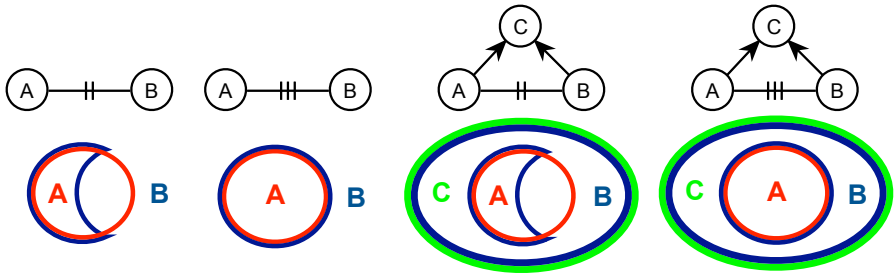


FIGURE 85. Mereology of context coexclusion and negation

The complementation is partial because A and B may overlap. If, as shown second from the left, there is both exclusion and coexclusion then the resultant negation is equivalent to full complementation.

[34]This and other ontological examples are greatly simplified to illustrate features of the protologic. In particular the ontological issues around the 'dead–alive' construct are complex and controversial [188].

The region corresponding to B in both cases is open, encompassing all the space outside B. Restricting the contexts of A and B to be enclosed within the mutual context of node C (Figure 85 right) a constrains the context of B to be a closed region.

Restricting negation by requiring it to be a relative to a context corresponds to the recommendation made by Aristotle to reduce semantic fragmentation (§2.6.4). Mereological complementation is known to introduce spatially fragmented structures through the possibility of creating mereological fusions of disconnected entities [360, §1.3.3].

5.3.4. Mereology of content inclusion in the kernel protologic. Glashoff's intensional model of the syllogistic relations through a pair of sets (§5.2.1) may be used to develop a mereological explication of the inclusion of content in the kernel protologic. As shown in Figure 86 left the content of node A is represented by a pair of regions labeled 'sA' and 'σA.' Given that σA may be a null region but not sA (because a node includes itself), there are two possible consistent configurations in which sA and σA do not overlap, and four possible inconsistent ones in which they do.

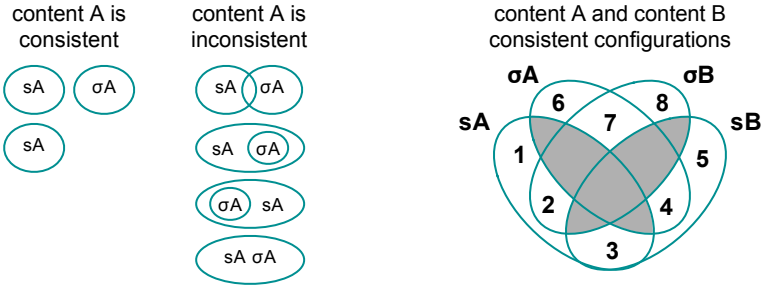


FIGURE 86. Mereology of content consistency in the kernel protologic

Figure 86 right shows a Venn diagram for the content of a pair of consistent nodes. The two shaded areas correspond to inconsistency, and the eight numbered regions being present or absent corresponds to various logical relations between the nodes.

This diagram may be used illustrate and generate all the possible spatial configurations of content for a pair of consistent nodes in the kernel protologic. There is a general constraint that sA and sB must not be null, corresponding to regions 1, 2 or 3, and 3, 4 or 5, being present, allowing 25 possible sub-configurations. Any of regions 6 through 8 may be present, allowing 8 possible sub-configurations. Thus, taking both into account, there are 200 possible spatial configurations of the content of two consistent nodes.

Figure 88 shows consistent configurations for the same nets as those shown in Figure 83. Considering the nets in terms of the regions of the Venn diagram, from the left: 3 must be present and 2, 4, 5 and 8 must be absent for A to include B; this condition must fail for A not to include B; 2 and 4 must be absent for A to

be compatible with B; this condition must fail for A to be incompatible with B; 3 must be present, 2, 4, 5, 8 must be absent and 1 or (6 and 7) must be present for A to properly include B.

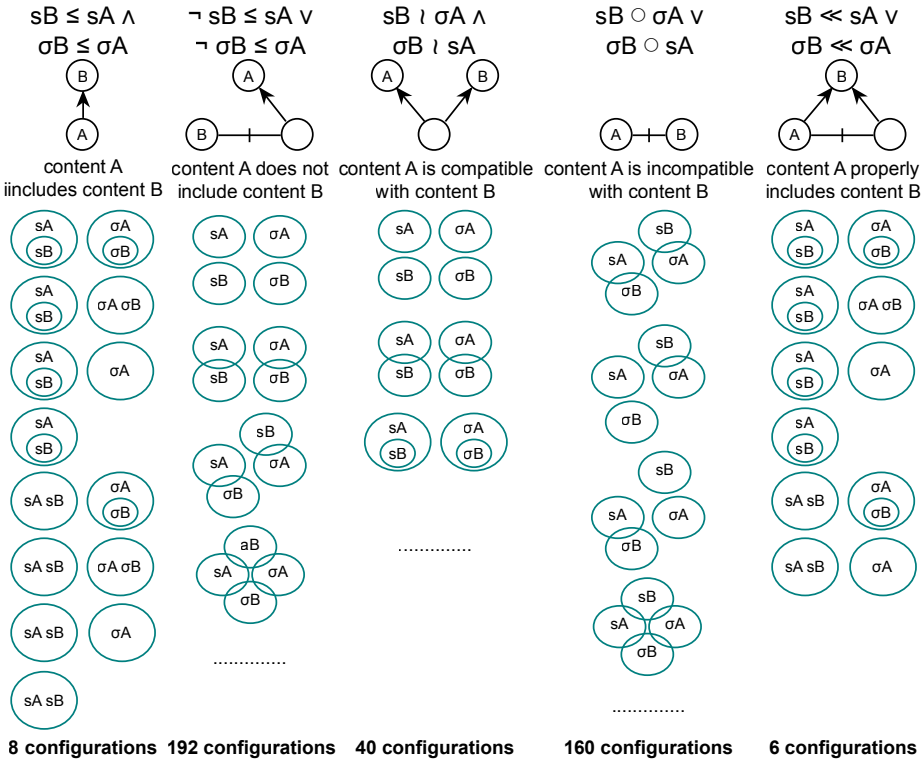


FIGURE 87. Mereological relations of contents in nets

At the top of Figure 88 are the mereological constraints corresponding to the nets, the first four of which are Glashoff's [155, §3] definitions in Hovda's [184] notation. On the left are the 8 possible configurations for content inclusion, and on the right the 6 for proper inclusion. Only a few distinctive exemplars are shown for the other nets where there are a large number of possible configurations,

Thus, there is a mereological model for content inclusion/exclusion or *meaning containment* having a spatial representation similar to that of Euler diagrams for context enclosure. However, it is substantially more complex and even more subject to Venn's criticism of requiring premature commitment to spatial configurations, although the diagrams can be useful in illustrating Glashoff's intensional analysis. The equivalent Venn diagram and the mereological definitions are also useful technically but less perspicuous.

5.3.5. Mereology of content with coexclusion. The intensional partition of nets with coexclusion links developed in §5.2.2 supports a mereological of content for

the kernel protologic extended with coexclusion. However, the complexity of the diagrams is greatly increased due to the combinatorial possibilities of σ and ω overlapping to represent negation links. For example, Figure 88 shows the eight consistent configurations for a single node with possible coexclusion links compared with the two (Figure 86) for a single node in the kernel protologic.

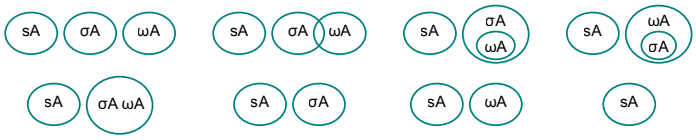


FIGURE 88. Mereology of content consistency with coexclusion

Figure 89 shows a six-set Edwards-Venn diagram [116, ch.4] for the intersections of the three sets providing an intensional partition of each of two consistent nodes, A and B. It comprises: a bottom rectangle for s(A); a right rectangle for s(B); a circle for σ(A); a barbell for σ(B); a four-toothed cogwheel for ω(A); an eight-toothed cogwheel for ω(B). The 39 shaded areas correspond to inconsistency, and the 25 numbered regions being present or absent corresponds to various logical relations between the nodes.

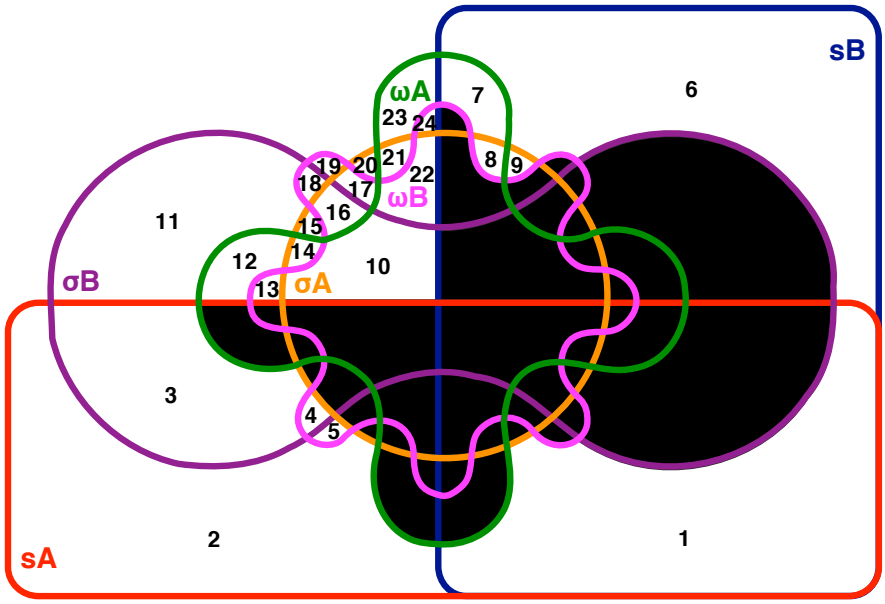


FIGURE 89. Mereological relations of contents in nets with coexclusion

This diagram may be used illustrate and generate all the possible spatial configurations of content for a pair of consistent nodes in the kernel protologic with coexclusion. There is a general constraint that sA and sB must not be null, corresponding to regions 1, 2, 3, 4 or 5, and 1, 6, 7, 8 or 9, being present, allowing 481

possible sub-configurations. Any of regions 10 through 24 may be present, allowing 32,768 possible sub-configurations. Thus, taking both into account, there are 15,761,408 possible spatial configurations of the content of two consistent nodes.

It is possible to enumerate the number of configurations corresponding to the basic logical connectives, for example 1024 corresponding to A→B. However, all that is apparent is that Euler-style diagrams for the spatial relations between the sets characterizing intensional partitions of a net with coexclusion are theoretically possible but practically intractable.

The mereological diagrams for both content and context also provide useful representations of the constraints introduced to represent structural connectives (§7.3.4).

6. Some significant translations

This section presents three significant translations of the protologic to other logical systems, such as description logics, the metalogic of the kernel protologic, and categorical syllogistics.

6.1. Translation to description logics

Description logics are fragments of first-order logic with known computational complexity, implemented inference algorithms and having well-defined semantics [20] that are widely used in practical applications, notably the *semantic web* [325]. Nets may be used to provide a description logic formal language with a graphical syntax similar to that of many *semantic networks* that is readily translation to standard linear expressions for description logics [140].

Nets representing structures in the protologics may be similarly converted to description logic expressions which, because they do not involve quantifiers, provide a compact logical translation of the nets in a form that is readily understood and is exportable to description logic inference engines.

Figure 90 shows two nets drawn from §2.7 and §2.6.7 in order to illustrate description logic translations.

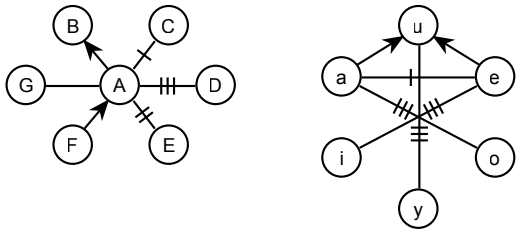


FIGURE 90. Translation of nets to description logic syntax

The net on the left exemplifies all possible links in the kernel protologic with coexclusion and translates to:

$$(F \sqcup G \sqcup \neg D \sqcup \neg E) \sqsubseteq A \sqsubseteq (B \sqcap G \sqcap \neg C \sqcap \neg D)$$
$$A \sqsubseteq G \sqsubseteq A$$
$$\neg A \sqsubseteq D \sqsubseteq \neg A$$
$$\neg A \sqsubseteq E$$
$$F \sqsubseteq A$$
$$A \sqsubseteq B$$
$$C \sqsubseteq \neg A$$

The disjunctive clause on the left of the first line is only a convenient syntactic grouping of those nodes that each imply A, a protodisjunction which nothing implies. Similarly, the clause on the right is only a convenient syntactic grouping of those nodes that are each implied by, a protoconjunction which implies nothing.

The translator can also represent the equivalences between A, G and ¬D in the first line more directly as:

$$(F \sqcup \neg E) \sqsubseteq A \equiv D \equiv \neg\ G \sqsubseteq (B \sqcap \neg C)$$

On the right a minimal canonical form of Blanché's hexagon §2.6.7) translates to:

$$(a \sqcup e) \sqsubseteq u \equiv \neg y$$
$$\neg i \equiv e \sqsubseteq (u \sqcap \neg a)$$
$$\neg o \equiv a \sqsubseteq (u \sqcap \neg e)$$

Additional and more complex examples of description logic translations are provided in §3.3.3, §7.1.2, §7.1.3 and §7.1.3. There are also translations from the description logic expressions to various inference engine knowledge interchange formats, first order predicate calculus and modal logic [20].

6.2. Metalogic of nets: metanets, metanodes and metalinks

The kernel protologic with coexclusion may be used to define a metalogic for the logical relations between nets by providing interpretations of such relations that conform with the definitions of inclusion, exclusion and coexclusion.

A *metanet* is a net comprised of *metanodes*, nodes whose labels are nets, and *metalinks*, links representing the metalogical relations between these nets. The nets considered are generally fragments of some larger net, *maximal subnets* such that all links between their nodes are present, or may be inferred from, the subnet.

Inclusion metalinks: a metanode representing some maximal subnet is defined to include another if the second net is part of a net logically equivalent (§2.2) to the first net, such as an ultra-conservative expansion of the maximal canonical form (§3.1) of the first net. If all the nodes in the second net are named, the part-of relation is simply defined as the links in the second net being a subset of those in the first net, or one logically equivalent to it. If the second net has anonymous nodes then this definition is extended such that one net includes another if there is a mapping of the anonymous nodes in the second net to any nodes in a net that is logically equivalent to the first net such that the subset criterion applies.

The inclusion link defined by the part-of relation complies with the inclusion repetition (§2.3), inclusion (§2.4) and loop (§2.4.2) inference patterns. It specifies

a partial order over nets (§2.4.4) that provides the foundations for a metalogic of nets based on the protologic represented in those nets.

Figure 91 exemplifies some inclusion metalinks from the subnet A→B. The many inclusion links between the other subnets in the metanet are not shown. The three metanodes to the left illustrate a sequence of derivations where that at the bottom derives from A→B by an ultra-conservative single node expansion pattern (§3.3) adding an anonymous node that includes A, that above it adds an inclusion link to B deriving from the inclusion inference pattern, and that above it deletes the inclusion link from A to B. The first two metanodes are metalogically equivalent to A → B but the third is just metalogically included in it.

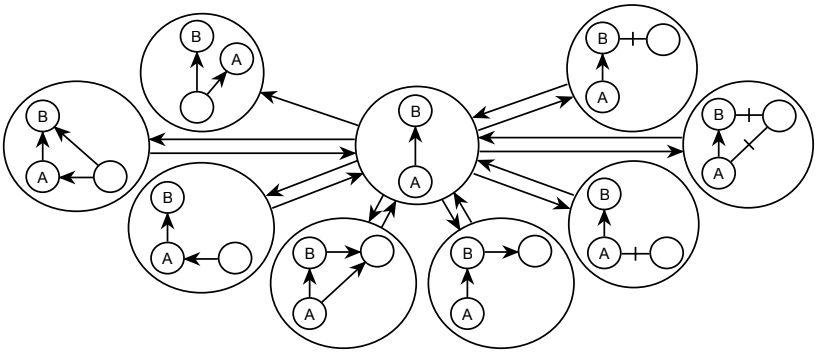


FIGURE 91. Examples of inclusion links between nets

Exclusion metalinks: a metanode representing some maximal subnet is defined to exclude another if there is no net that includes both and has no inconsistent links not present in, or inferable in, either net alone; that is, if merging the nets labelling two metanodes does not result in additional inconsistency.

This definition satisfies the exclusion repetition, exclusion and inclusion symmetry inference patterns (§2.5.1). It also enables inconsistency between metanodes to be defined independently of the consistency of the nets labelling them, thus supporting paraconsistency at both the logical and metalogical levels.

Figure 92 exemplifies some metanodes with exclusion metalinks to the subnet A→B together with the primary inclusion links between these metanodes. The sequence of four nets commencing with that on the left shows the inference of a metanode having an exclusion link between nodes A and B, A↠B, from one having an exclusion link between nodes that they include. The continuing sequence to the metanode A↞ ↠B at the top shows the inference of a weaker metanode inconsistent with A→B where the inconsistent link is in the net being merged rather than between A and B.

Coexclusion metalinks: a metanode representing some maximal subnet is defined to coexclude another if any metanode that excludes the first must include the second.

The definitions of metalogical inclusion and exclusion in terms of relations between the nets labelling the metanodes ensures that metalogical coexclusion is

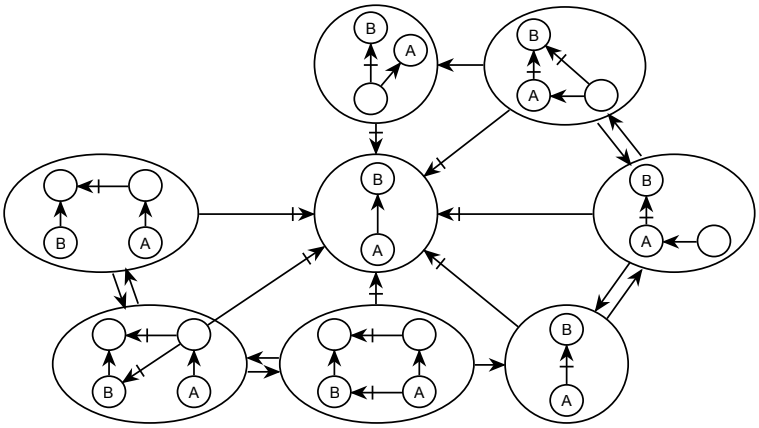


FIGURE 92. Examples of exclusion links between nets

always associated with exclusion and hence the link is necessarily one of negation (§7.1.1), that is *contradiction* in metalogical terms.

Disallowing metalogical contradictions leads to the *reductio per impossibile* inference pattern [192]: if adding the metalogical contradictory of a maximal net to another maximal net allows a new inconsistent link to be inferred then the first net may be inferred to be implicitly or explicitly part of the second net. An example of such reasoning is provided in §6.3.3.

The metanode A← ↠B at the top of Figure 92 is included by all the other metanodes that exclude the metanode A→B and hence one might expect it to have a negation metalink with that metanode. This relation was derived in §3.3.8 in that from the necessary absence, or inadmissibility, of the probe P3: A← ↠B may be inferred the necessary presence of the link A→B.

The metanode A← →B at the top of Figure 91 and A↠B are also be indicated to have a negation metalink through a similar sequence to that of Figure 91, and proved to have this relation because the exclusion link was defined to be so in §2.5. This also corresponds to the necessary presence of the link A↠B being inferred from the necessary inadmissibility of the probe P1: A← →B.

6.2.1. Square and hexagon of oppositions between nets. The metalinks between the four metanodes, A→B, A↠B, A← →B and A← ↠B, generate a metalogical square of oppositions (§2.6.6) as in Figure 93 left.

Rather than the simple ovals used for the previous diagrams, the metanodes are distinguished by a rectangular shape enclosing the associated net and tagged to be of type 'Meta.' The ovals were used to emphasize that the metalogic uses, at the meta level, the protologic already developed. However, it is useful to distinguish metanodes that are labelled by their nets, and whose specified links represent relations between those nets, from simple nodes whose labels are lexical and whose links are separately specified. The lexical labels, such as 'BaA,' of the metanodes

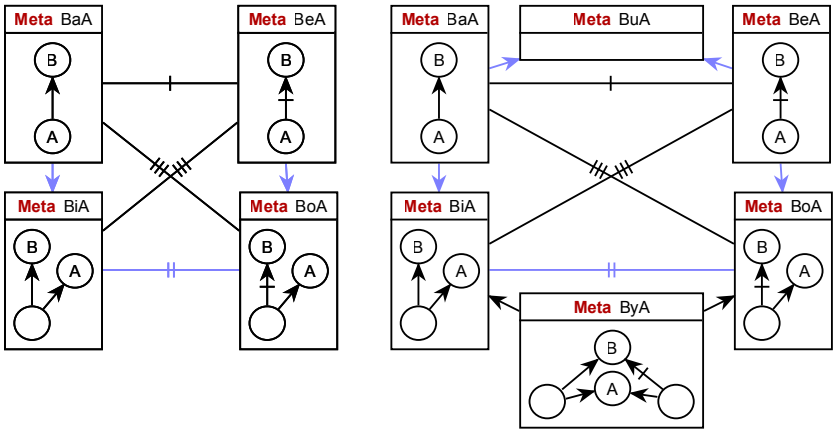


FIGURE 93. Square and hexagon of oppositions between nets

follow a syllogistic naming convention (§6.3.1) and are treated in the metalogic as aliases for the metanodes.

The square generalizes naturally to a Blanché hexagon (§2.6.6) when a metanode B← →A← ↠B merging A→B and A↠B is added at the bottom that includes the two bottom nodes of the square (Figure 93 right). This metanode, labelled 'ByA' in Blanché's terminology, is interesting as a representation of the process of diaeresis in which the genus A is divided by the differentiae constituted by B and a contrary.

However, there is no net representing the negation of this metanode, that is a net representing the impossibility of B providing differentia for A. The Blanché contradiction link is shown to a metanode without an associated net at the top, labelled 'BuA' in his terminology, which is constrained only by its negation link to that at the bottom. Thus, in the metalogic there may be ideal metanodes having well-defined logical relations to metanodes that represent nets in the kernel protologic but which do not themselves correspond to nets in that protologic.

6.3. A syllogistic interpretation

A simple and natural interpretation of the kernel protologic is provided by the categorical syllogistic of Aristotle's [15] *Prior Analytics* that both codified the argument forms of Socratic discourse [201, 85] and provided the framework for Aristotle's demonstrative science [124] and metaphysics [125]. Whilst syllogistic research declined as the mathematical logic frameworks of Hilbert, Bernays, Frege, Russell, Łukasiewicz, Tarski, *et al*, developed, it continued to evolve within those frameworks, and there has been a resurgence of technical developments in recent years that are relevant to the protologic.

The pioneers of modern logic provided translations of the syllogistic into the new formalisms, often extending it as they did so. Lewis' [229] 1918 *Survey of Symbolic Logic* details the algebraic representations and relational extensions of

the syllogistic by those often regarded as developing the precursors of modern symbolic logic, De Morgan, Boole and Peirce.

Frege [129, §6] represented Aristotle's inferential patterns (*Die Aristotelischen Schlussweisen*) in his 1879 *Begriffsschrift*. Bernays [33, p.369] in a tutorial address on theoretical logic in 1927 remarks that Aristotle's principles of inference constitute one of the most significant intellectual achievements, one of the few that form a permanent legacy to the fields of philosophical knowledge,[35] but are incomplete (*unabgeschlossen*).

Bernays [33, p.370] provides an interesting translation of the syllogistic relations to Hilbert's notation:

$$\text{a: } (x)\ P(x) \qquad \text{e: } (x)\ \overline{P(x)} \qquad \text{i: } (Ex)\ P(x)) \qquad \text{o: } (Ex)\ \overline{P(x)}$$

which, if the quantifiers were taken as ranging over the subject domain and P over the predicate domain, would provide a sound interpretation of the syllogistic in many-sorted logic [333, 271]. However, the formalization of many-sorted logic was still a decade away [318] and Bernays did not further develop his analysis of the syllogistic.

Łukasiewicz [238, §9-10] in his 1929 book on mathematical logic presents a translation of the syllogistic to Polish notation, and Bochenski [55] uses this to analyze various axiomatizations of the syllogistic. Eaton's [113, p.2] 1931 *General Logic* spans both syllogistic and mathematical logic, and emphasizes the continuity between ancient and modern logic, presenting Aristotle as developing the first "formal analysis of inferential relationships" but codifying only a limited selection of the "validating forms" of mathematical logic. In the current era Corcoran [94] terms these forms *panvalid patterns* and Boger [61] adopts this terminology in his detailed historical reconstruction of Aristotle's logic.

Technical interest in the syllogistic resurged in 1950s with Łukasiewic's [239] and Patzig's [273] translations of the syllogistic primitives as axioms for relations in first-order logic. This was followed by Corcoran [92, 93], Smiley [332], Clark [86] and Martin's [246] reframings of the syllogistic as a natural deduction system, Andrade and Becerra's [6] study of translations between Łukasiewicz and Corcoran's systems, Glashoff's [155] development of intensional semantics, Andrade-Lotero and Dutilh Novaes' [7] use of these studies to analyze KreiselÕs *squeezing argument* and Tennant's [353] extension of them to compare different strategies of natural deduction. These and related research provide a wealth of technical material applicable to the protologic so far defined, and to its further extension.

In addition, there are rich exemplars of the application of the syllogistic in the works of Aristotle and the massive secondary literature deriving from them. These include the seminal analysis of logical, ontological and epistemological issues by Galen, Avicenna, William of Sherwood, Petrus Hispanus, Abelard, Boethius, Duns Scotus, Buridan, Ockham, *et al*, complemented by recent studies of the detailed

[35] "eine der bedeutsamsten geistigen Leistungen bildet, und auch zu dem ganz wenigen gehört was als dauernder gesicherter Besitzstand im Bereiche der philosophischen Erkenntnis vorlieg"

argument forms in the *Prior Analytics* [61] and *Posterior Analytics* [124, 157], and Aristotle's notion of *definitions* [105, 81].

In order to map the continuities in the transition from conventional to modern logic, Clark [85] surveyed the historical innovations in ancient logic that prefigured those we now associate with Frege and the development of mathematical logic. Quine remarks in the preface that the "exaggerated notion of the cleavage between the old and the new fosters the very ignorance which engendered it" (p.4).

Kant's [199, Bvii] remark in the preface to the second edition of the *Critique of Pure Reason* that the syllogistic "appeared to be finished and complete"[36] and Russell's [315, p.38] in his *Art of Drawing Inferences* that it was "an obstacle to a sound understanding of logic" are neither unreasonable; the first because the patterns of reasoning that Aristotle codified are at the core of all logical systems; and the second because they need to be seen as exemplars of an indefinitely extensible collection of such patterns, not as a finished and complete system.

6.3.1. The syllogistic relations in the protologic. A syllogistic *term* is represented by a node with the same label. The copulae *belongs to every* and *belongs to no* are represented by the inclusion and exclusion links, respectively.[37] The inclusion and exclusion inference patterns of the protologic together with ecthesis (§3.3.6) are adequate to represent the four relations of Aristotle's categorical syllogistic.

Table 10 provides Aristotle's text[38], an English translation [15], and the modern a, e, i, o notation[39] for the four relations considered by Aristotle, followed by a translation into the protologic, and the same translation in the active voice.

Figure 94 shows the protologic representations as nets. On the left, the relations *belongs to every* and *belongs to no* are represented by inclusion and exclusion links, respectively. On the right, the relations *belongs to some* and *does not belong to some* are represented by an ecthetic node 'some A' with inclusion and exclusion links to B, respectively.

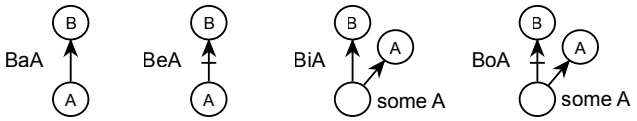


FIGURE 94. The four syllogistic relations represented in the protologic

[36]"Merkwürdig ist noch an ihr, daß sie auch bis jetzt keinen Schritt vorwärts hat tun können, und also allem Ansehen nach geschlossen und vollendet zu sein scheint."

[37]The alternative translations 'inheres in every' and 'inheres in no' are also perspicuous. 'Every' and 'no' are neutral to issues of intensionality/extensionality. As with 'some,' they may be interpreted in the protologic as 'every node included in' and 'no node included in,' respectively.

[38]Aristotle's use of the letters 'A' and 'B' is reversed for consistency with usage in this article.

[39]Boger's [61] term 'modern notation' is used because the converse notation was conventional in older literature, e.g. [205, p.62], prior to Patzig's [273, p.62] formalization of the syllogistic copulae as relations when he also reverted from Boethius' *omne A B est* to Aristotle's τὸ B παντὶ τῷ A ὑπάρχειν. The older notation also continues to be used [186], but Corcoran [92] and related literature uses a well-defined prefix notation for the relations to avoid ambiguity.

TABLE 10. Representing the four syllogistic relations

Prior Analytics	*English*	*Notation*
τὸ B παντὶ τῷ A ὑπάρχειν	B belongs to every A	BaA
	B is included in A	B←A
	A includes B	A→B
τὸ B μηδενὶ τῷ A ὑπάρχειν	B belongs to no A	BeA
	B is excluded from A	B↞A
	A excludes B	A↠B
τὸ B τινὶ τῷ A ὑπάρχειν	B belongs to some A	BiA
	B is included in some A	B← →A
	some A includes B	A← →B
τὸ B μὴ τινὶ τῷ A ὑπάρχειν	B does not belong to some A	BoA
	B is excluded from some A	B↞ →A
	some A excludes B	A← ↠B

The standard *conversions* of the first three relations are readily derived: BeA implies AeB follows from the symmetry inference pattern; BiA implies AiB follows because the anonymous node designated 'some A' may also be designated 'some B'; BaA implies AiB (and hence also BiA) because an anonymous node that includes A may always be added and, by the inclusion inference pattern, any node that includes A also includes B.

6.3.2. Syllogistic square of oppositions. The representation of Aristotle's analysis of the metalogical relations between the syllogistic primitives as a square of oppositions is attributed to Apuleius [232] in his *Peri Hermenias* (περι ἑρμηνείας) [249, 10] probably written in the second century but only preserved in much later medieval editions.

Figure 95 shows Apuleius' square as derived in §6.2.1 with each metanode labelled in the modern notation as an alias of the associated net. The metalinks are labelled with the terms that have been used for the relations in the square in various literatures, Greek, Apuleius' Latin, Boethius' Latin, and the protologic terminology.

Aristotle named the negations on the diagonals and the exclusion at the top which was sufficient for his purposes and from which those on the other sides may be derived. Apuleius, in the second century era of Ptolemy and Galen, was the first to name all four sides and the rationale for his terminology provides interesting insights into the logic of his times [158].

Apuleius' terminology was not widely adopted and Boethius [59] provided alternative Latin terms which became standard in the syllogistic. His 'contrarias' and 'contradictorias' are literal translations of Aristotle's ἐναντιον and ἀντίφασις. For the inclusion relation on the left and right sides because 'some A' is a sub-genus of A he adopted the Latin translation, 'subaltern,' of Greek term for subordinate, ὑπάλληλα, that Aristotle used to describe the relation between a sub-genus and a genus. For the inclusion relation at the bottom because it is under the exclusion

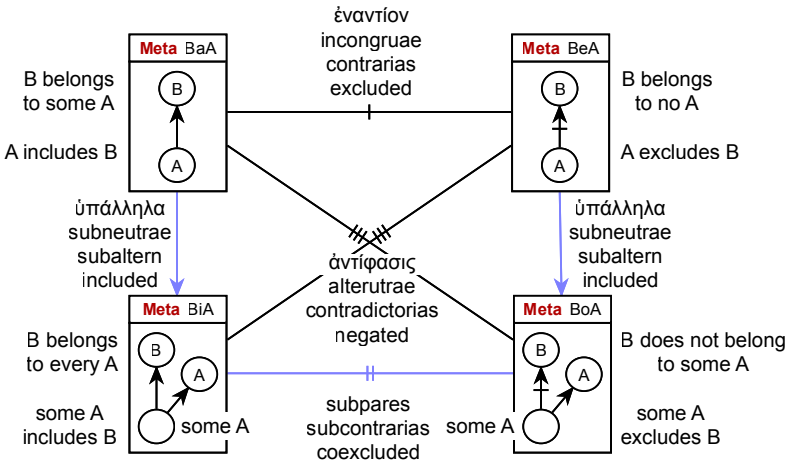


FIGURE 95. Square of oppositions

relation at the top he coined the term 'subcontrarias' by adding the Latin prefix 'sub' to contrarias.

Thus, the terms Boethius coined for the other three sides of the square are descriptive rather than metalogical. This distinction is significant when one analyzes later studies of extensions to the square that use his terminology and subaltern is used for subordination that does not correspond to metainclusion.

For example, William of Sherwood [231, 1.4.3 55] notes that singulars may be treated as particulars and that 'Socrates is running' is subaltern, according to Boethius' usage, to 'every man is running.' However, there is no logical derivation of the first from the second without the additional assumption that 'Socrates is a man.'

Following a suggestion of Lilje, Kretzmann [373, 32n39] uses Sherwood's rationale to add singulars to his square of oppositions creating another hexagon, but one in which some subalternations do not correspond to metalogical implications.

Czezowski [96] generates a similar hexagon in his analysis of similarities between singulars, universals and particulars in later developments of the syllogistic. He finesses the need for additional assumption by using 'this man' instead of 'Socrates' which could be interpreted as 'this is a man,' but his argument, in any event, only requires the subalternation not the metalogical implication.

6.3.3. Representing proofs of syllogisms. The two "perfect syllogisms" (συλλογισμὸν τέλιον [15, 25b35]), Barbara and Celarent, translate to the inclusion and exclusion inference patterns, respectively. Leibniz [225] developed a concise formal exposition of the syllogistic in terms of inclusion/exclusion, and captured these inference patterns as: "lncludens inclundentis est includens inclusi," "Includens excludentis est excludens exclusi." Łukasiewicz [239] treats the patterns as logical axioms, Corcoran [93] as rules of natural deduction and Martin [246] as "basic deductions."

It is well-established that the two perfect syllogisms and the conversions are adequate for the derivation of all valid categorical syllogisms. Bochenski [55] provides a comprehensive study of what combinations are adequate for the whole or part of the syllogistic.

Nets may be used to represent Aristotle's proofs of each syllogism. For example, Figure 96 shows his proof of Camestres. The premises are shown on the left, followed by application of the symmetry inference pattern (e-conversion), the exclusion inference pattern (Celarent), and the symmetry inference pattern, to infer the desired result.

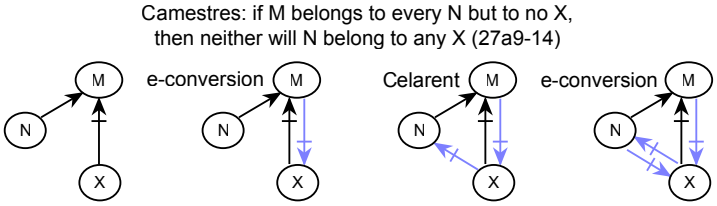


FIGURE 96. Proof of Camestres

Figure 97 top shows Aristotle's proof of Baroco by one of his preferred methods, *reductio per impossibile*. The premises are shown on the left, followed by the assumption of the metalogical contradiction of the conclusion (N includes X is contradictory to N does not belong to some X), application of the inclusion inference pattern (Barbara), leading to a metalogical contradiction (M includes X is contradictory to M does not belong to some X) refuting the assumption and allowing the conclusion to be inferred.

Figure 97 centre shows Smith's [336, p.227] proof of Baroco by ecthesis with the same argument structure as that for Camestres (with the node X now playing the role of some X). At the bottom is shown Galen's [144, p.41] simpler proof by ecthesis, where he takes the e-conversions as implicit; shown here by using the symmetric form of the exclusion link. Doing so significantly reduces the number of distinct forms of syllogistic reasoning, from 14 to 8 or from 24 to 11.

6.3.4. A generic inference pattern for the categorical syllogisms. Figure 98 left extends the generic inference pattern of Figure 25, providing additional nodes and using the symmetric exclusion link to encompass proofs of Aristotle's fourteen categorical syllogisms and ten other valid forms added later.

Node D is added to allow node A to represent 'some D' in various syllogisms; node F to represent an exclusion in Bocardo, Felapton and Fesapo; node E to represent an inclusion in Baroco; the anonymous node to represent 'some A' in the last four syllogisms with weakened conclusions. Node A also represents 'some B' and 'some C' in various syllogisms.

Table 11 shows the twenty four valid categorical syllogisms using the terminology of this article and the labelling of Figure 25. The top fourteen are those recognized by Aristotle and the remaining ten are those added by later writers to

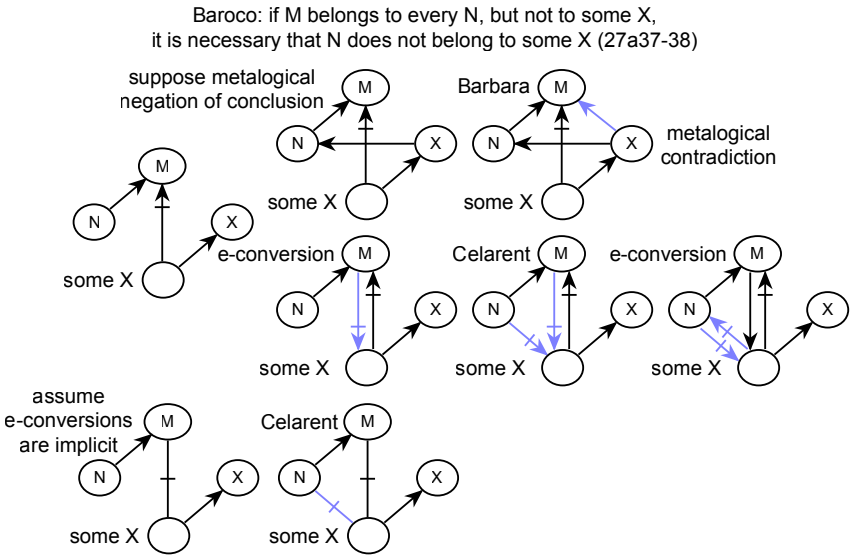


FIGURE 97. Three proofs of Baroco

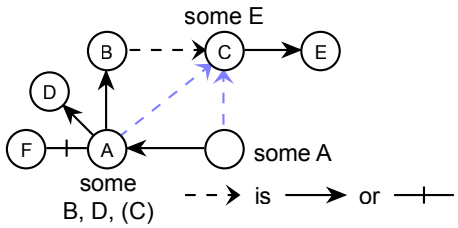


FIGURE 98. Generic proofs of categorical syllogisms

identify the full set of valid arguments with three assertions in syllogistic form. The second column of the table shows which of the two arrows should be substituted for the generic one in the net corresponding to a particular syllogism.

In larger nets the use of the inclusion and exclusion inference patterns involves only triples of nodes and hence falls under the argument forms of the syllogistic. Aristotle [15, ch.25] discusses larger structures and calculates the number of terms and conclusions that will be involved when multiple Barbara syllogisms with common terms are chained together.

6.3.5. Syllogistic reasoning in taxonomies. Taxonomic structures have a close relationship to syllogistic reasoning. All the valid categorical syllogisms except those involving particulars are instantiated in taxonomies having specified exclusion relations, for example Figure 62. All are instantiated in genus and differentia taxonomies where a lower genus may be termed 'some differentia.'

TABLE 11. The twenty four valid categorical syllogisms

->	*Mnemonic*	*Major premise*	*Minor premise*	*Conclusion*
		First Figure		
1.1 →	Barbara	B includes C	A includes B	A includes C
1.2 +-	Celarent	B excludes C	A includes B	A excludes C
1.3 →	Darii	B includes C	some D includes B	some D includes C
1.4 +-	Ferio	B excludes C	some D includes B	some D excludes C
		Second Figure		
2.1 +-	Cesare	C excludes B	A includes B	A excludes C
2.2 +-	Camestres	A includes B	C excludes B	C excludes A
2.3 +-	Festino	C excludes B	some D includes B	some D excludes C
2.4 +-	Baroco	A includes B	some E excludes B	some E excludes A
		Third Figure		
3.1 →	Datisi	B includes C	some B includes D	some D includes C
3.2 →	Disamis	some B includes D	B includes C	some C includes D
3.3 +-	Ferison	B excludes C	some B includes D	some D excludes C
3.4 →	Bocardo	some B excludes F	B includes C	some C excludes F
3.5	Felapton	A excludes F	A includes D	some D excludes F
3.6	Darapti	A includes B	A includes D	some D includes B
		'Fourth Figure'		
4.1 +-	Calemes	A includes B	B excludes C	C excludes A
4.2 →	Dimatis	some D includes B	B includes C	some C includes D
4.3 +-	Fresison	C excludes B	some B includes D	some D excludes C
4.4 +-	Calemos	A includes B	B excludes C	some C excludes A
4.5	Fesapo	F excludes A	A includes D	some D excludes F
4.6 →	Bamalip	A includes B	B includes C	some C includes A
		'First Figure' with weakened conclusion		
1.5 →	Barbari	B includes C	A includes B	some A includes C
1.6 +-	Celaront	B excludes C	A includes B	some A excludes C
		'Second Figure' with weakened conclusion		
2.5 +-	Cesaro	C excludes B	A includes B	some A excludes C
2.6 +-	Camestros	A includes B	C excludes B	some C excludes A

For example in Figure 63: Barbara may be used to infer that body belongs to every animal; Celarent that irrational belongs to no human; Datisi that living belongs to some rational; Bocardo that mineral does not belong to some substance; Baroco that human does not belong to some insensitive; and so on.

From a logical perspective, syllogistics may be viewed as providing inference patterns for reasoning about triples of nodes in taxonomic structures, which is not surprising since both syllogisms and taxonomies are based on the inclusion/exclusion inference patterns.

From a historical perspective, Clark [p.7][85] argues that Aristotle devised the syllogisms to systematize Plato's the dialectical method of Plato's *elenchos* (ἔλεγχος), where his use of *diaeresis* (διαίρεσις), his method of definition through collection and division, generates taxonomic schemata [153]), and based it on the

argument form of repeated use of the middle term in Eudoxus' theory of proportions [16, 1131a30-b7].

6.3.6. Mereology of syllogistic relations, Euler/Gergonne diagrams. The first four nets in Figures 83 and 88 correspond to the relations underling Aristotle's syllogistic and can be represented in mereological terms.[40]

In terms of the enclosure of contexts:

$$\mathrm{AaB} \equiv \mathrm{A} \leq \mathrm{B} \qquad \mathrm{AoB} \equiv \neg\ \mathrm{A} \leq \mathrm{B} \qquad \mathrm{AiB} \equiv \mathrm{A} \circ \mathrm{B} \qquad \mathrm{AeB} \equiv \mathrm{A} \wr \mathrm{B}.$$

These mereological definitions correspond to an extensional spatial model of the syllogistic as shown in Figure 83. This spatial analogy was used by Euler to explain syllogistic reasoning diagrammatically in his tutorial correspondence with a German princess [120, 14-24-Feb-1761], and developed formally by Gergonne [152].

A similar mereological model of the inclusion of content remained problematic until recently. Couturat [95, p.22] in collecting and editing Leibniz's papers saw the lack of an intensional spatial model of the syllogistic at that time as a reason for rejecting Leibniz's preference for intensional models of the syllogistic (as "réfractaire au schématisme géométrique").

Rescher [302] suggested that Leibniz had in mind the interpretation of a term as a set of properties. In reviewing Rescher's article, Dummett [107] noted that the contrary relation could not be interpreted as two sets of properties having nothing in common.

Ishiguro [189, ch.2§2] reported these discussions in her book on *Leibniz's Philosophy of Logic and Language*, commenting that his remark that no gold is silver because gold and silver are disparate could not be modeled in this way because gold and silver have overlapping properties.

In reviewing her book, Remnant [301, p.563] noted that Ishiguro and Dummett had "overlooked the possibility of attributing the disparateness of 'gold' and 'silver' to the fact that on analysis one of them is found to possess a property logically inconsistent with some property of the other." Remnant's suggestion was formalized independently some thirty years later in Glashoff's [155] intensional semantics for the categorical syllogistic (§5.2.1).

In terms of the inclusion of contents, the syllogistic relations can be represented as:

$$\mathrm{AaB} \equiv \mathrm{sB} \leq \mathrm{sA} \wedge \sigma\mathrm{B} \leq \sigma\mathrm{A} \qquad \mathrm{AoB} \equiv \neg\ \mathrm{sB} \leq \mathrm{sA} \vee \neg\ \sigma\mathrm{A} \leq \sigma\mathrm{B}$$
$$\mathrm{AiB} \equiv \mathrm{sB} \wr \mathrm{sA} \wedge \sigma\mathrm{B} \leq \wr\mathrm{A} \qquad \mathrm{AeB} \equiv \mathrm{sB} \circ \mathrm{sA} \vee \sigma\mathrm{B} \circ \sigma\mathrm{A}$$

These mereological definitions correspond to an intensional spatial model of the syllogistic as shown in the Glashoff diagrams of Figure 88.

Figure 99 illustrates Leibniz's [224] analysis of how assayers (*Docimastae*) might construe the similarities and differences between gold and silver in terms of common and contrasting properties. On the left is a net representing the two

[40] Kutschera [215] has analyzed the way in which these mereological relations are first discussed in some depth in Plato's *Parmenides*.

concepts, a common property, and a contrasting property, from which it can be inferred that the concepts gold and silver are incompatible or disparate. In the centre is the corresponding extensional Euler diagram, and on the right the corresponding intensional Glashoff diagram.

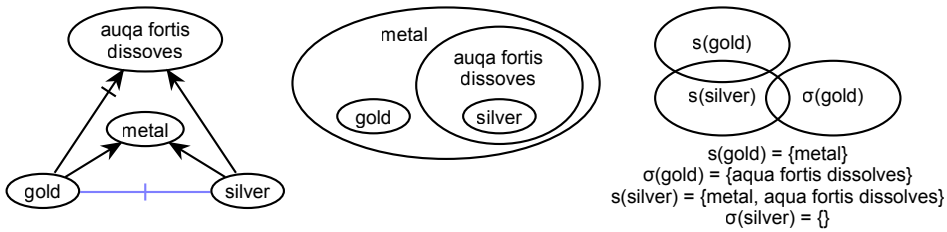


FIGURE 99. Net, Euler and Glashoff diagrams for gold-silver disparity

In her second edition Ishiguro [190] proposed to refute Remnant's suggestion with a counter-example, asking "are there any properties of an apple inconsistent with some properties of the number 3?" The abstract structures of Figure 99 may be used to model this disparity also but with the common and contrasting properties being very remote, for example, *entity* and *concrete versus abstract*, respectively.

Leibniz [223, 53-54] characterizes terms such as these whose common genus is remote as being *heterogeneous* (*Heterogeneas*), and notes that disparate concepts may have concepts in common provided they have some that are not common.[41]

The parallels between various passages in Leibniz's works and the intensional analysis of the syllogistic suggested by Remnant and formalized by Glashoff indicate that his intensional semantics were well-founded. They are summarized in *Definitiones logicae* [225] which provides concise and complete proof-theoretic intensional semantics for the syllogistic.

In any event, the spatial analogies resulting from the mereology of context and content provide a natural representation of the inference patterns of the protologic and the syllogistic.

7. Structural connectives

The focus of prior sections has been on logical structures and reasoning based on the basic inclusion and exclusion connectives in order to demonstrate the representational and inference capabilities of the foundational logical relations. This contrasts with specifications of logical systems that take structural connectives as primitive.

[41] Nobis verò in calculo sufficit duas res nullas ex quibusdam notionibus certis à nobis designatis habere communes, etsi alias forte communes habeant.

Structural connectives may be introduced in the protologic as *ideal elements* [176, 33] represented by the addition of nodes that are extremal relative to a set of links in the order relation associated with $\rightarrow$.

Koslow [206, 207] uses this approach to generalize Gentzen's introduction and elimination rules for logical connectives. He considers a general implication relation subject to the normal axioms for a consequence operator [40] and defines the usual structural connectives as extremal structures in the order relation of implication/consequence.

A similar technique may be used to define structural connectives in the protologic as graphical constructions based on the transitive relation of inclusion indicated by $\rightarrow$, taking into account the constraint upon its use defined by the symmetric relation of exclusion indicated by $\twoheadrightarrow$ or $\dashv\vdash$.

7.1. Structural conjunction

The links out of a node may be viewed as specifying a *protoconjunction* in that, if a probe includes that node then, from the inclusion and exclusion inference patterns, it will also include or exclude all the linked nodes. In the terminology of §2.7 this may be rephrased that if the node is marked *true* all the nodes that it includes will be marked *true* and those it excludes *false*.

However, the converse does not apply. One cannot infer that the node is true if some or all of the nodes to which it has outgoing links are marked appropriately. One can, however, define a constraint that specializes a node to be a full structural conjunction through an inference pattern that specifies this.

Definition: a node is the *conjunction* of a set of outgoing links iff any node with the same links includes it.

Figure 100 is a graphical representation of this definition.[42] The outgoing links defining the conjunctive node at the bottom left are distinguished by conjunctive variants of the base inclusion and exclusion arrows that have a heavier head. Any node, such as that at the bottom right, that has the same set of outgoing links may be inferred to include the conjunctive node.

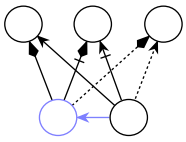


FIGURE 100. Structural conjunction inference pattern

If the bottom node on the right is regarded as a probe then the inference pattern implies that any probe that marks the nodes to which the conjunctive node has inclusion links *true*, and those to which is has exclusion links *false*, also

[42]The nets representing the structural connectives are less formal than those for the previous ones. The arrows should be parameterized and the dot notation, indicating an indefinite number, formally defined. However, this detracts from their readability and those shown are more readily comprehended.

marks node the conjunctive node as *true*. These are the converse truth conditions defining a full structural conjunction.

The inference pattern of Figure 100 parallels Koslow's [206, §13.1] approach to the definition of conjunction but takes into account exclusion as well as inclusion links. It specifies that, in terms of the order relation associated with →, the conjunctive node is *maximal* among all nodes having the outgoing links specified by the heavier arrows.

Arbitrary conjunctive nodes may be freely added as ideal elements to any net but they will not always be consistent; for example, if inclusion links are specified to two nodes between which there is an exclusion link.

7.1.1. Structural negation. As shown on the left of Figure 101, a single conjunctive exclusion link provides a structural negation inference pattern in which the tail node of any exclusion link includes the tail node of the negation of its head node.

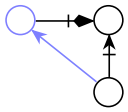


FIGURE 101. Structural negation inference pattern

This inference pattern captures the notion that a negation is implied by any contrary [68, p.126], and is a structural disjunction of all possible contraries [65, p.20].

This is not the definition of negation provided by Koslow [206, §12.1] which is in terms of explosion, that the assertion of a term and its negation implies any term. In the protologic the availability of the exclusion constraint makes it possible to provide a definition of negation not involving explosion that retains the paraconsistency of the protologic.

Like the exclusion link, structural negation is symmetric. However, a bare negation plays little or no role in the representation of useful structures in the protologic and it is not worthwhile to define a special symmetric link as was done in Figure 17 for the exclusion link.

7.1.2. Rule inference pattern. A node having both outgoing conjunctive and normal inclusion links may be regarded as implementing a *rule* such that the conjunction of the tail nodes of the conjunctive links entails the conjunction of the head nodes of the inclusion links. Figure 102 left shows such a rule having a set of premises on the left and a set of conclusions on the right.

Its description logic translation is:-

$$P1 \sqcap \neg P2 \sqcap ..... \sqcap Pn \sqsubseteq C1 \sqcap \neg C2 \sqcap ..... \sqcap Cn$$

The anonymous node in the centre representing the conjunction of the premises plays no role in the rule itself except to bind the premises and conclusions. However, it may be labelled to provide a name for the rule and, more importantly, it is

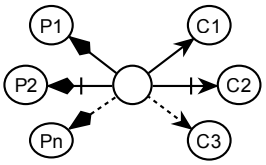


FIGURE 102. Rule inference pattern

available as a premise to other rules, allowing *rules with exceptions* [88, 137] to be represented. This is exemplified in §7.1.3.

A rule is not an ideal element that may be added freely without changing the logical interpretation of the existing net. A purely conjunctive node may be used to detect a condition in a net but leads to no inferences within the net. If the conjunction is extended to be a rule then the additional links beyond those required to specify the conjunction lead to additional inferences.

7.1.3. A rule-based solution to an expert systems problem. Figure 103 shows a rule-based solution to the contact lens prescription problem described in §4.3.2. It represents the linguistic description of the solution in terms of rules with exceptions given in §4.3.2. Knowledge acquisition in which general rules are elicited and then enhanced with exceptions is known to be natural to human experts [139, 89]. The determinable/determinate structures of Figure 76 are assumed and the case at the bottom has been marked true as in Figure 77. The inference pattern is distinctly different from the abductive pattern of Figure 77 but the outcome is the same.

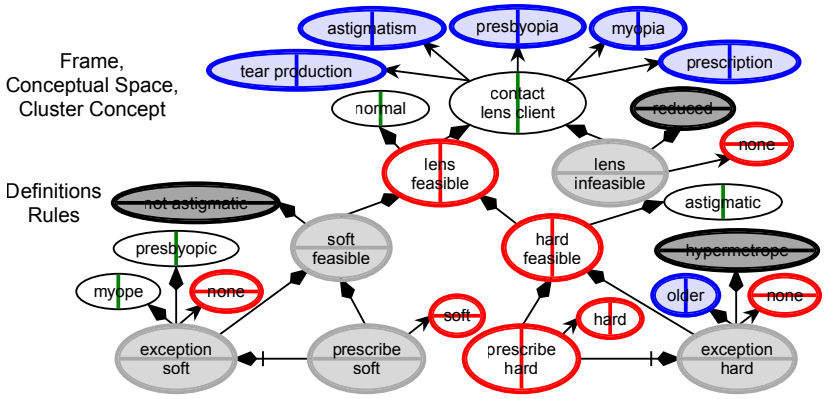


FIGURE 103. Rule-based solution of contact lens prescription problem

Table 12 shows the description logic translation of the net of Figure 103. Translation for use in a description logic inference engine is not required in order to implement the solution since the inference patterns of the protologic are implemented directly in the form specified in the preceding sections. However, it is also useful to be able to use the graphic language editor to prepare modules for other logical systems, and to be able to see the logical structures represented by the nets in familiar logical notations.

TABLE 12. Description logic translation of rule based solution

Description logic expression	Type
contact_lens_client ⊑ tear_production ⊓ astigmatism ⊓ presbyopia ⊓ myopia ⊓ prescription	frame
lens_infeasible ≡ contact_lens_client ⊓ reduced ⊑ none	rule
lens_feasible ≡ contact_lens_client ⊓ normal	definition
soft_feasible ≡ not_astigmatic ⊓ lens_feasible	definition
hard_feasible ≡ astigmatic ⊓ lens_feasible	definition
prescribe_soft ≡ soft_feasible ⊓ ¬exception_soft ⊑ soft	rule
exception_soft ≡ soft_feasible ⊓ presbyopic ⊓ myope ⊑ none	rule (exception)
prescribe_hard ≡ hard_feasible ⊓ ¬exception_hard	rule
exception_hard ≡ hard_feasible ⊓ hypermetrope ⊓ older	rule (exception)

This example provides an interesting situation in which to investigate the nature of the paraconsistency of the protologic in a practical situation. Figure 104 shows the inferences when the recommendation has been overridden to be none rather than hard. The changes are all ones that a person might interpret as reasonable. The only inconsistent node is 'lens feasible' which is true by the rules but false by the clinician's override. The node 'hard feasible' has changed from true to false which is a reasonable interpretation of the override. Other nodes have become irrelevant due to the override.

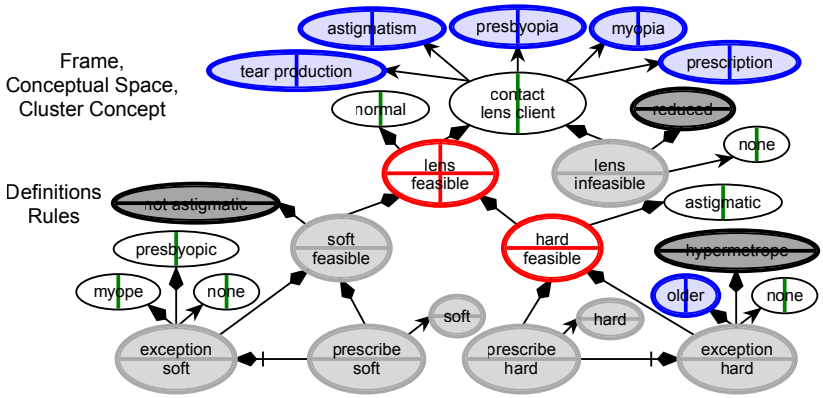


FIGURE 104. Overriding lens prescription recommendation

7.1.4. Rejection inference pattern. The rule inference pattern is the most common one involving conjunctions, but there is also another interesting one, a *rejection* inference pattern involving the exclusion of a conjunction, that is, asserting it to be false. A probe that includes a conjunction essentially reduces it to a proto-conjunction from which it may be inferred that its conjuncts are also included or excluded. However, one that excludes a conjunction leads to a rather more interesting inference pattern in which the state of affairs that is defined by the conjunction is rejected.

Stueber [346] has suggested that such a rejection inference pattern is the basis of apparent rule-following behaviour induced by social norms. He proposes that, rather than representing a social norm as a rule, we monitor situations that might violate the norm and reject behaviours that would create such violations. We internalize social norms through "self-monitoring dispositions that are sensitive to the violation of normative standards" (p.320).

Figure 105 left illustrates Stueber's (p.320) example of a soccer regulation that someone playing soccer who is not the goal keeper may not handle the ball as a rejection of any state of affairs in which the regulation is violated. The following three nets represent situations designed to assess human logical inference: Wason's [369, p.145-146] seminal *selection task* which stipulates that a card having a vowel on one side has an even number on the other; a variant based on an Italian postal regulation that a sealed letter must carry a 50 lire stamp [195]; and a further variant based on an American liquor regulation stipulating an age restriction on alcohol consumption [166].

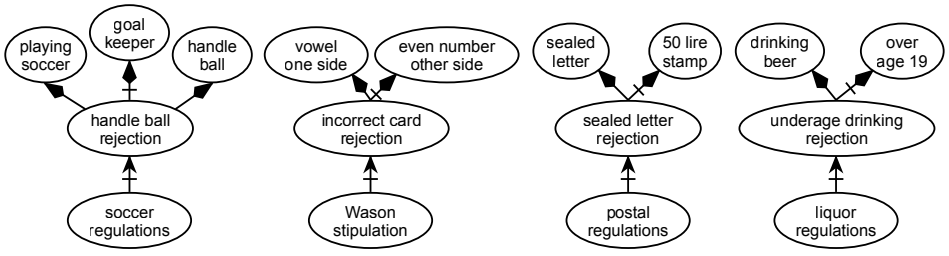


FIGURE 105. Rejection-based representation of social norms

When all the conjuncts are exclusions the rejection inference pattern corresponds to a structural disjunction over the tail nodes of the conjuncts and may be used to represent material implications and more complex formulae of the sentential calculus. For example, the two nets on the left of Figure 106 lrepresent the material implication A ⊃ B as a rejection.[43] If the node labelled ¬M is asserted false then at least one of the nodes of its conjuncts may be inferred to be true. That is, ¬A ∨ B is true, the classical interpretation of A ⊃ B as material implication.

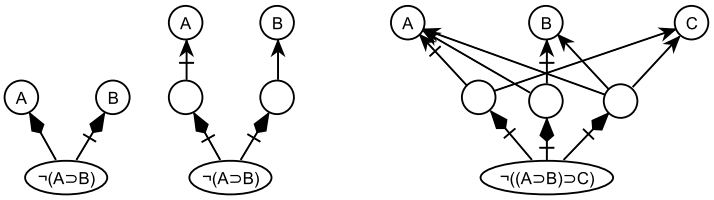


FIGURE 106. Rejection-based representation of sentential formulae

[43]The familiar symbol ⊃ is used to represent material implication and distinguish it from the arrow → of inclusion/entailment, rather than introduce some novel sign intended to take into account the variety of interpretations possible, some of which might involve the introduction of further symbols such as quantifiers.

Figure 106 right represents a more complex formula of the sentential calculus, (A ⊃ B) ⊃ C. More general examples are discussed in §7.2.6 in the context of the applications of structural disjunction.

7.1.5. A rejection-based solution to an expert systems problem. Figure 107 shows a rejection-based solution to the contact lens prescription problem described in §4.3.2.

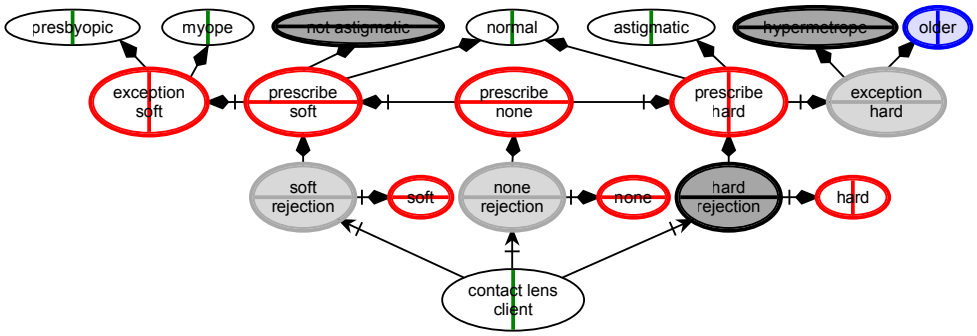


FIGURE 107. Rejection-based solution of contact lens problem

The determinable/determinate structures of Figure 76 are assumed and the case at the bottom has been marked true as in Figure 77. The inference pattern differs from the abductive pattern of Figure 77 and the rule pattern of Figure 103 but the outcome is the same.

7.2. Structural disjunction

As shown in §7.1.4 the rejection inference pattern may be used to represent a structural disjunction in the protologic. Structural disjunction may also be defined more directly and perspiscuously as the dual of structural conjunction.

The inclusion links into a node may be viewed as specifying a *protodisjunction* in that, if a probe excludes that node then, from the exclusion inference patterns, it will also exclude all the linked nodes. In the terminology of §2.7 this may be rephrased that if the node is marked *false* all the nodes that it includes will be marked *false*.

However, the converse does not apply. One cannot infer that the node is false if all the nodes from which it has ingoing inclusion links are marked false. One can, however, define a constraint that specializes the node itself through an inference pattern that allows this to be inferred.

Definition: a node is the *disjunction* of a set of incoming links iff it includes any node with the same links.

Figure 108 is a graphical representation of this definition. The incoming links defining the disjunctive node at the top left are distinguished by a variant of the base inclusion arrows that has a double head. Any node, such as that on at the

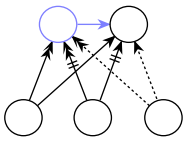


FIGURE 108. Structural disjunction inference pattern

top right, that has the same set of incoming links may be inferred to be included in the disjunctive node.

If a probe has exclusion links to the tail nodes of the links defining the disjunctive node then it may, without inconsistency, have an exclusion link to a an arbitrary node with inclusion links from these nodes such as that at the top right. Such a node also has an inclusion link from the disjunctive node so that an exclusion link may be inferred from the probe to the disjunctive node. That is, if the tail nodes of the incoming links defining the disjunctive node are all marked false then so is the disjunctive node. This is the converse condition defining a full structural disjunction.

The definition of structural disjunction specifies that, in terms of the order relation associated with $\rightarrow$, the disjunctive node is *minimal* among all nodes having the incoming links specified by the double arrows. This differs from Koslow's [206, §13.1] definition which specifies the maximality of a different construction, but the two definitions are equivalent.

The duality between disjunction and conjunction is not complete because the definition of disjunction involves only inclusion links. If one construes an exclusion link as being one in which if one node is marked true the other may be inferred false, then the dual is a link such that if one node is marked false the other may be inferred true.

This is not a common inference pattern but it can be represented in the protologic as a *preclusion link* (§**??**) that is dual to the exclusion link, in that the preclusion constraint is that the tail and head nodes cannot include the same node. However, such links do not appear to be useful in representing common logical schemata and are not regarded as a basic feature of the protologic.

7.2.1. Disjunctive judgement. The disjunction definition is less general than the conjunction definition in that it involves only inclusion links. However, it is adequate to extend the syllogistic inclusion and exclusion connectives to encompass many features of modern logic while avoiding the introduction of a bare structural negation. It may be seen as a natural extension to the syllogistic corresponding to the assumption of *taxonomic coverage*, that any entity under a genus is also under one of its species.

In his lectures on logic, Kant [200, §27-29] extends the syllogistic in this way with a pattern of reasoning requiring structural disjunction even though he asserted syllogistic logic to be complete [199, Bvii]. In his discussion of forms of judgement [233], while his categorical and hypothetical forms may be represented

using the syllogistic copulae, his third form, the *disjunctive judgement* may not.[44] He specifies that the possible judgements are disjoint and "taken together, equal to the sphere of the whole as parts of the sphere of cognition."[45] The first requirement may be represented by exclusion links but the second requires a structural disjunction.

Such complete coverage has been noted to be a tacit assumption in biological taxonomy [354, §4.3.3], and may be a long-standing one in ancient literature on genus and species. It can be represented in the protologic by replacing some inclusion links with disjunctive inclusion links. For example, in Figure 62 by replacing them all, and in Figure 63 by replacing those between the genera.

Figure 109 left shows a construct (§3.3.5) representing a binary disjunctive judgement such that any node that includes the top node must include one, and only one, of the bottom nodes. The two nets to the right of it extend this to ternary and quaternary judgements, and that on the right to a taxonomic structure representing an octary disjunctive judgement.

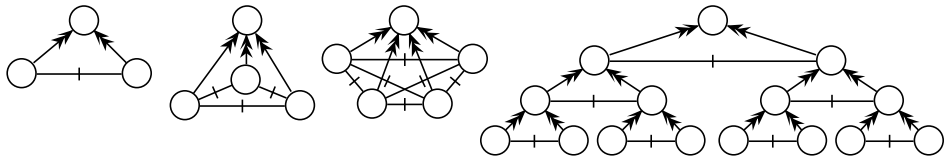


FIGURE 109. Representations of disjunctive judgements

Figure 110 top left represents Kant's example of a disjunctive judgement from his *Critique of Pure Reason*: "The world exists either through blind chance, of through inner necessity, or through an external case."[46]

Top right represents a hierarchical variation in which chance has been contrasted with cause to emphasize the commonality of the causal rationales. Bottom right represents the hierarchical variation as a genus/differentiae taxonomy in which the determinables of the general systemic determinable on the left have been used as differentiae. All three represent Kant's example as a disjunctive judgement conforming with his definition. They illustrate the role of the protologic in comparing various possible philosophical analyses of his meaning.

7.2.2. Induction as disjunctive reasoning. assadads adsadsdas adsasdds

7.2.3. Abduction as disjunctive reasoning. assadads adsadsdas adsasdds

Whereas the primary pattern of reasoning with a conjunctive node is to infer its truth value from those of the nodes to which it has outgoing conjunctive links,

[44]Kant's disjunctive judgement has been regarded as a logical curiosity because the disjunction cannot be represented by a binary logical operator. [276]

[45]"die Glieder der Disjunktion...sind wie Theile der Sphäre einer Erkenntniß, jedes des anderen Ergänzung zum Ganzen zusammen genommen der Sphäre des ersten gleich seien." [200, §29]

[46]"die Welt ist entweder durch einen blinden Zufall da, oder durch innere Notwendigkeit, oder durch eine äussere Ursache." [199, A74, B99]

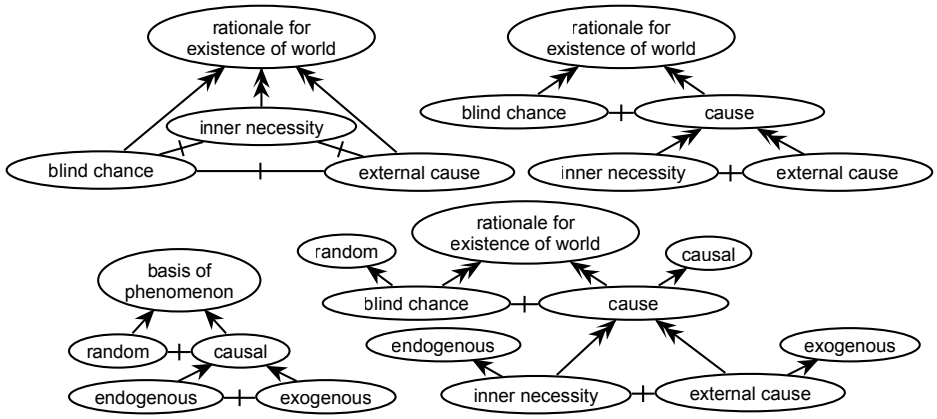


FIGURE 110. Kant's example of a disjunctive judgement

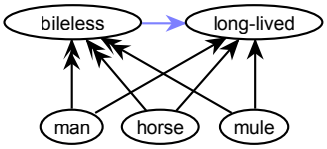


FIGURE 111. Representations of disjunctive judgements

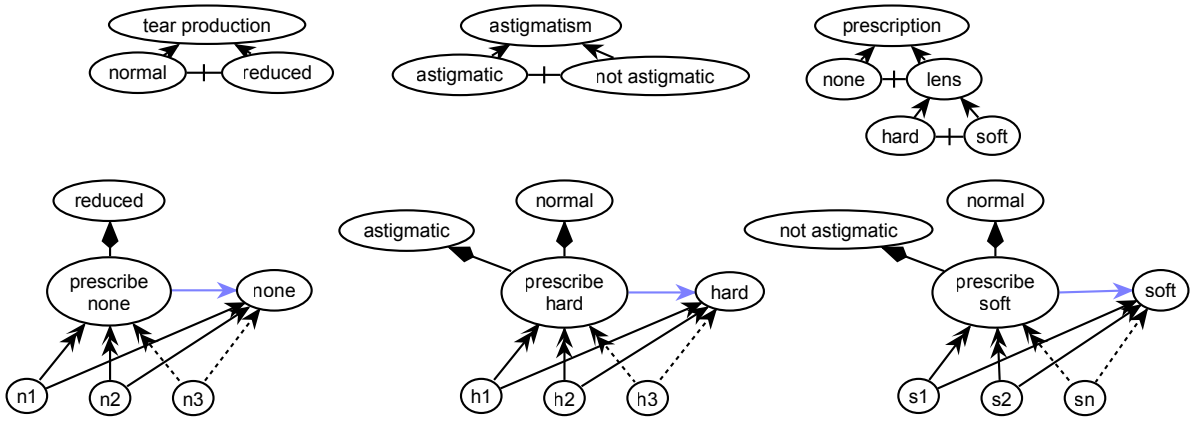


FIGURE 112. Representations of disjunctive judgements

that for a disjunctive node is to mark it true and infer what tail nodes of its incoming disjunctive links might make it so. If there is only one it can be inferred to be true. If there are several then none may be inferred true but the common inferences derived from asserting each node in turn to be true may be inferred.

This pattern of reasoning enables abduction to the best, or only, available prototype to be represented using structural disjunction. Thus, the metalogical

abduction discussed as a significant inference patterns for frames (§4.3.1) and individuals (8.4) may be pushed down to the protologic.

The abductive inference pattern is not simply disjunction but rather a specific pattern based on it as illustrated in Figure 113. The abduction hypothesis is a disjunction over a set of available *abducibles* that are distinguished by being bottom nodes of the net and having no incoming arrows. This constraint corresponds to the abducibles not being assertible to be true and only being derivable to be true through the abduction process.

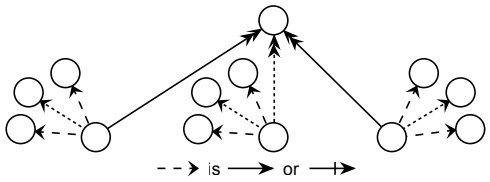


FIGURE 113. Abductive inference pattern

The abducibles may be seen as representing templets, prototypes, schemata, and so on, that may be fitted to some phenomenon, the features of which are specified by marking some of the nodes. They have inclusion or exclusion links to other nodes, and cannot be inferred to be true through these outgoing links. They may inferred to be false, that is, to be inconsistent with the specified state of affairs.

If only one of these nodes is not false then the disjunction inference pattern will infer that it is true. If several are not false then their common inferences may be inferred. In addition, if the capability is added to specify preferences between the non-false nodes such that only a subset is considered to potentially satisfy the disjunction, these patterns may be used to model defeasible reasoning (§8).

The nodes that the abducibles include or exclude need have no outgoing links and may represent what Wittgenstein [375, §4.21 et seq.] termed an *elementary proposition* (*Elementarsatz*) that is logically independent from any other [375, §5.134]. However, they may also be subject to logical constraints specified elsewhere in the net.

The rejection inference pattern (§7.1.4) may be regarded as representing disjunctions through exclusion links in conjunctions, and may also be used to implement the abductive inference pattern.

7.2.4. Abductive reasoning in the protologic. The abductive inference pattern may be used to push down to the protologic the metalogical disjunction implicit in the abductive reasoning exemplified in §4.3.1, §4.3.2, §9.0.1 and §9.0.2 by changing appropriate protodisjunctions to structural disjunctions.

For example, in Figure 114 the inclusion links from the prototypes to the node 'malt whisky' in Figure 75 have been replaced by disjunctive inclusion links. The inference that the only applicable prototype for 'whisky B' may be asserted true is made automatically through the disjunction inference pattern of the protologic rather than metalogically.

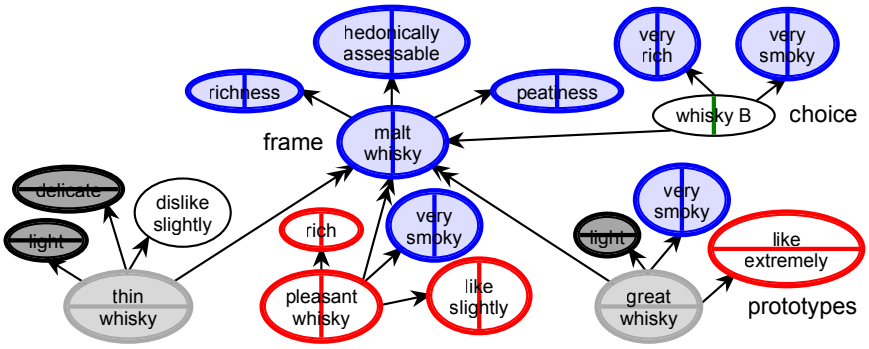


FIGURE 114. Disjunctive implemention of abductive inference

The same technique may be used in the other examples of abductive inference including the contact lens prescription problem of §4.3.2. However, more elegant and interesting solutions result from specifying a defeasible preference relation between the prototypes as discussed in §8.1 and illustrated in §8.2.2.

7.2.5. Defeasible, ampliative and creative abductive reasoning. sadsads adsdsad-saasd sadasdsdsaasdsda

7.2.6. Material implication and other sentential calculus formulae. The inclusion link, →, is appropriate to represent *entailment*, necessary, rather than material, implication. It specifies relations between terms prescribing their proper usage, conventions of language rather contingencies of a world. In addition the implicative conditional is not represented in a form subject to inference; it may only be *used* in the protologic, not *mentioned*.

However, the abductive inference pattern may be used to represent a material implication as a logical structure in which the implication is itself represented as a node subject to inference. Figure 115 left illustrates the simplest case. If node M is asserted true then at least of its disjuncts may be inferred to be true. Thus ¬A ∨ B is true, the classical interpretation of A ⊃ B as material implication. However, the conditioning on M means that what is represented is M ⊃ (A ⊃ B).

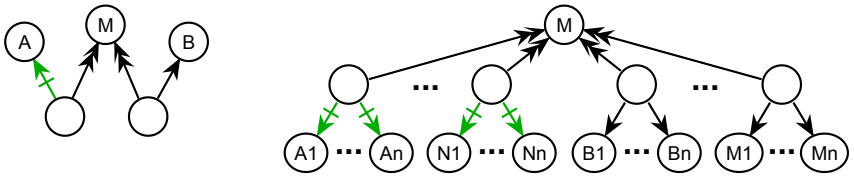


FIGURE 115. Representing material implication in the protologic

This non-truth-functional representation in terms of implication rather than equivalence avoids the so-called 'paradoxes of material implication' [115, §2.3]. The only situation in which the truth value of M is determined by those of A and B is

that it is inferred false if A is asserted true and B false, that is when the material implication does not hold.

Figure 115 right shows a more general form of material implication where there are multiple disjuncts of nodes with multiple outgoing links. This represents:

$$M \supset (((A1 \vee ... \vee An) \wedge ... \wedge (N1 \vee ... \vee Nn)) \supset ((B1 \wedge ... \wedge Bn) \vee ... \vee (M1 \wedge ... \wedge Mn)))$$

Thus a disjunctive node together with inclusion/exclusion links may be used to represent a material implication between complex clauses with premises in conjunctive normal form and conclusions in disjunctive normal form.

The most general form of the material implication structure of Figure 115 is one is which the tail nodes of the disjunctive links have both inclusion and exclusion links. The disjunction may be factored in a number of ways as a material conditional or nested material conditionals, that is, as formulae of a sentential calculus.

In addition, the disjunctive node, being explicitly represented, may have links from other nodes that control whether the material implication inference pattern is in effect, and this again has a sentential calculus representation. For example, Figure 116 shows a structure representing two rules, X and Y, that may be included as properties of any other node, with Y being one consequence of X. It represents:

$$X \supset (((\neg A \vee B) \wedge C) \supset (D \wedge (E \supset (F \wedge \neg G))))$$

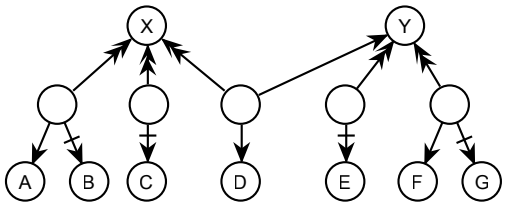


FIGURE 116. Example of an implication having another as a conclusion

From a semantic perspective a material implication inference pattern imposes no meaning relationships between the premises and conclusions or between the terms constituting them. This is what one would expect in a conditional that represents contingencies rather than conventions.

7.2.7. Translating nets from fixed to variable structure. Nets using inclusion and exclusion links to represent fixed structures may be translated to ones that represent variable, but logically equivalent ones, by restructuring them as a disjunction over the consistent probes that characterize the original net.

For example, Figure 117 left shows a construct (§4.1) that might be part of a taxonomy (§4.1.3). The top node represents a universe and the bottom nodes two mutually exclusive nodes that are included in the universe.

Figure 117 right shows a translation of the net to a material implication form represented as a disjunction over the three consistent probes that characterize the net. If the top node D is marked true then the logical relations between the nodes

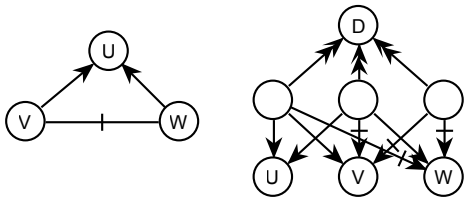


FIGURE 117. Converting fixed structures to variable ones

U, V and W will be the same as those of their counterparts on the left but if D is not so marked then the three nodes will be logically unrelated.

The basic material implication on the left of Figures 115 and 106 may be regarded as a translations of the simple entailment A→B, and it is interesting to see how the different structures respond to the inconsistent case in a state of affairs that includes A but excludes B, thai is, where A is asserted true but B false.

Figure 118 left shows, from left to right, the outcome with a necessary implication, and a material implication represented by a disjunction or a rejection. With necessary implication the violation leads to the premise and conclusion being inferred inconsistent, but violation of the material implication leads to it being inferred inconsistent rather than the assertions.

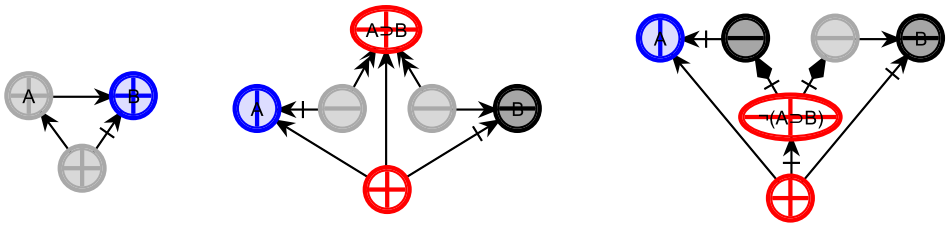


FIGURE 118. Inconsistency in necessary and material implications

These differing behaviours are consistent with the inclusion link, →, representing *entailment*, necessary, rather than material, implication, and specifying the semantic conventions of language rather than the contingencies of hypothesized laws of nature or prescribed rules of behaviour.

In all cases the node representing the state of affairs is also inferred inconsistent, that is inconsistency is not localized to the node representing the material implication. This more precise localization can be achieved by adding a preference relation to support defeasible inference (§8).

7.2.8. Mental models. Johnson Laird's [197, 196] *mental models* exemplify the role of disjunctive judgements in human reasoning. He hypothesizes that people represent a logical assertion as a disjunction over all its distinct models, the states of affairs that are consistent with it. Figure 119 from left to right illustrates mental models of A ⊃ B, a version of Wason's [368] card selection task and two variants of it [195, 166].

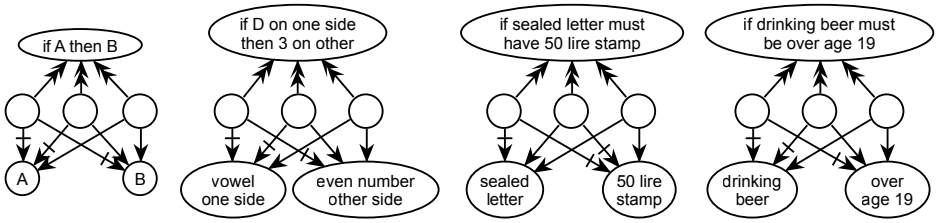


FIGURE 119. Mental models as disjunctive judgements

Disjunctive mental models were developed to account for some anomalous results in empirical studies of human reasoning [243, 2] that address the role of logic in human rationality [242, 141]. Wason [369, 368] found in early studies of the capability to reason logically about a simple material implication, A ⊃ B, that his subjects had about a 6% chance of reasoning correctly, a far poorer performance than expected. Variations on his experimental design in order to provide insights into this phenomenon have dominated the literature for some five decades [267, 342].

Types of error

An early "stunning result" [196, p.207] was that the *content* of the conditional had a major impact on performance, that the criterion that logical inference is about the form rather than the matter of reasoning was violated.

???? FINISH

7.3. Semantics of structural connectives

The structural connectives have been defined through variants on their base inclusion and exclusion links that specify that a node is extremal with respect to a subset of its links. This provides them with proof-theoretic semantics.

The variant arrows comply with all the inference patterns of their base arrows as specified in §2, and hence the various semantic models of §5 also continue to apply, but with some additional constraints upon nodes that are specified to be extremal. However, the variant arrows cannot be parameters of the generic inference pattern of §2.5.5; only their base arrows are 'copied' or 'inherited.'

7.3.1. Structural connectives as ideal elements. The nodes representing conjunction, negation and disjunction that are 'pure' in having only the structural links defining them are *ideal elements* in Hilbert's terminology. They can be freely added to any net without changing the logical interpretation of the existing net. In particular, as Hilbert [176, p.179] requires, they cannot make the existing net inconsistent,[47] although an added conjunctive node may itself be inconsistent.

Thus the semantic contribution of a pure structural connective that has no other links is limited to serving as an indicator that a certain condition exists in a net, for example, that a probe representing a state of affairs may be inferred to

[47]"die Erweiterung durch Zufügung von Idealen ist nämlich nur dann statthaft, wenn dadurch im alten engeren Bereiche keine Widersprüche entstehen"

include a structural node representing a conjunctive or disjunctive configuration of nodes.

This passive role changes significantly if, for example, a conjunctive node is specified to have additional outgoing links thereby constituting a 'rule' that does not just indicate a condition but also makes additional inferences from the existence of that condition (§7.1.2), or a disjunctive node is specified to be included in a situation implying that at least one of its disjuncts must also be included.

From a semantic perspective, a net without structural connectives may be seen as representing conventions in the application of terms, linguistic constraints on admissible descriptions of states of affairs that allow for concise descriptions that may be expanded if necessary. The structural connectives as ideal elements add to the vocabulary of such descriptions, but also provide the capability to encode constraints that go beyond conventions of language to express 'theories' about what kinds of states of affairs are possible or 'requirements' as which ones are allowed.

If a net represents the epistemic state of an agent then the non-structural connectives specify how terms used by the agent are related to one another, the extent to which the agent conforms with the linguistic conventions of a culture of communicating agents. The structural connectives that are not simply ideal elements represents the constraints that the agent applies to states of affairs in order to anticipate some aspects from knowledge of others, either predictively or normatively.

7.3.2. Truth tables for structural connectives. From a truth values perspective (§2.7), the truth tables for a binary conjunction, unary negation and binary disjunction may be determined by enumerating consistent probes. They are shown in Table 13 in terms of the vertical and horizontal marking of nodes (corresponding to inclusion and exclusion links from a probe).

Table 13. Truth tables for conjunction, negation and disjunction

$\sqcap$	○	⊖	◐	⊕
○	○	⊖	○	⊖
⊖	⊖	⊖	⊖	⊖
◐	○	⊖	◐	⊕
⊕	⊖	⊖	⊕	⊕

$\neg$	
○	○
⊖	◐
◐	⊖
⊕	⊕

$\sqcup$	○	⊖	◐	⊕
○	○	○	◐	◐
⊖	○	⊖	◐	⊕
◐	◐	◐	◐	◐
⊕	◐	⊕	◐	⊕

If the vertical marker is interpreted as *true* and the horizontal as *false*, these are the truth tables for Belnap's [30, p.14] *useful four-valued logic*, which plays a major role in the semantics of many logical systems [66] including relevant [245], paraconsistent [288] and nonmonotonic [57, ch.5] logics. From an algebraic perspective, nets involving the structural connectives are naturally represented by a *De Morgan algebra* [109, 127, 65].

7.3.3. Reconstructing nets with structural connectives. The results of §5.2 on the reconstruction of nets from partitions are graphical and carry over to nets with arrows representing structural connectives. However, as already noted, a node representing a structural connective no longer has an unspecified contribution to its content to its content or context.

The results of §5.2.5 on extensional semantics based on the reconstruction of the net from a set of probes carry over to nets with links representing structural connectives. However, the computational complexity of the reconstruction algorithm is substantially increased because the conjunction and disjunction inference patterns involve sets of nodes, and reconstruction requires a combinatorial search over all possible subsets of nodes.

Reconstruction is equivalent to the problem of machine learning from a complete set of correct cases which is known to be intractable [280], and to require heuristic assumptions in practical applications [202]. The extensional semantics of logical systems in terms of relations between sets of probes, or protoindividuals, does, however, provide significant set-theoretic foundations for the formal properties of those systems.

7.3.4. Mereology of structural connectives. In mereological terms the structural connectives are *closures*, *products* or *sum/fusions*, of the contents or contexts relative to which the nodes are defined to be extremal. The context and content of a node satisfying a structural constraint are determined by that constraint, and the node is no longer *primitive* having context and content particular to the node

From a contextual perspective (§5.3.1), the conjunction inference pattern specifies that the context of the conjunctive node is precisely what is common to the context (or, if exclusion, the complementary context) of each conjunct. The disjunction inference pattern specifies that the context of the conjunctive node is precisely the union of the contexts of the conjuncts.

Figure 120 left shows a net and an Euler diagram of the contexts for a protodisjunction, Q, and a protoconjunction, P, of A and B. On its right the structural forms are shown as the protodisjunction expanded to its minimum possible size and a protoconjunction to its maximum.

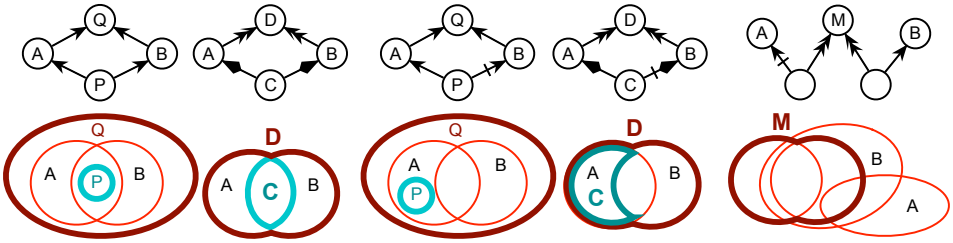


FIGURE 120. Context mereology of structural connectives

In the centre are shown similar diagrams when the protoconjunction excludes rather includes B. The inclusion of A plays an important role in constraining the

negation of B to be a small region within the context of A and outside that of B, rather than the indefinite/aorist region outside the context of B. The diagram illustrates that negation alone, not in the context of some other constraint, is problematic as Aristotle and De Morgan have noted (§3.3.5).

On the right is shown the representation of a material implication as a disjunction (Figure 115), together with a corresponding Euler diagram. It is apparent that the intersection of M and A is within B, that M implies that A implies B.

Figure 121 shows a net for the disjunction, D, and conjunction, C, of A and B, together with a Venn diagram showing the relation between the contents of D and C and those of A and B. The conjunction will be inconsistent if A and B are incompatible, that is if the regions of overlap of sA and σB or σA and sB are not empty.

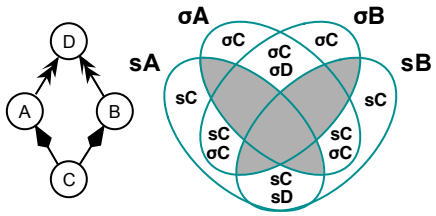


FIGURE 121. Content mereology of disjunction/conjunction

There is no simple representation of negation, or of conjunctions with exclusion conjuncts, in terms of content mereology. The reason for this is that the incompatibility relation is defined by the disjunction of two overlap constraints (§5.3.4), and satisfying the structural negation inference pattern (Figure 101) would require some new mereological construct, such as a disjunctive region.

However, many significant applications of logic do not require structural conjunction or negation and may be based on the inclusion/exclusion connectives together with structural disjunction. This has been demonstrated for material implication (§7.2.6) and abductive reasoning (§7.2.3), and is extended to defeasible reasoning in §8.

7.3.5. Extending syllogistics with structural connectives. The derivation of structural connectives as extremal cases of syllogistic copulae is simple. If P is any protoconjunction of A and B (Figure 120 left), then the definition of the structural conjunction of A and B, C, may be cast in pseudosyllogistic form:

AaP, BaP ∴ CaP

if A belongs to every P and B belongs to every P, then C belongs to every P

if P includes A and P includes B then P includes C

Similarly, if Q is any protodisjunction of A and B (Figure 120 centre) then the definition of the structural disjunction of A and B, D, is:

QaA, QaB ∴ QaD

if Q belongs to every A and Q belongs to every B, then Q belongs to every D

if A includes Q and B includes Q then D includes Q

The syllogistic relations in the net for material implication (Figure 120 right) are: AoM and BiM, and the minimality of M can be expressed:

AoY, BiY ∴ YaM
if A does not belong to some Y and B belongs to some Y,
then Y belongs to every M
if some Y excludes A and some Y includes B then M includes Y

This kind of analysis in antiquity could have brought about a synthesis of Aristotelian term logic and Stoic [247, 270] propositional logic,[48] but this did not occur [244, ch.5 §1.1]. Aristotle uses propositional argument forms in the *Prior Analytics* and other works [118]; it would be difficult not to do so. Mulhern [260, p.135] argues that "the evidence points to Aristotle's awareness of propositional logic but his rejection of it as an instrument fit for the purposes he intended." In particular, his rejection of structural negation as aorist (§3.3.5) implicitly rejects later syntheses that take it as primitive, such as those of Boole [62] and Venn [362].

The focus of Aristotle's formal development of logic was the proper form of the dialectical disputations as practiced by Socrates and the Sophists [201], and his syllogism is strictly a three-term argument chain [61] of a particular form. Even when he considers extension beyond three terms he only considers a chain, not more general uses of what have come to be termed the syllogistic copulae.

It is perhaps surprising that Medieval logicians, or Leibniz, Euler, Kant, De Morgan, Boole, Venn or Peirce with their innovative approaches to the syllogistic did not make the synthesis. Kant added structural disjunction (§7.2.1) to the syllogistic without appearing to realize that it was a major extension.

Venn [361] came closest to a synthesis when, in introducing his variant diagrams, he uses them to illustrate both syllogisms and propositional connectives, but he bases his algebraic system on structural negation. Eaton [113, p.2] suggested the basis for a synthesis, that Aristotle had provided a subset of the "validating forms" of mathematical logic. However, he goes on to present the two logical systems separately, without comparison, and fails to develop this insight.

The examples above of the definition of structural connectives within a syllogistic vocabulary show that the addition of the validating forms of sentential propositional logic as simple extensions of the syllogistic relations is not dependent on the advances in logical symbolism, algebra and set theory that we associate with Boole, Frege, Hilbert, Tarski, *et al.*

Neither are they dependent on notions of abstraction and axiomatic formulation that date back to Aristotle and Euclid, and became norms across scholarly disciplines, not just mathematics and logic. Major theories in ethics and psychology were presented in terms of postulates, theorems and corollaries, for example: Spinoza's [338] *Ethica ordine geometrico demonstrata*; Hull's [185] *Mathematico-deductive theory of rote learning*; Kelly's [204] *Personal construct psychology.*

[48] This notion is not quite as anachronistic as suggesting that Phidippides could have saved himself some 300 miles of running by using a cellphone to call Sparta and Athens; Aristotle's terminology does suggest a spatial analysis of syllogisms [165, §9.3].

However, Lachterman [216] suggests that modernity itself is associated with the widespread adoption of generalized Euclidian constructivism, a transition from the passive reception of ideas of the given to their active construction. Perhaps a synthesis would have required the constructivist reconceptions of logic that we now take for granted but were radical a century ago. In particular, that logical systems are themselves constructed as mathematical abstractions independent of any particular domain, and are pluralistic rather than monolithic.

Hilbert took logic to a new level of axiomatic abstraction where an inference could be applied in any domain that complied with the axioms whence the inference derived. As he noted in an 1899 letter to Frege [131, p.13], Pythagoras' theorem was not specific to Euclidean geometry but applicable to "love, law, chimney sweep..." (*Liebe, Gesetz, Schornsteinfeger*) if these were entities in domains that complied with the Euclidean axioms.

Béziau's [42] *Anthology of Universal Logic* commences in 1922 with Hertz's [**?**] *Uber Axiomensysteme für beliebige Satzsysteme* which provided simple formal constraints on a generic notion of logical implication deriving from *Principa Mathematica*. It may be that only from that era onwards there developed a metalogical perspective from which different logical systems could be analyzed as variants of common foundational notions rather than as rivals.

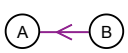
8. Defeasible inference

Paraconsistency in a logical system is desirable in that an inconsistency is localized and does not undermine reasoning outside that locale. However, such containment alone is inadequate to support reasoning *within* the logical system about the sources and consequences of the inconsistency. These aspects of the inconsistency remain at the metalogical level.

However, reasoning about inconsistency can be pushed down to the logical level by introducing nodes representing potential inconsistencies as anomalies, and playing no role unless nodes representing normal states of affairs do not apply. This form of inference may be introduced through a *preference relation* between the disjuncts of a disjunction such that a node is inferred to be false if any preferred node is not false.

For example, if a disjunct of a structural disjunction representing an anomalous condition is less preferred than the other disjuncts it will be inferred false and play no role unless all the other disjuncts are false. Then, being the only non-false node, it will be inferred to be true.

8.1. Preferential defeasible inference

The preference relation allows a partial order to be specified between the nodes of a structural disjunction. It is represented textually by 

the symbol, $<$,[49] with A$<$B specifying that node B is preferred to node A, and in a net by a link as shown on the right.

The associated inference patterns are shown in Figure 122, transitivity on the left, and exclusion if a preferred node is included or not linked (that is, it is not excluded) on the right.

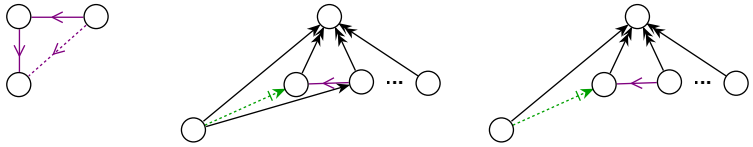


FIGURE 122. Preference inference patterns

In terms of contextual semantics, if the context of a node is part of that of a node that has sub-contexts then it is assumed to be disjoint with that of a less preferred sub-context if it is not disjoint with that a more preferred one.

These inference patterns are only applicable once all other inference patterns have been applied. They may lead to further inferences which are *defeasible* because they are based on the preference relation. The source of the defeasibility is apparent in the inference pattern on the far right which licenses an inference to be made when the truth value of a node is unspecified.

8.1.1. Defeasible material implication. For an example of the role of the preference relation, consider the basic material implication pattern (§7.2.6) of Figure 123 left and the various patterns of assertions and inferences to the right.

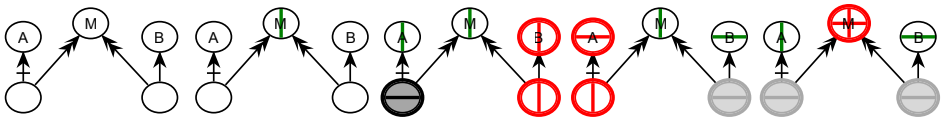


FIGURE 123. Material implication

From left to right: if node M is asserted to be true then A$\supset$B has been asserted but there will be no inference until truth values are asserted for A, B, or both; if nodes M and A are asserted to be true then node B will be inferred to be true; if node M is asserted to be true and node B false then node A will be inferred to be

[49]The predecessor, $\prec$, and successor, $\succ$, symbols are now commonly used in the social choice [163, p.26] and nonmonotonic reasoning [211, p.182] literature to represent preference orders. Unfortunately the usage is ambiguous because the specified order relation is reversed in the two literatures. Early studies of human preference used A$<$B to indicate that B is preferred to A [102, 317], but later studies use B$>$A [163]. Shoham's seminal study of preference-based nonmonotonic reasoning used A$\sqsubset$B [328, p.74], but later studies use B$\prec$A to indicate that B is preferred to A [211, 218]. When the relations between the logics of preference in social choice theory and in nonmonotonic reasoning were noted [220], this difference in conventions became a potential source of confusion. Bochman's [56, p.45n1] book explicitly reverses the convention of previous preference-based nonmonotonic reasoning literature and adopts one consistent with social choice theory.

false; if nodes M and A are asserted to be true and node B false then node M will be inferred to be inconsistent.

However, if, as shown in Figure 124, the anomalous condition leading to inconsistency is explicitly represented by an additional disjunct (labelled "?") that is less preferred than the other two then it will be inferred true and M will not be inferred inconsistent.

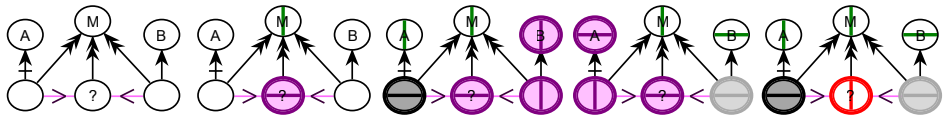


FIGURE 124. Defeasible material implication

The nets on the right of Figure 124 show the inferences when node M is asserted true and A and B asserted to have various truth values. The mauve colouring (light grey shading with mid grey surround in printed version) indicates a defeasible inference that may be changed without introducing inconsistency.

From left to right: if node M is asserted to be true then A defeasibly implies B has been asserted but again no truth values will be inferred for A and B; if nodes M and A are asserted to be true then node B will be defeasibly inferred to be true; if node M is asserted to be true and node B false then node A will be defeasibly inferred to be false; if nodes M and A are asserted to be true and node B false then node ? will be inferred to be true.

Thus the defeasible material implication makes the same inferences as the standard one except that it marks them as defeasible and explicitly represents the failure of the implication. Metalogical inconsistency has been pushed down to be indicated by the truth of a node indicating an anomalous situation from which further inferences may be derived. Inconsistency is no longer just tolerated and localized in the protologic but represented within it as a particular state of affairs.

8.1.2. Truth table for defeasible material implication. Table 14 shows truth tables for standard and defeasible material implication when node M has been asserted true. The third row exhibits the transition from inconsistency to anomaly representation.

TABLE 14. Truth tables for standard and defeasible material implication

M	A	B
◐	⊖	⊖
◐	⊖	◐
⊕	◐	⊖
◐	◐	◐

M	A	B	?
◐	⊖	⊖	⊖
◐	⊖	◐	⊖
◐	◐	⊖	◐
◐	◐	◐	⊖

8.1.3. Defeasible taxonomic coverage. Figure 125 left presents a second example showing how a violation of the assumption of taxonomic coverage can be represented as an anomaly rather than an inconsistency. Taxonomic coverage is asserted by setting the top node true, and the non-preferred node at the second level 'guards' the top node from becoming inconsistent if the coverage assumption is violated. The exclusion links may be specified or inferred from differentia so this technique is applicable to determinable/determinates, genera/differentiae and similar structures.

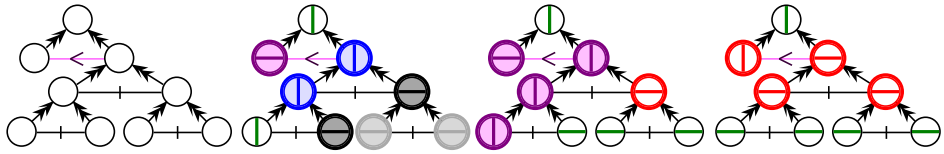


FIGURE 125. Representing violation of assumed taxonomic coverage

The other nets in Figure 125 show from left to right: the inference that other bottom modes are false when one it asserted true; the defeasible inference that a bottom node is true when all the others are asserted false; and the inference that the exception node is true when all the bottom nodes are asserted false.

The need to make inferences based on the detection of anomalies in the application of logical structures occurs across many disciplines. If the conditionals represent normative rules in a game or legal system then their violation usually has well-defined consequences. If they represent a conjectured scientific theory their refutation indicates a failure in the application of a theory which requires further investigation. In other disciplines, such as mathematics and philosophy, they may indicate inadequate specification of the universe of discourse to be rectified either by incorporating domain restrictions in the theory or by extending the domain through the addition of ideal objects that resolve the anomaly.

In all cases the logical representation of the conditional needs to support diagnosis of the source of the anomaly rather than the merely indicate that the system has failed through inconsistency.

8.1.4. Aristotle's defeasible reasoning from likelihood and signs. Aristotle's [15, 25a19-20] primary focus in the *Prior Analytics* is on necessary deduction in which "certain things being stated, something other than which is stated follows of necessity".[50] However, he also originated the formal study of non-deductive inference [72, p.193].

In the final section of the *Prior Analytics* and in his many other works that discuss practical reasoning, such as *Topics*, *Sophistical Refutations*, *Rhetoric*, and *Nicomachean Ethics*, Aristotle analyses defeasible but reputable (ἔνδοξος) argument forms with premises that are "only for the most part true" to "reach conclusions that are no better" [16, 1094b21-22].[51]

[50] συλλογισμὸς δέ ἐστι λόγος ἐν ᾧ τεσέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγης συμβαίνει τῷ ταῦτα εἶναι

[51] περὶ τῶν ὡς ἐπὶ τὸ πολὺ καὶ ἐκ τοιούτων λέγοντας τοιαῦτα καὶ συμπεραίνεσθαι

Aristotle [15, 70a3-b5] distinguishes between arguments from a *likelihood* (εικός) that a certain propositions is correct for the most part, and a *sign* (σημεῖον) that the already accepted conclusion of a proposition may be argued to be plausible evidence for its premise.

He terms an argument from likelihood or signs [240] an *enthymeme* (ἐνθύμημα) [73] and analyses various forms within a syllogistic framework, providing several example of plausible, but refutable, arguments. As Allen [4] notes, for Aristotle "an argument may be *reputable* without being conclusive. Burnyeat [73, p.96] argues that one should not translate Aristotle's συλλογισμὸς as "syllogism" because the latter term has come to be used in a much narrower sense than Aristotle's more general notion of constitutes a reputable argument.

Figure 126 represents one of Aristotle's examples of argument from likelihood, the reputable proposition that *the envious hate.*[52] The net of the left represents the proposition as certain using a universal material conditional. That on the right represents it as being only likely by including a non-preferred node to allow for the possible failure of the conditional if someone who envies does not hate.

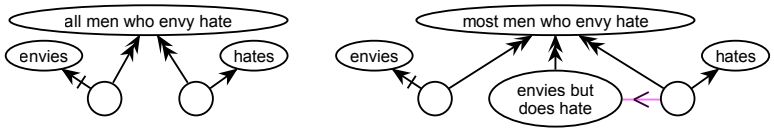


FIGURE 126. Aristotle argument from likelihood

The difference in behaviour of the two conditionals is that if the universal is asserted then a failure results in the assertion being marked inconsistent, whereas if the likelihood is asserted the assertion is still accepted but the anomaly of the failing case is inferred.

Figure 127 left represents one of Aristotle's examples of argument from signs, that *a women being sallow is a sign that she is pregnant.*[53] The non-preferred mode represents the possibility that a woman is sallow but not pregnant.

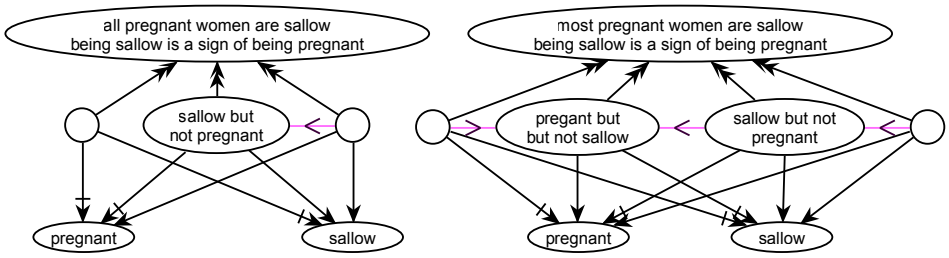


FIGURE 127. Aristotle argument from signs

It is extended on the right to one from both likelihood and signs with an additional non-preferred node representing the possibility that a woman may be

[52] τὸ μισεῖν τοὺς φθονοῦντας

[53] ἐπεὶ γὰρ ἕπεται ταῖς κυούσαις τὸ ὠχρόν

pregnant but not sallow. The non-preferred nodes represent the two possible anomalies: left, that the argument from likelihood fails; right, that the argument from signs fails.

8.2. Patterns of defeasible inference

Aristotle's argument forms are specific examples of generic schemata for defeasible inference which may be viewed as the extension of the representations of material implication, and more complex sentential constructions, as a disjunction over a set of bottom nodes representing epistemic templets (§7.2.6). In general, defeasible reasoning may be represented through a preference relation on the applicable templets that is used to make a choice between them rather then simply infer their common inferences if each is considered in turn to be true.

Figure 128 centre shows how the material conditional on the left can be extended to represent Reiter's [300] *normal default*, that if A is known then B may be defeasibly inferred provided there is no reason to infer that B is false. On the right the net is extended to represent Lehmann's [218, §6] preferred variant in which, in order to imitate the *modus tollendo tollens* of a standard material implication, if B is known to be false then A may be defeasibly inferred to be false.

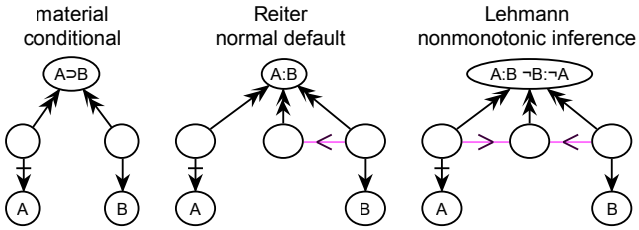


FIGURE 128. Defeasible material implication

Figure 129 left extends Reiter's normal default to represent Peirce's abductive inference as the defeasible affirmation of the consequent [283]. If B is known and A implies B then A will be defeasibly inferred. In the centre Lehmann's variant is extended to represent abduction so that, in addition, if B is known to be false then A may be defeasibly inferred to be false. On the right it is extended to defeasible equivalence, that the truth values of A and B are the same.

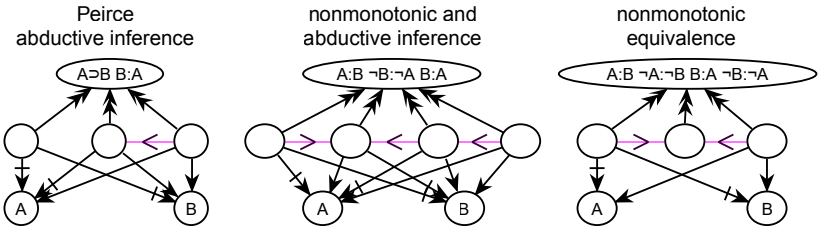


FIGURE 129. Defeasible abductive material implication

Figure 130 shows two variants of a more complex conditional with two conflicting defaults: A normally implies C but A and B normally implies not C. The second default is more specific than the first and the conflict is resolved through the usual convention of specificity [218, §3.1(4)] that the more specific default takes precedence over the less specific one, that is, the specified defaults are interpreted as A∧¬B:C and A∧B:¬C.

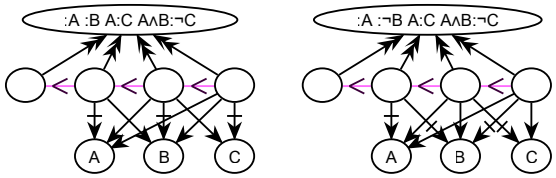


FIGURE 130. Multiple defaults

On the left the normal defaults are added that A is true and B is true and A implies C. The behaviour is that: with no information A and B are assumed true and C inferred false; if B asserted false then A is assumed true and C inferred false.

On the right the defaults are changed such that B is normally false, and the behaviour becomes that: with no information A is assumed true, B false, and C inferred false; if B asserted true then A is assumed true and C inferred false.

These behaviours are all defeasible and any truth values may be asserted for A, B and C without inconsistency.

8.2.1. Multiple interacting defaults. The examples in the previous section essentially compile multiple defaults into a preference relation over a single disjunction of epistemic states. This has been shown to be a sound basis for well-defined and well-behaved patterns of default reasoning [211, 56]. However, in practice, defaults may be specified independently, for example as components of ontologies or theories intended to be combined with others.

While it has been a major topic for research, there is no generally agreed procedure for compiling multiple defaults to a preference relation over epistemic states, or otherwise processing them in a rational fashion [58]. There is also no consensus what constitutes rational nonmonotonic reasoning although there are many plausible criteria [211, 218].

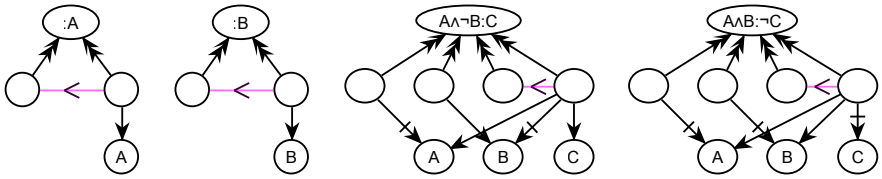


FIGURE 131. Separate multiple defaults

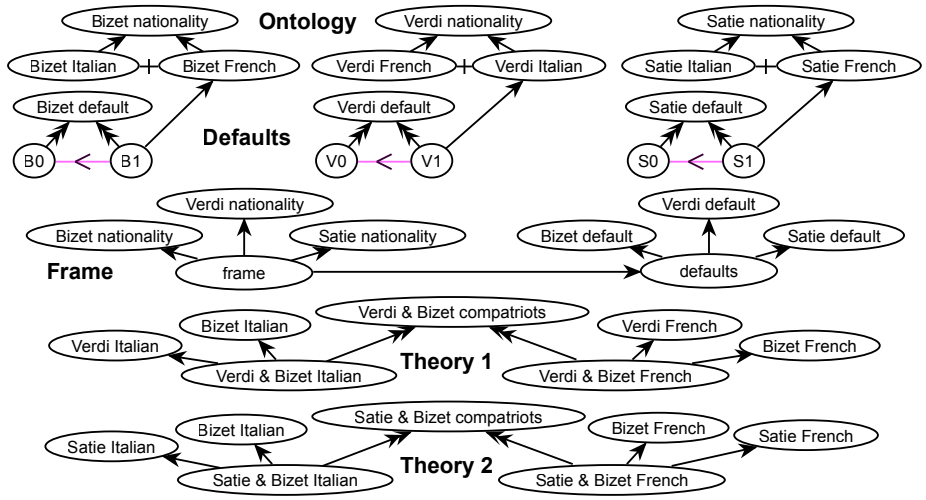


FIGURE 132. Stalnaker's test case

8.2.2. A defeasible abductive solution to an expert systems problem. Figure 133 shows a disjunction used to implement an abductive solution to the contact lens prescription problem described in §4.3.2. It represents the linguistic description of the solution in terms of templets with defaults given in §4.3.2. The determinable/determinate structures of Figure 76 are assumed and the case at the bottom has been marked true as in Figure 77. The inference pattern is similar to that of the rules with exceptions in Figure 103 and the outcome is the same.

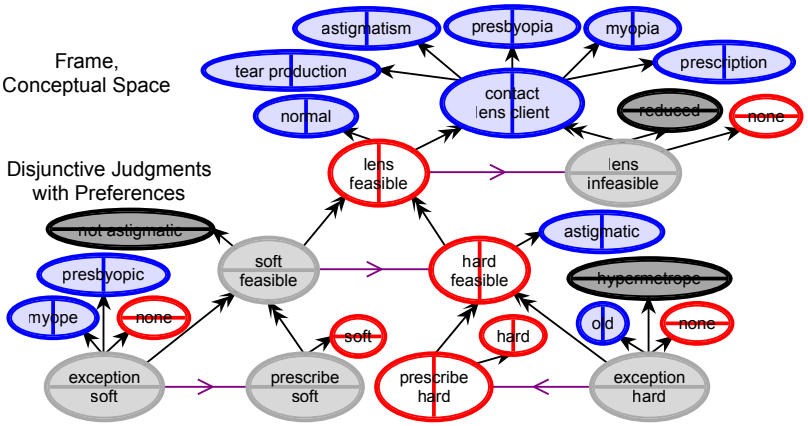


FIGURE 133. Disjunctive judgement used to recommend lens prescription

IMPACT OF DEFAULTS – CENDROWSKA's REQUIREMENTS – Q A IN ES

Table 15 shows the description logic translation of the net of 133.

TABLE 15. Description logic translation of disjunctive solution

Description logic expression	Type
contact_lens_client ⊑	frame
tear_production ⊓ astigmatism ⊓ presbyopia ⊓ myopia ⊓ prescription	
contact_lens_client ≡ lens_feasible ⊔ lens_infeasible	disjunction
lens_infeasible ≺ lens_feasible	preference
lens_feasible ⊑ normal	templet
lens_infeasible ⊑ reduced ⊓ none	templet
lens_feasible ≡ soft_feasible ⊔ hard_feasible	disjunction
hard_feasible ≺ soft_feasible	preference
soft_feasible ⊑ not_astigmatic	templet
hard_feasible ⊑ astigmatic	templet
soft_feasible ≡ exception_soft ⊔ prescribe_soft	disjunction
prescribe_soft ≺ exception_soft	preference
exception_soft ⊑ myope ⊓ presbyopic ⊓ none	templet
prescribe_soft ⊑ soft	templet
hard_feasible ≡ prescribe_hard ⊔ exception_hard	disjunction
prescribe_hard ≺ exception _hard	preference
prescribe_hard ⊑ hard	templet
exception_hard ⊑ older ⊓ hypermetrope ⊓ none	templet

8.3. Default reasoning and logics of fiction

[219]

[178]

8.4. Individuals

Individuals play a major role in most ontologies and logical systems generally provide the means to represent and reason about such a notion. However, the very wide range of explicata proposed for the notion of an individual, and the controversies associated with them [345, 161, 97], indicate that it would be inappropriate to treat the notion as primitive, even though this is a feature of many major logical systems.

A universal protologic should be capable of representing the diversity of models of individuals without bias, including being able to capture the logical aspects of those systems which take the notion as primitive.

8.4.1. Nature of individation. The diversity of analyses of the notion of an individual stem from issues that go beyond those of metaphysics and logic. Millikan [254, ch.1] provides an ontologically minimal pragmatic explicatum for the individuation of substance as supporting a human need to reidentify relevant, replicative phenomena in order to anticipate them.

From this perspective, the significance of individuals in our ontologies derives from the types of phenomena that affect our existence in our particular subuniverse where matter tends to aggregate in clumps and life is based on discrete cells. Even in our world there are significant "natural complexes" [70] that are not readily individuated, such as clouds, trees and societies.

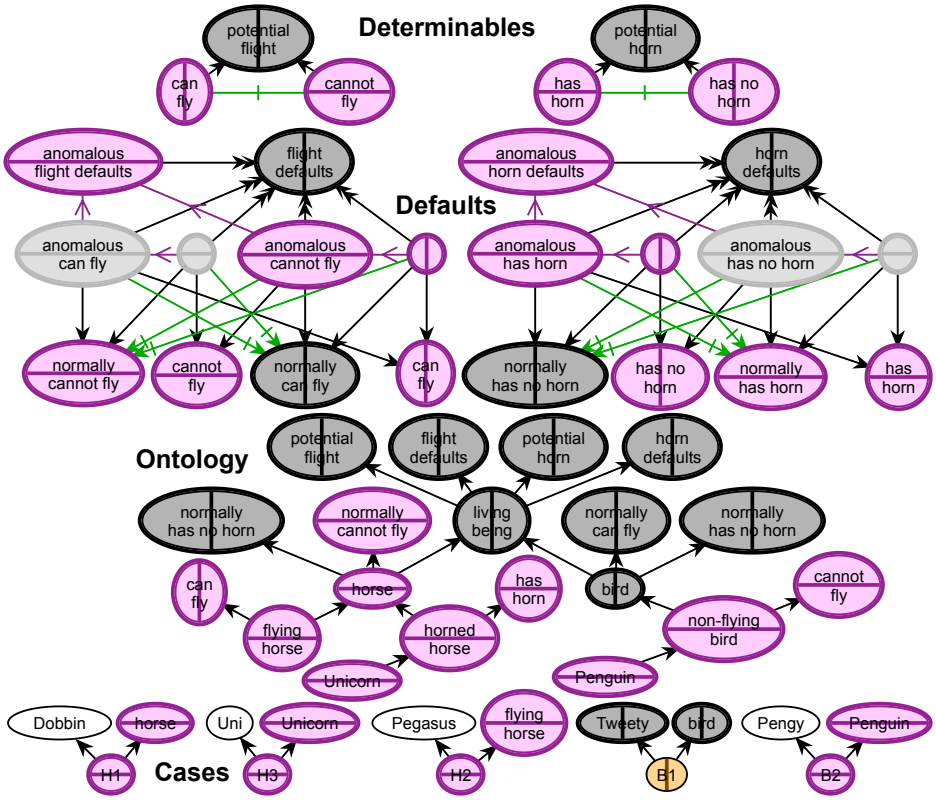


FIGURE 134. Animal taxonomy with defaults

Strawson [345, p.214] argues that the basic criteria of individuality requiring logical support are those of *distinctness* and *reidentification*. The first requires a stock of unique labels and the second the support of any logical constraints placed upon the application of these labels to phenomena; that is, issues of *reference* [121, 298]. Gracia [160] argues that *impredicability* is the essential characteristic of the entity representing an individual, that is, that it is a bottom node in the protologic.

As Frege and Russell developed modern logic they modelled individuating labels, or *proper names*, as abbreviations for explicit, or implicit, *descriptions* [312, 344] that could be logically represented. However, this analysis became contested as Marcus and Kripke developed the notion of *direct reference* [187] within a modal logic framework.

Recent studies have integrated these approaches by treating a proper name as itself a descriptive term, 'the entity named...,' an index to a *file* representing phenomena associated with what is imputed to be the 'same' individual [21, 269].

9. Representing states of affairs, individuals and relations

The protologic as developed above does not require the addition of individuals as primitive entities for any of the technical purposes for which they are normally required. Set-theoretic extensional semantics have been provided in §5.2.5 based on *probes* simply defined as bottom nodes in §3.3.7.

Bottom nodes satisfy Gracia's impredicability criterion for an individual and could be termed *protoindividuals* but they need not be distinct nor refer to any other entity. Extensional semantics are a technical feature intrinsic to the protologic involving no other connotations or denotations.

Figure 135 is a simple illustration of the representation of an individual based on the well-known example that Ramsey [296] provided to counter Russell's argument that there was an essential logical distinction between universals and particulars [313]. The anonymous bottom node represents a templet for a state of affairs in which an individual named 'Socrates' exhibits behaviour that can be characterized as 'wise.'

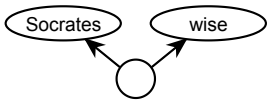


FIGURE 135. Representation of an individual

The nature of the Russell-Ramsey debate may be analyzed in terms of Figure 135. Ramsey's position corresponds to the anonymous node being symmetrically 'some Socrates' entity and 'some wise' entity. However, in its role of representing reference to the state of an individual, it may be distinguished from the other nodes in being an impredicative bottom node corresponding to the focus in Russell's article on singulars as states of affairs.

The bottom node satisfies Gracia's criterion for an individual if it is intended to represent a particular situation. If it is intended as a generic templet that applies to particular instances of Socrates then it does not, but those instances of which it is predicated will satisfy the criterion. That is, impredicability is not an absolute distinction but rather a pragmatic feature of use.

Strawson's criterion of reidentification captures the essence of the intent behind the use of proper names, to track what is imputed to be the 'same' individual in different situations. That is, reference to an individual is intended to be a *rigid designator* and this is again a pragmatic rather than logical distinction [27].

The bottom node will satisfy Strawson's criterion of distinctness if the term 'Socrates' is always used to identify what is intended to be a single individual, but, if it is applicable to several individuals, then additional identifying terms may be required to track the intended individual. That is, proper names may require contextual information to disambiguate their intended application [275, p.295].

A similar diagram to that of Figure 135 may be used to illustrate Wittgenstein's [378, p.25] suggestion to Russell in his letter of January 1913 that 'Socrates is mortal' be represented logically as "Socrates and $(\exists x)x$ is mortal."

In terms of Perry's [278, 21, 299, 298] analysis of proper names as functioning to index *mental files* of information about an individual, it may be that a combination of name and context is required to disambiguate the file access. One may generalize this to any combination of terms, including names, being used to provide an unambiguous path to the information about an individual, which corresponds to neo-descriptivist models of reference [269].

Access though indexing does not correspond to a basic logical function because specifying the node Socrates as true leads to no inferences. Castañeda [79] models individuation as requiring an additional *individuator operator* providing the indexical access.

One can model the individuator operator as one that infers which bottom nodes would make the specified indexing nodes true, and this may be implemented through the form of abductive inference discussed in §4.3.1 and §4.3.2. That is, given the assignment of truth values to the indexing nodes, one considers the disjunction of all the bottom nodes that have not been inferred to be false and draws the inferences common to them.

This operation is illustrated in the following sections through examples with multiple bottom nodes representing several individuals, states of a single individual, or substance templets [254] to be fitted to states of affairs representing experience of individuals. The same logical analysis serves for all three types of situation and their variants.

9.0.1. Venus as morning star and evening star. Figure 136 represents the two perceived states of Venus that Frege [130] used to exemplify his distinction between *sense* and *reference*. The planet appears as a bright celestial object near the rising or setting sun at dawn or dusk. Early Greek astronomers regarded the two phenomena as arising from different celestial bodies and named them differently [359, n.23].

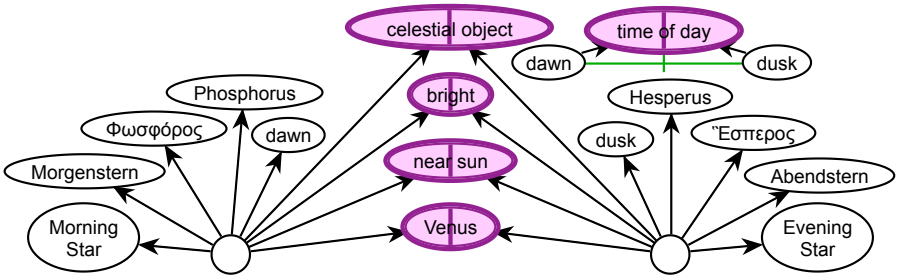


FIGURE 136. Representation of perceived states of Venus

Venus has many other perceived states, far from the sun, a dark object transiting the sun, and so on. They are relevant to the full representation of our perceptions of Venus in a semantic network, but not to Frege's discussion, and subsequent literature deriving from it, which is concerned with the formal issues of using the equality symbol in logic between names having the same referent.

The net illustrates the major issues. Many names may be used to reference the same physical object, and someone may not be aware that they do not refer to different ones; removing the node for Venus illustrates this. In addition someone may know some of the names and that they have a common referent but not know the corresponding names in another language; removing some of the state-specific names illustrates this.

Adding links to the net of Figure 136 that make any or all of the names equivalent creates no inconsistency. However, equivalencing the two states make the net inconsistent because they are incompatible through the exclusion relation between dawn and dusk.

Abductive inference after marking the nodes representing dawn or dusk as true leads to all the morning star names and Venus being marked true but none of the evening star names, and *vice versa*. That is, it is reasonable to have different names that reference the same entity in different contexts.[54]

9.0.2. Representing multiple individuals and their states. Figure 137 represents part of the semantic network of a student taking a course, PHIL227, on the lives of some ancient Greek scholars. It illustrates the representation of multiple individuals and their multiple states

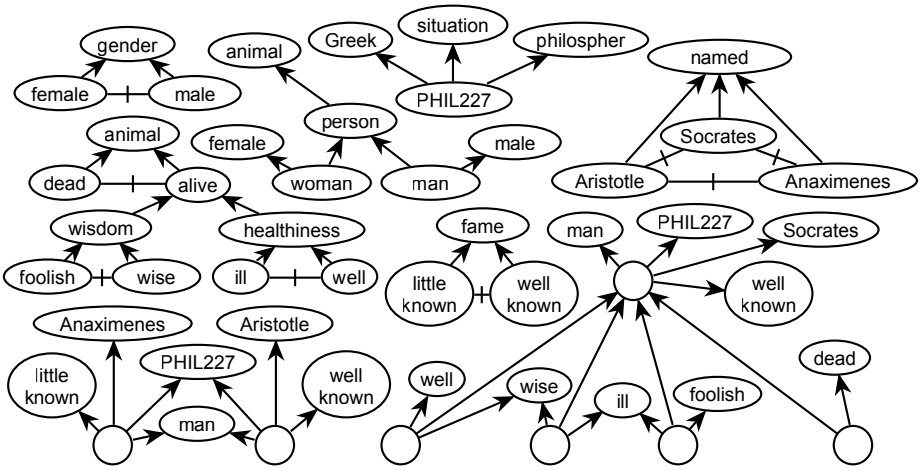


FIGURE 137. Representation of multiple individuals and states

At the top left are determinables for some of concepts used in the course. Top centre represents that all the situations under consideration are the topics of the course, individuals who are Greek philosophers. Top right represents the names of the three distinct individuals considered. At the bottom are templets for the individuals and, in the case of Socrates, for a number of his possible states with the common features factored out.

[54]Particularly if, in considering Frege's example, one takes into account that some ancient cultures viewed Venus as a god that changed gender between dawn and dusk. [359, p.172-173]

Abductive inference from the node situation takes the disjunction of all the non-false bottom nodes that include that node and concludes the inferences that they have in common. On the net as shown it infers that all situations are male Greek philosophers considered in PHI227 and that their fame is relevant but no determinant of fame may be inferred.

It does not infer that their wisdom and healthiness are generally relevant because the situation of Socrates being dead is included where they are not and there are no determinates of these determinables attributed to states of Anaximenes and Aristotle. If the nodes Socrates and alive are marked true then wisdom and health are inferred relevant and well known true. If, in addition, the node foolish of a state of Socrates is marked true it is inferred that he is ill. However, if the node ill is marked true it is not inferred that he is foolish.

Thus the logical constraints represented in the net lead to inference patterns that are often attributed to rules but arise through abductive reasoning asserting the disjunction of the logically possible situations given the information specified.

9.0.3. Variations on the representation of individuals. The preceding sections illustrate one approach to the representation of individuals in the protologic. There are many possible variations corresponding to those adopted in particular logics.

The representation conforms with Mill's [253, §5] dictum that "proper names are not connotative: they denote the individuals who are called by them; but they do not indicate or imply any attributes as belonging to those individuals." In the protologic this corresponds to names being represented by top nodes that include no others and only exclude other names.

However, there is nothing in the protologic to prevent names being given connotations if this is deemed appropriate. Those in Figure 137 have been give the general connotation that any individual to which they are applied has been 'named.' This could be extended to more specific connotations that are deemed *essential* to the application of a particular name and would have the effect that the application of the name to an individual having an incompatible connotation would be inconsistent.

Specifying specific connotations for names is problematic unless the lexical structure of the names used is extended so that each is a unique identifier intended to be denote only one individual, as is attempted for medical records [9]. An alternative approach is that illustrated in Figure 137 by the anonymous node at the left middle that factors out some characteristics that are postulated to be common to every state of Socrates. Such a node might be labelled 'stereotypical Socrates.'

Figure 138 shows some of the variations possible: on the left is the basic representation of a state of affairs representing Socrates as wise and mortal; in the centre mortal is treated as essential to the individuator Socrates; on the right mortal is treated as included in the notion of 'Socrates the person' and, to contrast with this, Socrates made immortal as the protagonist of Plato's philosophical

writings is represented as 'Socrates the literary figure.' The historic and literary figures are linked through their inclusion of the same individuator.

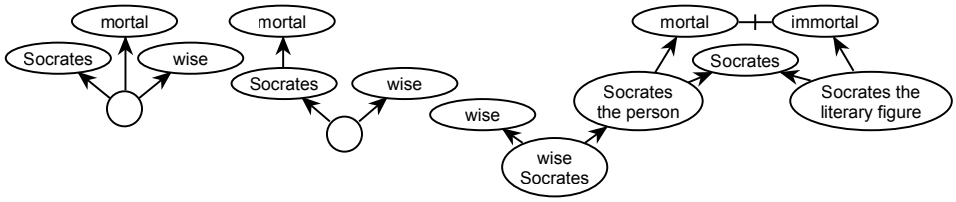


FIGURE 138. Representing the essence and the state of an individual

Each of these representations is logically viable and an illustration of how different ontological distinctions may be represented in the protologic. Essence, accident, state condition

Figure 139

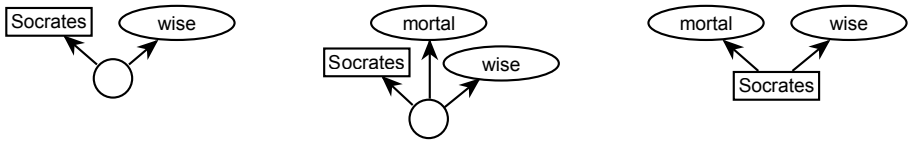


FIGURE 139. Representing individuators

In first-order logic and related systems, such as *description logics* [20], individuals are represented primitives that are distinct from predicates. This can be done in the protologic by introducing a variant node that is constrained to be a bottom node. This was a feature of a version of the graphical notation used in this paper that was designed to provide a visual language for description logics [140, §3.6].

A disadvantage of treating individuals as logical primitives is that representation of the same individual in multiple states becomes problematic and requires the introduction of further primitives to support, for example, possible worlds and/or temporal reasoning [19, 330]. This parallels the human need for the concept of different situations/contexts and/or time in order to account for changes in what is imputed to the 'same' entity.

A natural way to introduce time in the protologic is to note that time intervals may be regarded as determinates of the determinate *duration*, having inclusion and exclusion relations that may be represented as described in §4.1.3. Nodes representing durations may be included in nodes representing states of affairs to provide a context that indexes them just as does any other form of context. This does not reduce the complexity of temporal reasoning but brings it within the general framework of reasoning with other forms of contextual information.

Most issues of *reference* [121, 298] are not ones of logic, but rather ones of epistemology, psychology and sociology. Millikan's [254] pragmatic, constructivist

account of the human need to track and anticipate replications of substance provides a general systemic grounding for the wide variety of issues, including the emergence of symbolic language [347].

Issues of *reference fixing* are those of *symbol grounding* [171] in general, how we come to apply any symbol in a consistent and useful way. The developmental processes involved in symbol grounding have been studied extensively [26, 149]. Its computational emulation has become a major practical problem in artificial intelligence studies [348] as technologies that interact with the world are designed to communicate with people in natural language.

Logic has no direct role to play in resolving issues of reference and symbol grounding. However, it plays a major secondary role in formalizing the recognition of incompatibilities in the application of symbols.

Such differences may reflect errors in the application of symbols, changes in the entity to which they are applied, differences in individual or cultural usage, and so on. How these incompatibilities are to be resolved is largely extra-logical, but, as exemplified in §2.5.4, logical models may be used to identify the source of the inconsistency and the minimal changes that would resolve it [3].

9.1. Relations

10. Other topics

11. Conclusons

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Context – how it implement qua? – how do we filter out what is irrelevant? – applicable sub-contexts

Taxonomies as definitions

Granger sees A as using determinable/determinate

Mention supposition – relation to existence?

Disjunction as a basis for negation – Aristotle's several assumptions of this – Kant's disjunctive judgement

Art example, link back to semantic fragmentation

τὸ μὲν Ἑλληνικὸν ὡς ἓν ἀπὸ πάντων ἀφαιροῦντες χωρίς σύμπασι δὲ τοῖς ἄλλοις γένεσιν ἀπείροις οὖσι καὶ ἀμείκτοις καὶ ἀσυμφώνοις πρὸς ἄλληλα βάρβαρον μιᾷ κλήσει προσειπόντες αὐτὸ διὰ ταύτην τὴν μίαν κλῆσιν καὶ γένος ἕν αὐτὸ εἶναι προσδοκῶσιν

τὸ δ΄ οὐκ ἄνθρωπος οὐκ ὄνομα ... ἀλλ΄ ἔστω ὄνομα ἀόριστον

τὸ τί ἦν εἶναι essence, what it is for a thing to be

συμβηκος accident

Other papers:-

Logic of art

Rational reconstruction of knowledge engineering

Visual languages for conceptual structures

George Kelly's mathematical psychology

Logic should provide a formal system for the precise encoding of the representational and inferential frameworks of ontology and epistemology. It should do so as a mathematical system independent of both that is capable of providing specialized structures that do not distort or add extraneous content to those frameworks. It should mirror them accurately in order to provide insights into the notions underlying frameworks and their applications that are relevant to, and comprehensible within, the originating disciplines.

Introduce notion that logical existence covers other cases by specification of universe of discourse.

have some papers of coexclusion in logic using sub-contrary terminology – need to mention sub-contrary as just meaning below contrary somewhere, unfortunate terminology

Text held over

The nature and role of negation in language and logic was recognized as being problematic by ancient philosophers such as Parmenides, Plato and Aristotle [180], and the issues continued in the development of modern logic as Russell, Demos and Wittgenstein debated the nature of 'negative facts' [309, 268, 272]. The problems may be analyzed as ones introduced by the coexclusion component of negation.

Ferejohn [123] has developed a very detailed analysis of the issues with negation that Plato discusses in *Sophist* and *Statesman* in terms of the *semantic fragmentation* that can occur through the unrestricted negation of a concept, and the solution proposed by Plato and Aristotle that negation should always be relative to a context. These considerations are also relevant to the modern debate on the status of negative facts, and it is interesting to determine whether they can be usefully represented in the kernel protologic.

Aristotle criticized an unrestricted negation such as 'not man' as an indefinite (ἀόριστος, aorist) term not designating anything as does a name.[55] Plato exemplified the problem in *Statesman* by considering the tendency in Greece to divide the human race into 'Greek' and 'not Greek' and name the 'not Greek' as 'barbarian' as if that were a single kind despite the disparity of origins, cultures and languages.[56]

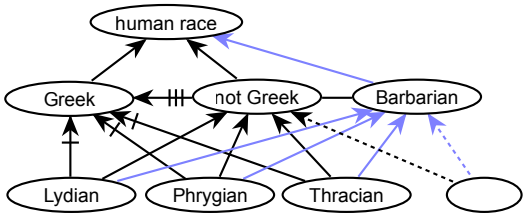


FIGURE 140. Plato's example of semantic fragmentation

The semantic fragmentation that he criticizes may be seen as arising through the coexclusion component of negation, that every node that excludes its head most include its tail (Figure 26). From a meaning containment perspective, this implies that every node that is

[55] τὸ δ' οὐκ ἄνθρωπος οὐκ ὄνομα ... ἀλλ' ἔστω ὄνομα ἀόριστον [14, 16a30-32]

[56] τὸ μὲν Ἑλληνικὸν ὡς ἓν ἀπὸ πάντων ἀφαιροῦντες χωρίς σύμπασι δὲ τοῖς ἄλλοις γένεσιν ἀπείροις οὖσι καὶ ἀμείκτοις καὶ ἀσυμφώνοις πρὸς ἄλληλα βάρβαρον μιᾷ κλήσει προσειπόντες αὐτὸ διὰ ταύτην τὴν μίαν κλῆσιν καὶ γένος ἓν αὐτὸ εἶναι προσδοκῶσιν [281, 262d]

incompatible with a given node contains the same meaning but it is not apparent that the races that include 'not Greek' have anything in common.

Aristotle's technique for managing semantic fragmentation was to require that the exclusion components of a negation be expressed relative to a common context. He notes in *Categories* [14, 14a19-20] that "contraries should in all cases either belong to the same genus or belong to contrary genera, or be themselves genera," and in *Metaphysics* [17, 1054b23-26] that a contrary, in order to express a *difference* (not just 'other than'), needs to be relative to a *likeness*: "But difference (διαφορὰ) and otherness (ἑτερότης) are distinct (ἄλλο). For, while the other and that which it is other than are not necessarily other in something (for everything that is some being is either other or the same), the different differs from something in something, so that it's necessary for there to be something the same in which they differ."

Aristotle provided examples where the context was so narrow that a negation corresponded to a unique well-defined notion. Figure 141 left shows his example of the term 'not odd' qua 'number' corresponding precisely to the term 'even' [15, 73b23-24].

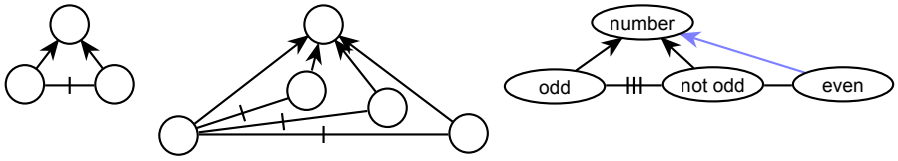


FIGURE 141. Aristotle's example of semantic coherence

This example becomes logically sound if the protodisjunction of 'odd' and 'even' is replaced with a structural disjunction of the term 'integer,' and Aristotle assumes the available of that construction in various discussions in his *Organon* although it is not available in his development of syllogistics. Kant [200, §27-29] added it explicitly as a 'disjunctive judgement' in his *Lectures on Logic* (§7.2.1).

De Morgan [100, p.379-380], in his reconstruction of syllogistic reasoning, termed the common context that ameliorates the aorist aspects of a negation a *universe*. He discussed his notion of a universe divided by a contrary with his protégé Boole, who re-conceptualized in psychological terms it as a *universe of discourse* [334, p.102] determined by the "state of mind of the speaker."

In developing theoretical foundations for his *personal construct psychology*. Kelly [204, p.8-9] also argues for the basic structure of cognition being a difference relative to a likeness. He terms this structure a *construct* [142, p.193], describing it as a *templet* that we create and fit to experience in order to give it meaning and attempt to anticipate further experience. He termed the top node the *range of convenience* and the bottom nodes *dichotomous poles*.

The issues managed through a common context specifying the universe of discourse or range of convenience are apparent if one neglects that provided by 'human race' in Figure 140, and considers 'not Greek' as applied to literature, cuisine, mountains, and so on. What is the common component of meaning to this diversity of kinds that accounts for their being 'not Greek'? Within the context of 'human race' it is reasonable to envision that there might be a simple definition of being 'Greek,' such as having a parent who is Greek, that leads to an equally simple definition of being 'not Greek.'

From this perspective the issues of

In many cases the issues with negation are diagnostic of heterogeneity in the meaning of the term being negated. This is not apparent in Plato's example where a suitable definition of 'Greek' might make 'not Greek' a term with a well-defined meaning that could be construed as the essential component of the meaning of each non-Greek race, even though the aggregation of such would be a heterogeneous collection having no other common attributes. That is, only the negation, not the original term, is semantically fragmented.

Contrast this with a notion such as 'perfect dinner party' where one person might classify such an event as 'not perfect' because the food was of good quality, another because certain people failed to attend, another because an unpleasant argument arose, and so on. Figure 142 represents these situations and it is apparent that each type of failure excludes 'perfect dinner party' and hence includes its negation, but that the reasons for this are the inferred exclusions of very different types of failure that have little semantic content in common. The semantic fragmentation of the notion of 'poor dinner party' stems from the semantic heterogeneity of the many aspects of the notion of 'perfect dinner party' and is not just a case of negation in itself classing together several kinds of entity having few attributes in common.

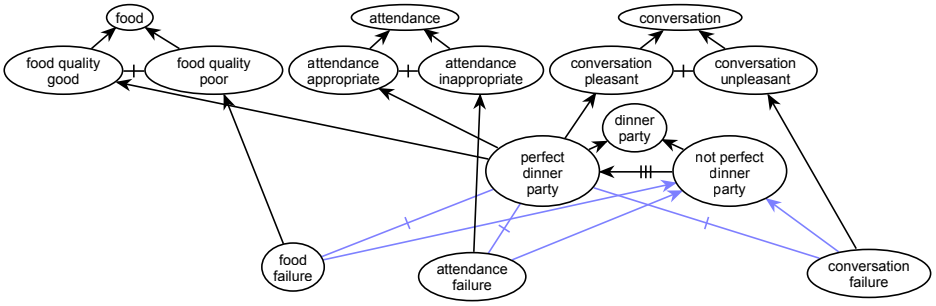


FIGURE 142. Negation of a heterogeneous concept

Many significant notions in a wide range of disciplines have this characteristic of semantic heterogeneity through their inclusion of diverse terms having no commonality of meaning, with their only connection being their joint involvement in certain states of affairs. They present problems of semantic fragmentation that make a coherent account of their natures problematic. A good example is the controversy in the literature on the philosophy of art about the definition of an 'art object' [98, 76] (§4.3.3).

This section has shown that the kernel protologic supplemented by coexclusion is sufficient to represent the issues involved in semantic fragmentation. In particular, the use of the weak logical structures developed through the resemantification of → and the avoidance of stronger ones than needed indicates that the logical framework is minimal and required by the situations being analyzed, and does not introduce notions stemming only from the logical system that are not inherent in those situations.

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