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A Minimal Framework for Knowledge Representation Sufficient to Support Analogical Reasoning and Other Forms of Inductive Inference

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MOTIVATION

This paper attempts to establish the role of knowledge representation (KR) in inductive reasoning by separating the formal operations of inductive algorithms from their operation in a specific KR scheme, and establishing minimal constraints on a KR scheme for these operations to be viable. It is important in this analysis to ensure that the operations and constraints derived are adequate to carry out recognizably effective inductive reasoning in realistic domains. We apply the results in detail to an analogical reasoning tool (Leishman 1989) that uses well-defined algorithms to emulate significant examples of analogical reasoning studied by analogy researchers such as Gentner (1983, 1987), Indurkhya (1985), and Russell (1987). We also apply the results to algorithms for empirical induction showing that the operations and constraints are adequate to support their operation, and that the variations in their application throws light on the similarities and differences between analogy and empirical induction. We link the results to foundational studies of the role of generalization in induction (Niblett 1988, Krawchuk 1989).

KNOWLEDGE REPRESENTATION

Since we wish the framework presented in this paper to cover many forms of induction and knowledge representation we will introduce a minimal structure that is adequate for our purposes leaving the primitives as little defined as possible. Our basic primitive is a class of knowledge structures (KSs) which are undefined but can be thought of as statements about a world. To be able to derive relations of analogy between KSs, that is structure-preserving partial correspondences, we need a few simple operations. To establish a correspondence, we need the capability to test that two KSs are the same in some sense. To allow the correspondence to be partial at a structural level, we need the notion of one KS being 'part-of' another in some sense, and the capability to be able to select the parts. To allow the correspondence to be partial at an informational level, we need the notion of one KS being a 'generalization' of another, that is that of losing existing information and not adding any new information. It is possible to retain generality yet formalize these notions by embedding them in a category-theoretic framework. We will, however, present them less formally at this stage in order to retain clear links to computational and psychological connotations of inductive reasoning. We are not primarily concerned in this paper about issues of decidability and consistency for some of the 'large' structures that come under our unrestricted definitions, and will use a set-theoretic vocabulary for the primitives needed.

Thus consider:

1. A set K, of possible *knowledge structures*
2. An idempotent, symmetric, transitive relation of *equality* on K, =, such that is possible to determine for all k,e∈ K if e=k, that is whether e is equal to k
3. A relation of *part-of* on K, π, such that it is possible to determine by some procedure for all k,p∈ K if pπk, that is whether p is a part-of k, and which is consistent with the relation of equality:

$$\forall k,k' \in K,\ k=k' \Rightarrow \forall p \in K:\ p\pi k,\ \exists p \in K:\ p=p' \qquad (1)$$

that is, equal structures have equal parts.

4. The *part-of* relation also defines a natural partial order on K, $\supset$, such that:

$$\forall k,j \in K,\ k \supset j \Leftrightarrow \forall p \in K:\ (p\pi k \Rightarrow p\pi j) \vee (\exists i \in K:\ k \supset i \wedge i \supset j) \qquad (2)$$

The second term is put in to obtain a transitive closure since π may not itself be transitive.

5. A relation of *generalization* on K, γ,such that is possible to determine for all k,g∈ K if gγk, that is whether g is a generalization of k, by the following test:

$$\forall k,g \in K,\ g\gamma k \Leftrightarrow \forall g' \in K:\ g'\pi k',\ \exists k' \in K:\ k'\pi k \wedge g'\gamma k' \qquad (3)$$

That is, a is a generalization of k if every part of a is a generalization of some part of k, that is, if Figure 1 holds.

6. The *generalization* relation also defines a natural partial order on K, ≥, such that:

$$\forall k,g \in K,\ g \geq k \Leftrightarrow g\gamma k \vee (\exists b \in K:\ a \geq b \wedge b \geq k) \qquad (4)$$

That is the relation, ≥, is the transitive closure of the relation, γ.

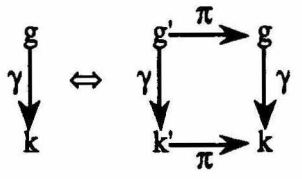

Figure 1 Diagram defining generalization

Note that we have defined generalization very generally as *any* relation that is inherited by every part of the generalization —this corresponds to the very wide range of meanings of the colloquial notion of generalization as a transformation that loses information and adds none. We have subsumed all aspects of structure-preservation in the *part-of* relation applied to knowledge structures. Note that this subsumes a variety of structures within the world which the KS represents. For example, if a logical calculus were being used for knowledge representation, its entailment relation, $\rightarrow$, would be a natural choice for the definition of π, that $p\pi k \Leftrightarrow k \rightarrow p$.

Note that we have not required the basic relations *part-of* and *generalization* to be transitive. It is often the case that components are treated as indivisible at one level, and divisible at a lower level. We may not wish to think of a very high level generalization as being in itself a generalization of something very concrete, but rather think of the chain of generalizations joining them. In both cases the order relations obtained by transitive closure of the *part-of* and *generalization* relations correspond to our intuitions that something about these relations should be transitive. In the later discussion we will not distinguish between the relation and its transitive closure unless strictly necessary.

FORMATION OF ANALOGIES

The formation of an analogy between two knowledge structures may be expressed simply in terms of the primitives defined. An analogy is a structure-preserving, partial correspondence between two dissimilar KSs, the analogs. Generalization is used to reduce the dissimilarity until a common generalization for both analogs is found. Since, in analogical reasoning, one analog is generally assumed better known than the other and used to suggest the existence of non-evident structure within it, it is usually required that the generalization is minimal. Minimal generalizations are not necessarily unique. Thus a KS, a, is an analogy between two KSs, a' and a", if it is a mutual generalization and there is no generalization strictly lesser than it satisfying this condition, that is:

$$a\gamma a' \wedge a\gamma a'' \wedge \neg(\exists b \in K: a\gamma b \wedge \neg(b\gamma a) \wedge b\gamma a' \wedge b\gamma a'') \qquad (5)$$

This establishes a partial correspondence between a' and a" because, from (3), to each part-of the generalization, a, there corresponds a part of the analog, a', and a part of the analog a", so that we may regard these parts in correspondence through a relation, «a», such that:

$$\forall p', p'' \in K: p'\pi a' \wedge p''\pi a'', p'\langle\langle a\rangle\rangle p'' \Leftrightarrow \exists p \in K: p\pi a \wedge p\alpha p' \wedge p\alpha p'' \qquad (6)$$

Figure 2 shows these results in diagramatic form.

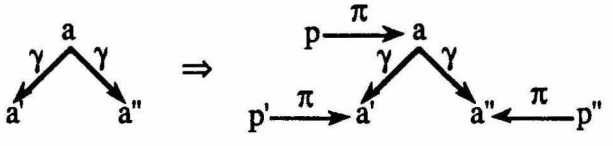

Figure 2 Analogy as mutual generalization and the implied partial correspondence

Thus the formation of analogies can be expressed in terms of the weakly constrained constructs which we have defined for a very general notion of a knowledge structure. The following section shows that this formulation is adequate to allow powerful analogical reasoning when conceptual graphs are used to instantiate our notion of a KS.

APPLICATION TO AN ANALOGICAL TOOL

Leishman (1989) presents an analogical tool which instantiates the minimal requirements of a KR defined above using the conceptual graph model (Sowa, 1984). The equality test, part-of relation and generalization relation, have a natural basis in the conceptual graph model and meet the specified constraints.

A single conceptual graph graph states a proposition and contains concept nodes, relation nodes, and arcs between them. This structure allows analogs to be represented as finite bipartite graphs. Concept nodes represent entities, attributes, states and events, while relation nodes specify relationships between concepts. Each concept in a conceptual graph is part of a lattice of concepts. This lattice supports inheritance, and concepts higher in the lattice

are more abstract than lower ones. Graphs can also express more complex sentences which contain individuals, modality, nested propositions and co-references. Nested propositions refer to concepts of type 'proposition' whose referent field contains the conceptual graphs being asserted by the proposition. Co-reference links show anaphoric references and express an equality relation between concepts. Sowa (1984) defines a 'generalization' operation over conceptual graphs that combines the part-of (subgraph) relation and the type hierarchy abstraction exactly as in (3). Any subgraph is a generalization of the original graph; replacing a type label with a supertype generalizes a graph; deleting an individual from the referent field generalizes a graph. The analogical tool derives a set of minimal common generalizations using these operations.

The analogical tool has been shown to perform effectively on a range of examples of analogy from the literature—from: the CYC project (Lenat et al 1986); Gentner's work (1983, 1987); Indurkyha's PhD thesis (1985); Russell's PhD thesis (1987); Gick and Holyoak (1983). Figure 3 shows an example of the tool in operation taken from Gentner's description of a heat-water analogy used by Carnot (Gentner 1987). This example is significant because it contains a pitfall for over-simplistic analogy formation processes and requires an extension to the analogy formation process defined in (6). At the top of Figure 3 is a representation a person might have of a water flow situation. The graph at the bottom of Figure 3, represented by dashed nodes with connecting arcs, is a description of what that same person might know about heat flow. These two analogs are presented to the analogical tool. The tool derives two analogies with equal measures. Both analogies have formed the same correspondences between concepts except that in one *Temperature* has been matched to *Pressure*, and in the other, *Temperature* has been matched to *Diameter*. The tool uses an *extensibility* principle to select the analogy which matches *Temperature* with *Pressure*. That is, this analogy is more extendable because it enables the tool to transfer both of the proposition boxes from the source water analog as well as the result relation between them over to the target heat analog, whereas the other can only transfer one of them.

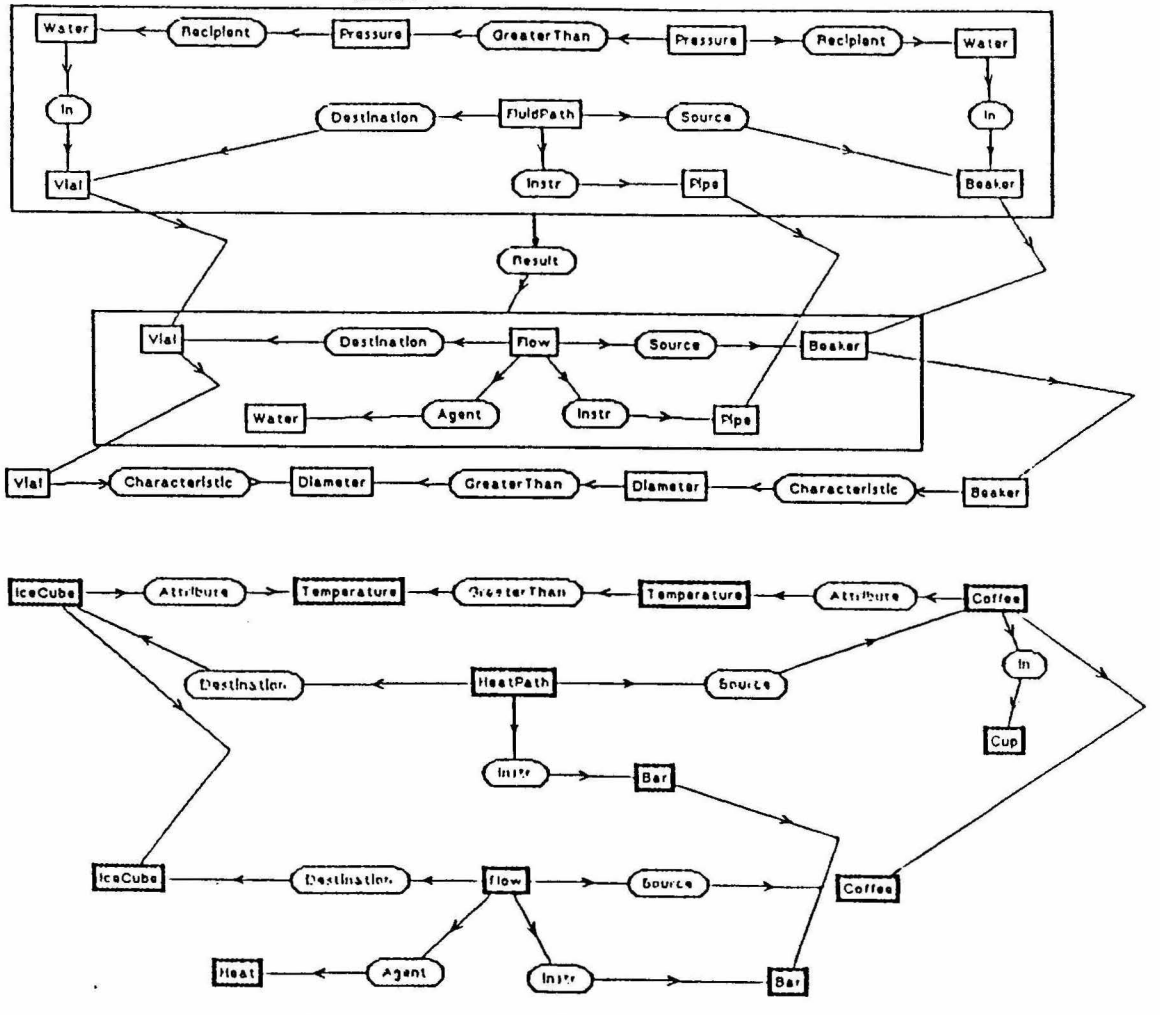

Figure 3 Example of a heat-water analogy

The *extensibility* principle can be defined in terms of our existing primitives by noting that it looks for an analog which is a lesser generalization than that defined in (6) by allowing the target analog about which an inference is to be made, a" say, to be extended in such a way as to make this possible. Figure 4 is a variation on Figure 2 showing an extended analogy, a, involving an extended analog, a", which must be maximal (with respect to a particular a) to avoid introducing structure that is not in a'. The part-of relations on the right are similar to those in Figure 2 but now involve the possibility that p" is *not* part of a" but rather a hypothesized extension of it.

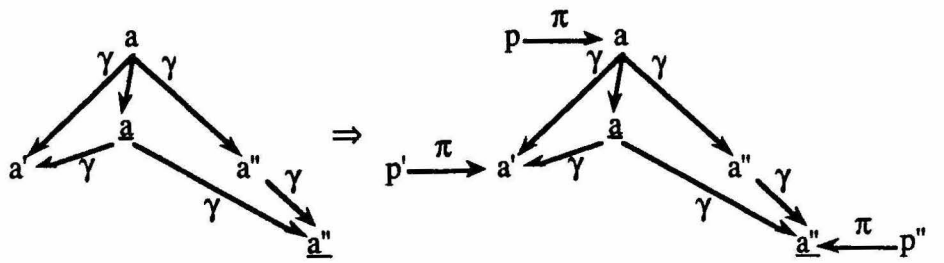

Figure 4 Extensibility of an analogy

OTHER APPLICATIONS

The definition of generalization given in (3) corresponds to that used by Niblett (1988) and Krawchuk (1989) in their analyses of a variety of inductive systems and machine learning techniques. The primitives we have defined seem adequate to encompass many forms of induction. In terms of the analysis in this paper, many techniques for empirical induction, for example, appear as the formation of maximal generalizations that discriminate separate sub-classes. The full version of this paper will contain analyses of various inductive techniques within this framework.

CONCLUSIONS

Very weak constraints on the notion of a 'knowledge structure' are sufficient to support powerful algorithms for inductive reasoning. These constraints are so weak and arise so naturally in inductive reasoning processes that it is reasonable to suppose they are also 'necessary' in the sense that without them such reasoning would not be possible. The sufficiency of these constraints implies that the notion that 'rich' knowledge representations are essential to effective machine learning is misleading if it is taken to suggest representation- or domain-dependence in the basic techniques. The implication is more subtle, that our knowledge representation scheme must support the semantics of our domain including both deductive and inductive inference processes. It is conventional to accept this requirement for deductive inference because we take that as being simply defined in terms of a limited number of very general inference procedures. What this paper suggests, together with related work on the formalization of inductive reasoning, is that inductive inference is also simply defined in terms of a limited number of very general inference procedures. These procedures are deductive up to the point a hypothesis is made based on them, for example, that the extension of a" in Figure 3 represents a true statement about the world described by a". The *plausibility* of the hypothesis is independent of the knowledge representation scheme used. The *truth* of the hypothesis depends, as it must, on the *correspondence* betwen the knowledge representation scheme and the world it represents.

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