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K. FLEISCHMANN, B. MAHR and D. SIEFKES, *On t-bounded concatenation, a uniform method for proving lower complexity bounds.*

The essence of Quine's paper *Concatenation as a basis for arithmetic*, this JOURNAL, vol. 11, is to describe computations, using e.g. arithmetic terms or Turing machines, by words over a finite alphabet. Recently this idea has been used to prove lower complexity bounds for logical theories by describing computations of time- and/or tape-bounded Turing machines within these theories. Following a suggestion of A. R. Meyer we introduce t-bounded concatenation theory as a general method for such proofs. Let t be a total recursive function. Then t-BCT is like ordinary concatenation theory where all atomic formulas $x = yz$ are bounded by $lx \leq tn$. Let t be monotonic, growing fast enough (roughly $(tn)^2 \leq t(2n)$ a.e.), let B be a set s.t. $t(e \cdot n) < \mathrm{Time}(B) \leq tn$ for some $e > 0$. Then for some $c > 0$, $t(c \cdot n)$ is a lower Turing machine time bound for t-BCT. The same bound holds for any theory to which t-BCT is poly-lin reducible. This method yields all the known lower bounds for logical theories which are exponential, hyperexponential, or nonelementary (Meyer, Fischer, Rabin, Rackoff). It therefore shows the general structure of all these proofs. It also strengthens the results to network complexity. For weaker theories we use analogously t-bounded word theory, t-BWT, with atomic formulas 's is the ith letter of y' and without quantifiers for words. Here we get lower bounds of e.g. $2^{c \cdot n/\log n}$ (Meyer, Rackoff). Polynomial-bounded word theory is NP-complete and is useful in proving NP-completeness.

CHRISTIANE ROUSSEAU and MICHAEL P. FOURMAN, *Parameters in complex analysis and the logic of topoi.*

Validity in topoi gives continuity in parameters. Consider the interpretation in the topos of sheaves on the parameter space. By a proper choice of model we can use topos validity to give more information, for example, holomorphy in parameters.

The sheaf $\mathbf{C}_M$ of continuous complex functions on a complex manifold M acts as the complex field in $\mathrm{Sh}(M)$, the topos of sheaves on M. In the internal logic, the subsheaf H_M of holomorphic functions is a Cauchy complete subfield of $\mathbf{C}_M$ with $\mathbf{Q} \times \mathbf{Q} \subseteq H_M$ and thus forms a suitable domain for analysis. Internal holomorphic functions from H_M to H_M interpret as actual holomorphic functions on $M \times \mathbf{C}$. In fact we have a bijection $\mathrm{Hol}(H_M, H_M) \cong \mathrm{Hol}(M \times \mathbf{C}, \mathbf{C})$.

Statements of one variable analysis interpret in $\mathrm{Sh}(M)$ as statements with parameters. Theorems are provable in the logic of topoi only if they are true holomorphically in parameters. The problems of dealing with parameters correspond precisely to the problems of giving constructive proofs (in the logic of topoi). The following one variable theorem is valid in the logic of topoi, and interprets as the classical implicit function theorem.

THEOREM (APPROXIMATION OF ZEROS). *Let $f: H \to H$ be holomorphic, then $f'(0) \neq 0 \to f(0) \# 0 \vee \exists \delta > 0\, \exists!\ y(|y| < \delta \wedge f(y) = 0)$.*

This is a special case of the next theorem, which interprets as the classical many variable division theorem.

THEOREM (WEIRSTRASS DIVISION THEOREM IN ONE VARIABLE). *Let $g, f: U \to H$ be holomorphic, where U is a neighbourhood of 0 in H, and suppose that $f^{(p)}(0) \neq 0$. Then, either $\exists i < p$ such that $f^{(i)}(0) \neq 0$, or $\exists W \subseteq U$, a neighbourhood of zero such that there exist a unique holomorphic $h: W \to H$ and a unique polynomial P of degree $< p$ with $g = h \cdot f + P$.*

B. R. GAINES, *Some experience with a confirmation machine.*

The study of inductive logics suffers from the lack of case studies rich enough to test all aspects of

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theories and yet not so complex as to themselves be imprecise. The determination of the structure of a probabilistic automaton from a sample of observations of its behaviour may be taken as an exceptionally well-defined situation in which to study inductive logics and scientific inference. However, until recently this problem was not well-formulated, let alone solved. This paper briefly outlines a general formulation of behaviour→structure inference that encompasses this and related problems. Its solution has been implemented as a computer algorithm and tested on a variety of problems including artificial and real-world data. The methodology and results obtained are examined as an explicit example of an inductive "confirmation machine" providing exemplary material for discussions of the scientific process by Carnap, Popper, Kuhn, et al.

The formulation is in terms of a set of models ordered statically by *complexity* and dynamically by their degree of *approximation* to an observed behaviour. The minimal elements in the product of these two orders are *admissible* models whose approximation is not bettered except by models of greater complexity. For the case of probabilistic automata, complexity is defined in terms of number of states of the model, and approximation in terms of Finetti/Savage probability-eliciting measures.

This formulation reveals: (a) the *arbitrariness* of the order relation of complexity — if too many real-world phenomena are complex then there will be a "revolution" in which models are re-ordered; (b) the *nonuniqueness* of solutions — all admissible models are equally valid and there is an essential choice to be made between accuracy and simplicity; (c) the variety of *validation strategies* possible — a modeller may choose to validate a simple model by searching for behaviour that it predicts but which has not been observed, or to falsify a complex model by searching for behaviour that it predicts cannot occur. The discussion is illustrated by examples of admissible model sets for a variety of probabilistic systems, obtained from the modelling strategy implemented as a computer program.

ROBIN GRAYSON, *Some separation principles in intuitionistic topology.*

The standard topological separation principles as classically formulated are not valid in the constructive real numbers. So we rephrase "T_0", "T_1", "T_2", as follows ($x, y, z \ldots$ are points, $U, V, W \ldots$ are open sets, of a space, X):

(T_0) $\forall U(x \in U \leftrightarrow y \in U) \rightarrow x = y$.

(T_1) $\forall U(x \in U \rightarrow y \in U) \rightarrow x = y$.

(T_2) $\forall U, V(x \in U \wedge y \in V \rightarrow \exists z \in U \cap V) \rightarrow x = y$.

A *separated* topological space has a relation, $\#$, satisfying (i) $\neg(x \# x)$; (ii) $\{y \mid x \# y\}$ is open; (iii) $x \in U \rightarrow (y \in U \vee x \# y)$.

Now the condition on a topological space,

($T_2^{\#}$) $x \in U \rightarrow (y \in U \vee \exists V, W(x \in V \wedge y \in W \wedge V \cap W = \varnothing))$ makes the definition $[x \#_2 y$ iff $\exists V, W(x \in V \wedge y \in W \wedge V \cap W = \varnothing)]$ determine a separated topological space.

Continuous functions between separated topological spaces are "strictly extensional", that is $f(x) \# f(y)$ implies $x \# y$. Given a set with an apartness, $(X, \#)$, define A is *open* iff $x \in A \rightarrow (y \in A \vee x \# y)$, for all x. The continuous functions from this separated topological space into any other, are exactly the strictly extensional maps.

The classical duality between closed and open sets fails intuitionistically; we give a counterexample to the identity, $\bar{A} \cup \bar{B} = \overline{A \cup B}$, in the model of the real numbers in sheaves over the real line. So we reformulate

(Regular) $x \in U \rightarrow \exists V, W(x \in V \wedge U \cup W = X \wedge V \cap W = \varnothing)$.

(Completely Regular) $x \in U \rightarrow \exists f: X \rightarrow^{\mathrm{cts}} [0,1]$. $(f(x) = 0 \wedge \forall y(f(y) < 1 \rightarrow y \in U))$.

(Normal) $U \cup V = X \rightarrow \exists U', V'(U \cup U' = X = V \cup V' \wedge U' \cap V' = \varnothing)$.

Then Tychonoff's Theorem holds, and, assuming Dependent Choices, the following.

URYSOHN'S LEMMA. *X is normal iff* $U \cup V = X \rightarrow \exists f: X \rightarrow^{\mathrm{cts}} [0,1] \forall x[(f(x) < 1 \rightarrow x \in U) \wedge (f(x) > 0 \rightarrow x \in V)]$.

TIETZE'S EXTENSION THEOREM. *If X is normal, $f: X \rightarrow R$, and* $\forall x[x \in U \vee (f \mathrm{ct}^s at\, x \wedge |f(x)| < a)]$, *then* $\exists g: X \rightarrow^{\mathrm{cts}} R$. $\forall x \forall n \in N[x \in U \vee |g(x) - f(x)| < n^{-1}]$.

The real numbers in sheaves over the real line are normal; also, intervals are connected and closed intervals are compact. We note that (T_0), (T_1), (T_2), ($T_2^{\#}$), and (Regular), are valid in any sheaf of topological spaces, whose stalks all have the property.

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