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DERIVING THE CONSTRUCTS UNDERLYING DECISION

Mildred L.G. SHAW and Brian R. GAINES
York University

In this paper we argue for the key role of the lowest level of the epistemological hierarchy in decision analysis. The source system of distinctions or constructs is a fundamental determinant of the operation, success or failure, of all higher levels. We show how the constructs being used at this lowest level may be elicited from a person through an interactive computer program. We also give fuzzy set semantics for construct systems that bring in the notions of opposition and relevance that seem to play major roles in human decision processes.

1. Introduction

Increasingly computers are being used as tools for aiding knowledge acquisition and decision-making, either alone or in some form of man–machine symbiosis. It is possible to give a system-theoretic framework for much of the operation of such computer-based systems enabling suitable algorithms to be formally derived. However, most practical systems commence with some inbuilt vocabulary for the problem area for which they are designed and this itself is not subject to modification either by the computer or by the user. Some degree of pre-supposed terminology is common to all problem-solving environments even when automation is not involved. If we take a problem to an "expert", then we first of all have to express it in such a way that he can understand it. We may well find, however, that this task is itself the most difficult of all.

In this paper we argue first of all for the importance of this terminological level in any form of knowledge acquisition and decision-making system, and show its significance in terms of Klir's [15] *epistemological hierarchy*. We relate this to Kelly's [13] work on *personal construct* systems and to Shaw's [22] techniques for *computer-based elicitation* of constructs and their analysis using Zadeh's [29] *fuzzy sets*. The operational value of this approach is demonstrated through an example of the computer program PEGASUS applied to a decision-making problem.

2. Levels of knowledge and fuzzy sets

One of the most useful frameworks within which to discuss decision analysis is Klir's [15] *epistemological hierarchy*. Fig. 1 shows the cycle of *events* → *modelling* → *prediction / decision / action* → *events* passing through this hierarchy.

The lowest level is one of *source systems*, effectively one of data terminology definition whereby the way in which any observed behaviour of a system about which a decision is to be made is defined and agreed. The next level is one of *data systems*, effectively one of system observation whereby the actual behaviour of some system is described in terms of the agreed domain of discourse at level zero. Levels above these are concerned with models of data and their interrelationships. Establishing any system for decision analysis involves making a number of presuppositions at each level of this hierarchy (Gaines [5]). From a formal point of view the presuppositions taken at each level are quite distinct from one another and they are also quite arbitrary. From a practical point of view, however, the presuppositions at different levels interact with one another and are certainly not arbitrary since they determine the success of a specific decision-making system. If it is not explicitly realized that such presuppositions are being made and are necessary to any operational system,

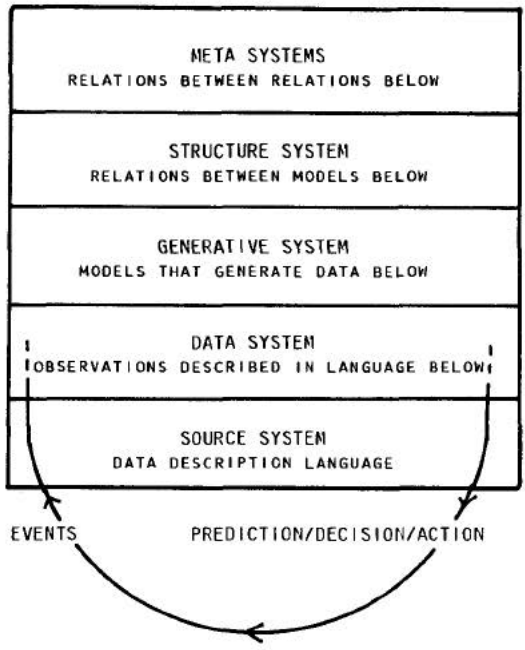

Fig. 1. The epistemological hierarchy (Klir [15]).

then it is easy to become confused about the reasons for the success or failure of any particular system. It is also possible to generate fruitless debates about the merits of different approaches to decision-making systems which are based on the failure to distinguish between presuppositions made at different levels in the hierarchy.

One presupposition that originates from the physical sciences and is often inapproprate to decision anaysis is that of *precision* in the source system and its application to create the data system. In the physical sciences the expected and preferred source system in which to represent data is one of exact measurement. We use a source system of physical quantities and their precise measurement and when we come to generalize this in decision analysis we forget that the underlying constructs of physics have been derived and refined over a very long period and are themselves of a peculiar and, perhaps, unique nature. The existence of continuous and limitless scales for physical variables of length, time, mass, charge, and so on, is an important phenomenon that marks out the constructs involved as being different from those in many other sciences. However, the existence of refined measuring schemes for some constructs should not blind us to their close relationships to other constructs for which no such physical measurement exists. For example, compare the concepts of *tallness* and *beauty*. The concept, the perception, of *tallness* exists in a more primitive sense than does the measurement of *height*. We are able to generate and follow arguments involving *tallness* without having any concept of inches, centimetres, or any other metric scales. Whilst a *scientific* analysis might conclude that there is a wide and ill-defined range of physical phenomena that combine in an extremely complex fashion to produce the subjective impression of beauty, in everyday reasoning it is as primitive a term as tallness. We certainly do not distinguish between them in statements such as: "this ladder is too tall to put in the kitchen", and "this vase is too beautiful to put in the kitchen". We are capable of decision-making such as: "he likes girls that are tall and beautiful; Mary is not very tall but very beautiful; he will probably like Mary."

Fuzzy set theory (Zadeh [29], Gupta, Saridis and Gaines [12], Mamdani and Gaines [18]) provides a model of constructs at the source system level which is more primitive and generally applicable than the quantitative scales of the physical sciences. Under certain conditions measuring schemes may be available which preserve all the order relations of the fuzzy sets and obey additional axioms. Deriving these additional, higher-order constraints, is itself an important task of empirical science. However, one should not presuppose their existence and the general assumption at the source level can be only that there may be predicates relevant to the problem under consideration that define fuzzy sets in relation to the entities involved. The links between the basic constructs used at the source level and fuzzy set theory have already been established. Goguen [11] takes a formal axiomatic approach to the notion of a

"concept" in natural and artificial languages and shows within a very general category-theoretic framework that one obtains generalized fuzzy sets. Gaines [7] shows that the modelling techniques based on measures of complexity and approximation that are effective for stochastic data readily generalize to data that is both fuzzy and stochastic and leads to *possibilistic* models.

Thus, fuzzy set theory provides a syntactic basis for source and data systems that enables decision analysis to be generalized to the imprecise data of everyday life. There are, however, possible artefacts that arise from the *semantics* of the source and data levels and are often unrecognized as major determinants of system success or failure. The *terminology* used in describing data is tacitly accepted and its derivation is not taken to be a significant part of establishing the decision-making system. However, in the literature of both arts and sciences it will be found that descriptive terminology changes both organically and discontinuously as disciplines develop and undergo various forms of *revolution* (Kuhn [16]). The literature on knowledge acquisition and decision-making is most often concerned with the upper levels of Klir's hierarchy, such as the relationship between a data system and the generative system that models it, rather than the language in which the data itself is described. However, all of these upper levels, where refined modelling techniques may be applied, derive their significance, if any, only from the significance of the lower levels. No amount of technique at the upper levels can compensate for lack of significant information expression at the lower levels. In the next section we discuss an approach to the operational analysis of these lower levels.

3. Personal constructs and fuzzy sets

One of the most detailed studies of the operational role and significance of the source system level on personal decision-making arises from the psycho-philosophical work of the clinical psychologist George Kelly. In his seminal work, *The Psychology of Personal Constructs*, he notes that:

> Man looks at his world through transparent patterns or templets which he creates and then attempts to fit over the realities of which the world is composed (Kelly [13, pp. 8–9]).

Kelly emphasizes the epistemological role of these constructs in predicting and controlling the world and their ontological status as personal conjectures rather than reality-derived absolutes:

> Constructs are used for predictions of things to come, and the world keeps rolling on and revealing these predictions to be either correct or midleading. This fact provides the basis for the revision of conestructs and, eventually, of whole construct systems (Kelly [13, p. 14]).

This capability to revise construct systems in the light of experience and the pluralism of possible construct systems was the basis of Kelly's *constructive alternativism*: that we are not necessarily bound by our construct systems but can revise them to see the same problem from a new viewpoint.

At a fundamental level we can see Kelly's constructive alternativism as being based on the primitive operation that Spencer Brown calls *making a distinction*. Brown commences his seminal work on *Laws of Form* by noting:

> a universe comes into being when a space is severed or taken apart... The act is itself already remembered, even if unconsciously, as our first attempt to distinguish different things in a world where, in the first place, the boundaries can be drawn anywhere we please. At this stage the universe cannot be distinguished from how we act upon it, and the world may seem like shifting sand beneath our feet... Although all forms, and thus all universes, are possible, and any particular form is mutable, it becomes evident that the laws relating such forms are the same in any universe (Brown [2, p.v]).

Thus, for both Kelly and Brown, all models at higher levels are fundamentally dependent on the distinctions made – the constructs used – at the source level. In decision analysis we have a basic requirement to determine what these distinctions are, or should be, before proceeding with the analysis. Kelly terms this the *elicitation* of a construct system.

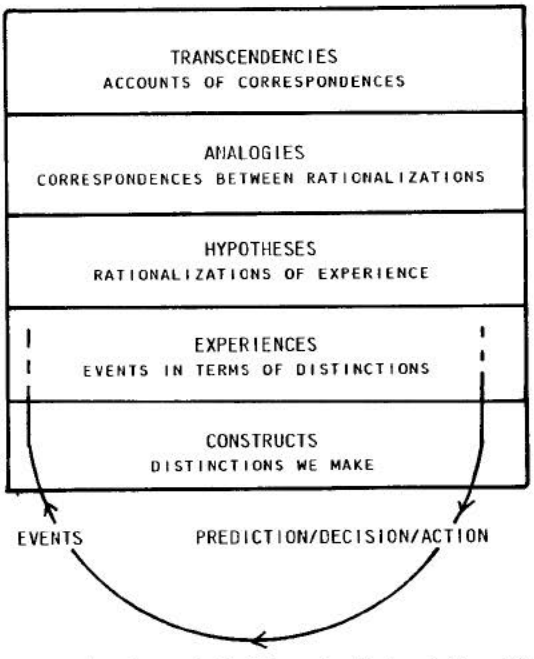

Fig. 2. The epistemological hierarchy (Gaines & Shaw [9]).

To show the significance of this in terms of the epistemological hierarchy it can be redrawn with Klir's terminology of a source system replaced by Kelly's of a *construct* system.

Fig. 2 shows this together with a more psychological interpretation of the upper levels of the hierarchy (Gaines and Shaw [9]) which is useful in understanding an individual's knowledge acquisition and decision-making processes, i.e. those of the *personal scientist* (Shaw [22]). In this figure the lowest level is one of the *constructs* or the *distinctions* made in interacting with the world. The next level is one of the *experiences* of events which happen to us, and we make happen, in terms of the distinctions already made. Levels above these are *hypotheses* which are rationalizations of experience, *analogies* between these rationalizations and *transcendencies*. Interaction with the world is, therefore, mediated through the construct system to produce experience which is modelled through the higher levels and leads to predictions, decisions and actions again mediated through the construct system.

Brown defined a distinction as a binary cut and Kelly also presented constructs as binary categories and based his own methodology for eliciting them on this. However, later workers found the need for shades of grey between the two poles of a construct and Kelly notes that this is consistent with his notion of a construct:

> The construct, of itself, is the kind of contrast one perceives... while constructs do not represent or symbolize events, they do enable us to cope with events, which is a statement of a quite different order... They also enable us to put events into arrays or scales, if we wish (Kelly [14, pp. 13-14]).

The interpolation within the basic bipolar distinction that has been made can be generated through logical operations that correspond to paradoxes of *self-reference* in classical logic. Varela [27] shows how a three-valued logical calculus arises from acceptance of a single self-referential form in Brown's calculus of distinctions. Gaines [4] shows how such "primitive paradoxes" may be iterated to give an indefinite number of distinctions between the poles of that originally made, and hence how the fuzzy truth value of an arbitrary proposition may be derived through a Dedekind section.

Kelly himself developed an elaborate but fairly informal epistemology based on his constructive alternativism and its consequences stated as a fundamental postulate and a set of "corollaries". He also developed techniques for eliciting the personal construct structure of particular individuals and it is this aspect of his work which has been primarily developed by later workers, mainly in the context of clinical psychology, that is, of people with abnormal construct systems. In recent years applications of Kelly's ideas in management science have led to an interest in the personal construct systems of professional staff, and interactive programs have been developed that elicit an individual's

construct system within a definite context (Shaw [22,23]). Shaw and Gaines [24] have discussed such systems within the framework of general knowledge acquisition and the personal computer, and have given a fuzzy semantic model for the personal construct structures. In the next section we give an actual example of someone using interactive computer elicitation of constructs in personal decision-making.

4. Interactive elicitation of a construct system

Kelly's [13] *repertory grid* is a way of representing a construct system used to make distinctions amongst a number of alternatives. For example, in deciding a career choice a person may have a number of possible jobs open, the *elements*, and may distinguish between them by different criteria, the *constructs*. For example, one construct might be *done before – totally new*. To allow for the shades of grey between the two extremes of the distinction, or *poles* of the construct, it is usual to have a rating scale, say from 1 to 5. In fuzzy set terms a rating of 1 is a degree of membership of 1.0 to the left-hand pole and 0.0 to the right-hand pole, and a rating of 3 (on a 5-point scale) would be a degree of membership of 0.5 to each pole.

```
                    * 1  2  3  4  5  6  7  8  9
                   ******************************
                    *
 know what it is 1 * 1  1  5  5  3  2  5  2  1 * 1 unknown
                    *
        exciting 2 * 5  1  2  5  1  1  2  1  3 * 2 doesn't excite me
                    *
  changes little 3 * 1  1  1  3  5  1  5  3  1 * 3 changes everything
                    *
     done before 4 * 1  4  5  5  4  2  4  4  2 * 4 totally new
                    *
can work at home 5 * 4  4  1  5  4  3  3  2  1 * 5 have to go out
                    *
  work with team 6 * 1  1  2  2  1  5  5  2  4 * 6 on my own
                    *
  more free time 7 * 3  3  3  5  1  1  1  1  1 * 7 not much free time
                    *
   probably dull 8 * 1  4  3  1  4  2  5  5  3 * 8 varied & interesting
                    *
                    * *  *  *  *  *  *  *  *  unemployment
                      *  *  *  *  *  *  *  *  ideal job
                      *  *  *  *  *  *  *  consultancy US
                      *  *  *  *  *  *  consultancy UK
                      *  *  *  *  US teaching
                      *  *  *  government
                      *  *  company
                      *  another department
                      my job now
```

Fig. 3. Grid elicited by PEGASUS from person thinking of changing job.

Fig. 3 shows a repertory grid elicited from a person contemplating a change of exployment: the elements are different jobs and the constructs are significant distinctions between them. A 1 in the matrix indicates that the left-hand description on the row applies to the element on that column; a 5 that the right-hand description applies; and intermediate values that the element lies between the two descriptions. This grid was derived by the computer program PEGASUS (Program Elicits Grids And Sorts Using Similarities, Shaw [22]) which elicits a grid interactively from an individual, simultaneously acting as a psychological reflector for heightening his awareness and deepening his understanding of himself and his processes. This is done by continual commentary on related elements or constructs, together with the encouragement to differentiate between them.

Before choosing his elements the user is asked to think about his purpose for eliciting the grid. Here it was stated to be "to decide whether to change my job". This is of great importance for the interaction which is to follow, since the elements are dependent on the purpose and the constructs are dependent on the elements. By using combinations of reviewing the purpose and adding and deleting constructs and elements, a depth of interaction can be achieved which could not at first be envisioned. The user is given the opportunity to reflect on his understanding of the topic and to examine and explore his thoughts and feelings in an atmosphere of heightened awareness of personal knowing. His perception may be changed in a way which by other means can take much time to accomplish.

The program is divided into six main sections and these are illustrated here through the elicitation of the grid in fig. 1. The first section is the *Basic Grid* in which explanation of grids and the use of the terminal are given, and the first four constructs elicited. Before the user chooses his elements he is asked to think about his purpose which defines the universe of discourse and enables him to choose elements which are relevant and representative of the topic area since the choice of elements largely determines the depth of interaction which can be achieved. This is shown in the dialogue with PEGASUS in fig. 4 in which computer output is in upper case and the responses in lower case.

The conventional method is then used to elicit the first few constructs, i.e. to present a triad of elements to be divided into a pair and a singleton indicating the poles of the construct. After the user has named the poles the computer assigns a 1 to the pair and a 5 to the singleton, and he has then to assign ratings from 1 to 5 to each of the other elements. These are then retyped in groups to highlight the relative position of each element with respect to the others, and an option to change the ratings is given. This is shown in the dialogue with PEGASUS in fig. 5.

The second section, *Construct Match*, provides feedback when two constructs are highly related. The user is first asked to add an element which is either at pole 1 on the first construct and pole 5 on the second or vice versa. If

he can add a new element it must be rated on all the constructs so far elicited, but if he cannot split the two constructs this way he is asked if he would like to delete a construct, combine two constructs into one, or just carry on. Feedback from a construct match is shown in the dialogue with PEGASUS in fig. 6. It is interesting that the user here does not do as the computer asks in the ratings assigned but does add a new element. This illustrates the role of the feedback in stimulating the user's own thought processes rather than directing them in ways determined by the program.

When four constructs have been entered, the program moves into the third section, *Element Match*, and begins to calculate matching scores between elements. Each element is matched with every other on the basis of the ratings

```
THIS IS A PROGRAM TO ELICIT A KELLY REPERTORY GRID.
PLEASE READ CAREFULLY EVERYTHING THAT IS PRINTED AND
MAKE SURE YOU UNDERSTAND WHAT YOU HAVE TO DO.
A REPERTORY GRID IS A TECHNIQUE DEVISED BY KELLY TO
HELP YOU EXPLORE THE DIMENSIONS OF YOUR THINKING.

YOU MUST DECIDE ON A PURPOSE FOR DOING THE GRID AND
KEEP THIS IN MIND WHEN YOU CHOOSE THE ELEMENTS--THE
THINGS YOU ARE GOING TO THINK ABOUT DURING THE PROGRAM.
THESE ELEMENTS WILL THEN BE USED TO ELICIT CONSTRUCTS.

THIS PROGRAM INCORPORATES FOUR VERSIONS OF PEGASUS.
 1. A PEGASUS GRID ELICITATION STARTING A NEW GRID;
 2. A STRAIGHT KELLY REPERTORY GRID ELICITATION
    WITHOUT COMMENTARY (MIN-PEG);
 3. A PEGASUS GRID ELICITATION USING A STORED BANK
    OF CONSTRUCTS (PEG-BANK);
 4. A PEGASUS GRID ELICITATION WITH PART ALREADY
    ELICITED BY YOU RECENTLY (PRE-PEG).
WHAT IS THE NUMBER OF THE VERSION YOU WISH TO USE ?1

TYPE IN ON ONE LINE YOUR PURPOSE FOR DOING THIS GRID

 ?to decide whether to change my job

NAME SIX ELEMENTS.
YOU MUST CHOOSE A SET OF SIX ELEMENTS KEEPING IN MIND
WHY YOU WANT TO DO THIS GRID. THEY COULD BE PEOPLE, EVENTS,
PIECES OF MUSIC, PICTURES, BOOKS, OR WHAT YOU WANT BUT
WHATEVER YOU CHOOSE THEY MUST BE OF THE SAME TYPE AND
EACH MUST BE WELL KNOWN TO YOU. TRY TO CHOOSE SPECIFIC
THINGS. NOW TYPE EACH ONE AFTER EACH QUESTION MARK.
DO NOT FORGET TO PRESS THE RETURN KEY AFTER EACH.

ELEMENT  1  ?my job now
ELEMENT  2  ?another department
ELEMENT  3  ?company
ELEMENT  4  ?government
ELEMENT  5  ?US teaching
ELEMENT  6  ?consultancy UK
```

Fig. 4. Introductory dialogue with PEGASUS.

```
TRIAD FOR ELICITATION OF CONSTRUCT 1
 1 my job now
 2 another department
 3 company

NAME THE PAIR

CAN YOU CHOOSE TWO OF THIS TRIAD OF ELEMENTS WHICH ARE IN
SOME WAY ALIKE AND DIFFERENT FROM THE OTHER ONE.
TYPE IN THE NUMBERS OF THE PAIR ONE AFTER EACH QUESTION
MARK. DONT FORGET TO PRESS THE RETURN KEY AFTER EACH.
 ?1
 ?2

NAME THE POLES OF YOUR CONSTRUCT

NOW I WANT YOU TO THINK ABOUT WHAT YOU HAVE IN MIND WHEN YOU
SEPARATE THE PAIR FROM THE OTHER ONE.HOW CAN YOU DESCRIBE
THE TWO ENDS OR POLES OF THE SCALE WHICH DISCRIMINATE
my job now AND another department FROM company
JUST TYPE ONE OR TWO WORDS FOR EACH POLE TO REMIND YOU WHAT
YOU ARE THINKING OR FEELING WHEN YOU USE THIS CONSTRUCT.

LEFT POLE RATED 1 -- ?know what it is
RIGHT POLE RATED 5 -- ?unknown

TYPE IN THE RATINGS

NOW IF my job now AND another department ARE
ASSIGNED THE VALUE 1 AND company IS ASSIGNED THE VALUE  5
ACCORDING TO HOW YOU FEEL ABOUT THEM PLEASE ASSIGN TO EACH
OF THE OTHER ELEMENTS IN TURN A PROVISIONAL VALUE FROM 1 TO 5

 1 my job now          1
 2 another department  1
 3 company             5
 4 government          ?5
 5 US teaching         ?3
 6 consultancy UK      ?2

POLE 1 --know what it is

 1 my job now          1
 2 another department  1

 6 consultancy UK      2

 5 US teaching         3

 3 company             5
 4 government          5

POLE 5 --unknown

DO YOU WANT TO CHANGE ANY OF THESE VALUES ? no
DO YOU WANT TO CHANGE THE POLE NAMES ?no
```

Fig. 5. Triadic elicitation of constructs.

used, and a comment made on the highest match if it meets the set criterion. The first choice offered is to add a new construct on which the two elements are placed at opposite poles, and then all the elements must be rated in the usual way. Alternatively, an element may be deleted, or no action may be taken. Feedback from a element match is shown in the dialogue with PEGASUS in fig. 7.

Section four allows the user to *Finish* at this stage in the cycle. Section five, *Review*, gives the choice of adjusting or redefining the purpose, and altering the level of match on which feedback commentary is given. There is an opportunity to delete any element or construct which is felt to be unsatisfactory in any way. In the sixth section, *Alternative Elicitation*, the user is given the freedom to add an element or to add a construct without using a triad if he so wishes. For example, in the elicitation shown the element *ideal job* was added in this way.

When the grid reaches the maximum size allowed, or if the user chooses to finish before that, the results can be analysed using one or more of a variety of techniques such as FOCUS, ENTAIL, INGRID or QARMS (Gaines and Shaw [10]). The analysis using the FOCUS procedure is shown in fig. 8. This is a two-way fuzzy hierarchical cluster analytic technique which systematically reorders the rows of constructs and columns of elements to produce a focused grid which shows the least variation between adjacent constructs and adjacent elements. The printout shows the trees of the elements and constructs as well

```
THE TWO CONSTRUCTS YOU CALLED
 1 know what it is--unknown
 4 done before--totally new
ARE MATCHED AT THE  66 PERCENT LEVEL.
THIS MEANS THAT MOST OF THE TIME YOU ARE SAYING
know what it is YOU ARE ALSO SAYING done before
AND MOST OF THE TIME YOU ARE SAYING
unknown YOU ARE ALSO SAYING totally new

THINK OF ANOTHER ELEMENT WHICH IS EITHER
know what it is AND totally new
OR done before AND unknown
IF YOU REALLY CANNOT DO THIS THEN JUST PRESS RETURN AFTER
THE FIRST QUESTION MARK BUT PLEASE TRY. THEN YOU MUST GIVE
THIS ELEMENT A RATING VALUE ON EACH CONSTRUCT IN TURN.
AFTER EACH QUESTION MARK TYPE A VALUE FROM 1 TO 5
WHAT IS YOUR ELEMENT ?consultancy US
RATINGS :

know what it is--unknown ?5
exciting--doesn't excite me ?2
doesn't affect other things--changes everything ?5
done before--totally new ?4

DO YOU WANT TO CHANGE ANY OF THESE VALUES ?no
```

Fig. 6. Element elicitation through construct match.

```
THE TWO ELEMENTS  2 another department AND  6 consultancy UK
ARE MATCHED AT THE  80 PERCENT LEVEL.
THIS MEANS THAT SO FAR YOU HAVE NOT DISTINGUISHED
BETWEEN another department AND consultancy UK
DO YOU WANT TO SPLIT THESE ?yes
HELP ?yes

THINK OF A CONSTRUCT WHICH SEPARATES THESE
TWO ELEMENTS AND THEN KEEPING THIS IN MIND
ACCORDING TO HOW YOU FEEL ABOUT THEM PLEASE ASSIGN TO EACH
OF THE OTHER ELEMENTS IN TURN A PROVISIONAL VALUE FROM 1 TO 5

NAME THE POLES OF YOUR CONSTRUCT

LEFT POLE RATED 1 -- ?work with team
RIGHT POLE RATED 5 -- ?on my own

TYPE IN THE RATINGS

 2 another department   1
 6 consultancy UK       5
 1 my job now          ?1
 3 company             ?2
 4 government          ?2
 5 US teaching         ?1
 7 consultancy US      ?5

POLE 1 --work with team

 1 my job now          1
 2 another department  1
 5 US teaching         1

 3 company             2
 4 government          2

 6 consultancy UK      5
 7 consultancy US      5

POLE 5 --on my own

DO YOU WANT TO CHANGE ANY OF THESE VALUES ?no
```

Fig. 7. Construct elicitation through element match.

as the focused grid with the element and construct labels. It can be seen that the element *ideal job* matches most closely to *US teaching* and that *unemployment* and *consultancy UK* have similar characteristics. It can also be seen that constructs 7, 2, 8, 4 and 1 form a related cluster with 5, 3 and 6 only loosely linked in. Since the PEGASUS feedback encourages the separation of constructs very close matches are rare.

As a PEGASUS elicitation proceeds this FOCUS algorithm is used to offer to the user a possible explanation and interpretation of his meaning system in the terms of the similar patterns he uses in supposedly different circumstances. Cross-references are mapped across the grid and exhibited to the user in such a

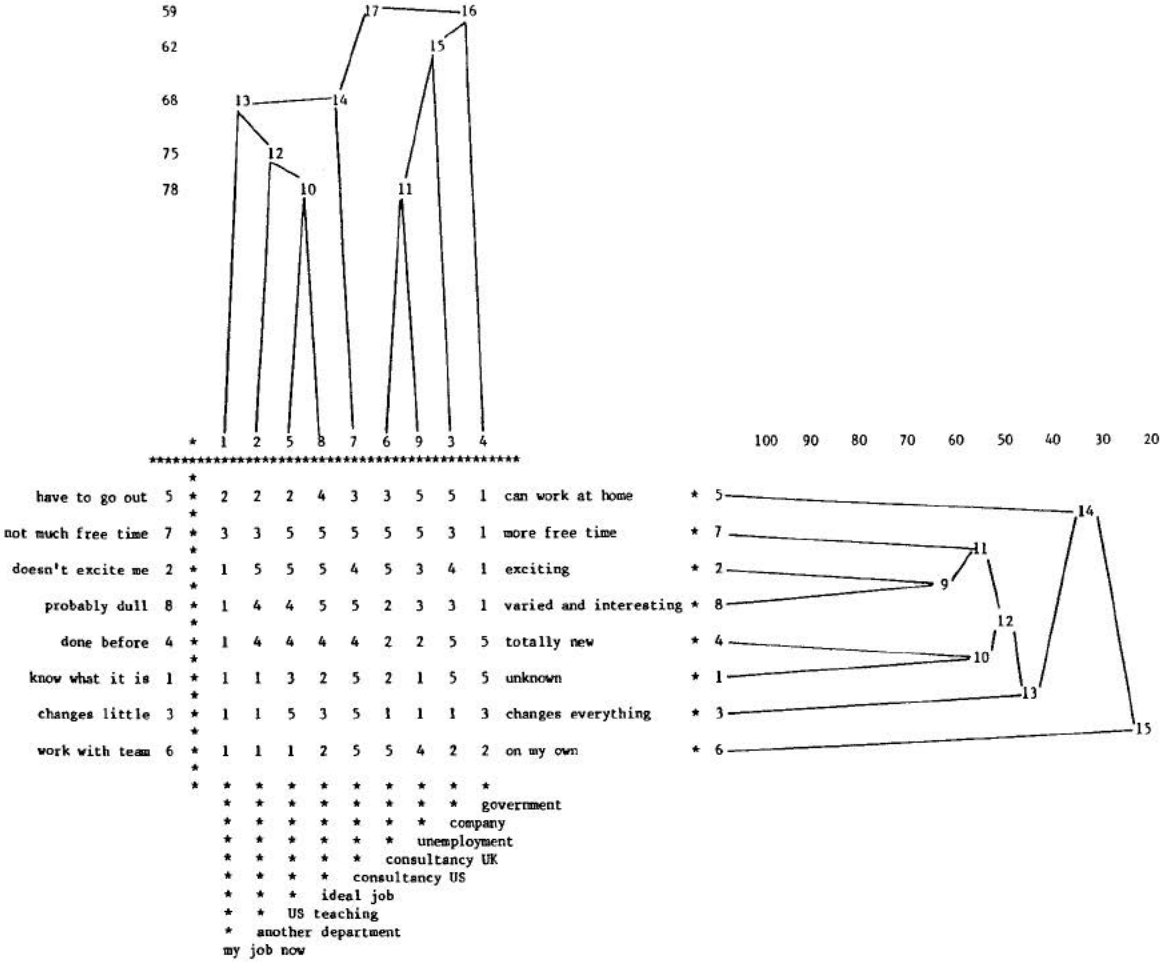

Fig. 8. Grid sorted by FOCUS to give a fuzzy hierarchical cluster analysis.

way as to offer him the facility to reconsider and change anything he feels to be inappropriate, which in turn enables him to be more aware of the links he is implicitly holding in his cognitive model. Here the conversation can be seen as a feedback loop with the computer acting as an emancipator by re-presenting the content of the conversation and the implicit links within it. In this way the participative anlysis extracts and presents the essence of the personally meaningful relationships in the grid. It may well be impossible for a person to derive such relationships themselves without help – for example, the studies of Michalski and Chilausky [19] on expert systems show that inductive algorithms are better at extracting the rules for an expert's decision than the expert himself is at expressing them.

5. Further development of fuzzy set semantics for personal constructs

There are features of Kelly's personal construct psychology that go beyond the basic fuzzy set interpretation of the construct system developed above. In particular he emphasizes the bipolarity of constructs and we have incorporated this only incidently, and he also brings in notions of relevance. These extensions seem particularly important to decision analysis and in this section we place them within the framework of fuzzy set theory.

In previous sections we have essentially assumed the working hypothesis:

Hypothesis 1. The basis underlying personal constructs is that each pole of a construct is a *predicate* defining a fuzzy set such that a construed element has a *degree of membership* to that set.

Thus, we may take the pole of a construct as a predicate, PRED, and write PRED x as the degree of membership of the element x to that predicate. For example, we may consider the fuzzy set of *exciting* jobs and then ask the question; "What degree of membership does *my job now* have to the set of exciting jobs?" Kelly's original binary allocation then appears as the use of classical set theory and multipoint rating scales appear as the use of some form of fuzzy set theory in which graded set membership is allowed.

Note that there is no assumption that different people make use of the same construct labels or constructs, and that fuzzy sets in which the same elements have different degrees of membership are essentially different fuzzy sets and hence define different constructs. Thus, the personal nature of constructs is inherently allowed for in this model. However, the assumption of defined *degrees of membership* is a strong one since it enables us to compare different fuzzy sets in a variety of ways that well-order them. These in themselves generate fuzzy sets, and hence possible constructs at a meta-level, e.g. the fuzzy set of constructs similar to construct A on some criterion. This is particularly important in the extension of interactive construct elicitation systems such as

PEGASUS because it enables feedback about the analysis of construct structures to be couched in the same terminology as was used in eliciting them.

Hypothesis 1 takes us only a small step towards a *calculus of constructs* because it says nothing about the way in which the fuzzy sets may be manipulated to give us fuzzy reasoning and hence a model underlying construct generation, reasoning with constructs, etc. There are a number of possible foundations of fuzzy reasoning (Gaines [4]) and we shall assume for purposes of this paper:

Hypothesis 2. The fuzzy sets underlying constructs are manipulated according to an *uncertainty logic* (Gaines [6]).

The logics discussed in the paper referenced include those used by many authors in decision and control studies (Gaines and Kohout [8], Mamdani and Gaines [18]) and also subsumes probability logic. It will not be necessary in the following discussion to use more than a fragment of the logics common to most of them.

One apparent problem with fuzzy sets in catching the nature of personal constructs is that their definition contains no reference to the *bipolarity* which Kelly felt to be a particularly important feature of personal constructs. Hypothesis 1 may appear to lead to precisely those defects of formal logic that Kelly warns against:

> Now conventional logic would say that black and white should be treated as separate concepts. Moreover, it would say that the opposite of black can only be stated as not black, and the opposite of white can only be stated as not white. Thus the person whose field we mentioned would have shoes which would be just as much not white as the time of day, and he would write on paper which would be just as not black as the distance to his office (Kelly [13], p. 106]).

Part of the problem that Kelly is discussing here is one of *relevance*. The standard predicate calculus fails to distinguish between *not* and *not relevant*. It is only in recent years that formal logics accounting for relevance have been established (Anderson and Belnap [1]). However, what he also brings out is the psychological role of the concept of *opposite* which has no logical counterpart; it is related to negation but not identical to it.

By considering only bipolar constructs, however, Kelly rules out the use of construct theory itself to analyse both logically and psychologically the nature of the concept *opposite*. Logics based on fuzzy set theory *do* allow us to make a distinction between *negation* and *opposite* and to relate this to *relevance*, thus providing the tools that Kelly needed to explicate fully these key distinctions. First, it is worth noting that Zadeh in presenting fuzzy set models of verbal reasoning makes precisely the distinction between opposite and negation that Kelly makes: the fuzzy set representing *young* is quite distinct from the fuzzy

set representing *not old* (e.g. Zadeh [30, p. 290]). Thus, Kelly's requirement that this distinction be made is satisfied – it is also worth noting that his argument for the intrinisic bipolarity of constructs is borne out by the extent to which such fuzzy predicates are presented in the literature as pairs of opposites.

Equally importantly, Kelly's insistence on bipolar constructs does not allow him to treat relevance within a uniform framework – he has to introduce it separately in terms of constructs having a *range of convenience*. By considering an element to have quite distinct degrees of membership to the two poles of a construct, i.e. to a construct and its *opposite* we can also capture the concept of relevance – i.e. a construct is irrelevant to an element if its degree of membership to both poles of the construct is zero. Kelly could have captured this in his binary model of constructs by noting, for example, that the construct *good–bad* is irrelevant to something which is both *not-good* and *not-bad* (it is clearly debatable still whether this explication of relevance captures all its psychological connotations – we would suggest not, only that it captures some key ones). Those who extended his bipolar notion to allow for multipoint rating scales also failed to allow for relevance when they made the scales a one-dimensional interpolation between the two poles of a construct.

Clearly, whilst the notion of *opposite* is not the same as that of *negation*, it does have some relationship to it – the *opposite* predicate is constrained by similar but weaker constraints than the *negated* predicate. We would certainly require a construct and its opposite to be symmetric in their properties since the label of *opposite* could be applied to either relative to the other. We have suggested (Shaw and Gaines [24]) that if OPP PRED is the opposite of PRED we would have the following substitution rule:

Sub. 1. A valid formula remains valid under the simultaneous uniform substitutions: OPP PRED → PRED and PRED → OPP PRED.

By using Sub. 1 in the equivalence, OPP PRED ≡ OPP PRED, we obtain:

$$\forall x \text{ OPP OPP PRED } x \equiv x, \tag{1}$$

which emphasizes the similarity of OPP to negation, NOT. We would also expect symmetry with respect to negation so that the negation of an opposite is the opposite of the negation, i.e. a second substitution rule:

Sub. 2. A valid formula remains valid under the substitution anywhere of NEG OPP for OPP NEG, and vice versa.

We would probably also feel that a predicate has to imply the negation of its opposite even it is not equivalent to it, i.e.:

$$\forall x \text{ PRED } x \supset \text{NOT OPP PRED } x, \tag{2}$$

which gives us in terms of degrees of membership:

$$\forall x \text{ PRED } x \leq 1 - \text{OPP PRED } x$$

or

$$\forall x \text{ PRED } x + \text{OPP PRED } x \leq 1, \tag{3}$$

whereas for negation, NOT, we have:

$$\forall x \text{ PRED } x + \text{NOT PRED } x = 1, \tag{4}$$

so that a negation is an opposite, but not necessarily vice versa.

Constraint (2) does not completely capture the semantics of *opposite* since we would also want implicational relationships between different predicates on the same element to be preserved but reversed in taking opposites:

$$\forall x \, (\text{PRED1 } x \supset \text{PRED2 } x) \supset (\text{OPP PRED2 } x \supset \text{OPP PRED1 } x). \tag{5}$$

This, together with substitutions using Sub. 1, gives us the strong result:

$$\forall x \text{ PRED1 } x - \text{PRED2 } x = \text{OPP PRED 2 } x - \text{PRED1 } x, \tag{6}$$

that opposite preserves the interval structure on degrees of membership.

We may now also define the *relevance*, REL, of a predicate to an element as the degree to which inequality (3) is an equality, i.e.

$$\forall x \text{ REL PRED } x \equiv (\text{NOT OPP PRED } x \supset \text{PRED } x), \tag{7}$$

which in terms of degree of membership gives:

$$\forall x \text{ REL PRED } x = \text{PRED } x + \text{NOT OPP PRED } x. \tag{8}$$

This also corresponds to the extent to which OPP and NOT are the same:

$$\forall x \text{ REL PRED } x \equiv (\text{OPP PRED } x \equiv \text{NOT PRED } x). \tag{9}$$

Yager [28] has recently developed these constraints into a fuzzy calculus of measures of opposition, relevance and extremism in constructs.

The definitions above show the close relationships between opposite, negation and relevance that Kelly hints at in his work but did not have the formal tools to define. The discussion of this section demonstrates that Hypothesis 1 although omitting specific mention of bipolarity, actually gives one adequate tools with which to explicate both the notions of *opposite* and *relevance* in personal construct theory. It further suggests that in the elicitation of personal

constructs the bipolarity does have a significant role to play for, without information on the degree of membership to both poles of a construct, we cannot distinguish between its not applying and not being relevant. However, this also requires a change in the methodology of construct elicitation since current techniques, such as those used in PEGASUS, by forcing elements to be rated against constructs on a linear scale are automatically assuming relevance. This is not unreasonable given that both elements and constructs are elicited from the user to be mutually relevant to the context of a specifically selected topic. We have suggested (Shaw and Gaines [24]), however, that in a large-scale system with wide ranging applications such that uniform relevance cannot be taken for granted it would be appropriate to allow users to ascribe separate ratings to the two poles of a construct. Roberts [21] has recently reported some experiments on such unipolar rating in repertory grid elicitation.

The analysis of data from experiments on construct elicitation is a major task in its own right. One may regard it as a problem of attempting to determine the entailment structure for a class of constructs defined extensively by the degrees of membership of a set of elements to various fuzzy sets. There is also the converse problem of determining possible relationships between elements given their co-ordinates as members of a class of fuzzy sets. We generally need first of all to define some similarity measures such as the degree of equivalence, EQUIV, between two construct poles:

$$\mathrm{EQUIV}(A, B) \equiv \forall x (Ax \equiv Bx), \tag{10}$$

so that in terms of degree of membership:

$$\mathrm{EQUIV}(A, B) = 1 - \max_x |Ax - Bx|, \tag{11}$$

i.e. it is determined by the largest difference in the ratings of the same element on the two different constructs.

This is a reasonable measure of similarity but not a very robust one against errors in the data collected. If it were not possible to check unusual discrepancies by asking the user to re-rate the appropriate constructs then it might be felt appropriate to hedge the notion of equivalence by defining the similarity measure USUALLYEQUIV as:

$$\mathrm{USUALLYEQUIV}(A, B) \equiv \textit{Usually}\ x (Ax \equiv Bx), \tag{12}$$

where the quantifier *Usually* is defined through a fuzzy set in which the 100% occurrence of the predicate has a degree of membership 1 and the 0% has 0 with intermediate frequencies having intermediate degrees of membership. The effect of this is to allow for some major variations with only slight loss of truth value – it is similar to the use of *quartiles* in statistics and the concept of a *set*

of measure zero in probability theory.

Equivalence between constructs is one important measure in allowing them to be clustered in terms of similarity. However, it is also of interest to investigate the implicational relationships between them, and we may define similar measures to (10) by replacing $Ax \equiv Bx$ with one of the four distinct implications possible: $Ax \supset Bx$, $Bx \supset Ax$, $Ax \supset$ NOT Bx, or $Bx \supset$ NOT Ax. This gives us a directed fuzzy graph of implications between constructs that may itself be clustered to determine the main entailment structure for the constructs. We have reported studies of such logical analyses of fuzzy repertory grid data using the ENTAIL program, and compared them with previously used techniques such as the cluster analysis program FOCUS, the principal component analysis program INGRID and the Q-analysis program QARMS (Gaines and Shaw [10]).

One major advantage of the fuzzy set approach is that there is an associated linguistic semantic model in terms of Zadeh's fuzzy *hedges*, and this may be used to avoid the use of numbers in the discourse with the user if desired. For example, instead of having the user allocate jobs to the construct poles *exciting – doesn't excite me* on a five-point scale one might instead allow them to make statements of the form:

my ideal job is very exciting,

where *very* represents one of the hedges: *absolutely, extremely, very*, pred, *rather, fairly,* pred *and not* pred, *fairly not*, etc. (where pred is inserted to show a null hedge and an ambivalent hedge, respectively). These are all interpretable as degrees of membership for internal computation but allow the user to operate in *natural language* rather than some imposed technical framework.

Such linguistic representations may also be used for meta-level communications with the user, e.g. two constructs *are very similar* rather than *matched at the 75 percent level.* Even the analysis of construct relationships and entailment structures might be best presented in this way – Rhodes and Klinger [20] have demonstrated how effective such fuzzy semantics can be in allowing natural person—computer dialogue in a decision-making situation. It may even be possible to use this approach non-interactively to elicit construct and decision structures from transcripts of discourse (Linde and Goguen [17]).

6. Conclusions and related developments

In this paper we have argued for the key role of the lowest level of the epistemological hierarchy in decision analysis. The source system of distinctions or constructs is a fundamental determinant of the operation, success or failure, of all higher levels. We have shown how the constructs being used at

this lowest level may be elicited from a person through an interactive computer program. We have also given fuzzy set semantics for construct systems that bring in the notions of opposition and relevance that seem to play major roles in human decision processes.

In papers elsewhere we have linked the techniques described here with Checkland's [3] *soft systems methodology* and the extraction of *root definitions* (Shaw and Gaines [26]). We have also shown how the techniques can be applied to the multiple construct structures of decision-making groups and linked this with various psycho-social theories (Shaw and Gaines [25]). Programs related to PEGASUS have been developed for the analysis of group data such as SOCIOGRIDS (Shaw [22]) which produces a *social network* based on mutual construing together with a *mode grid* of consensual construing.

Fuzzy set theory as a syntactic framework for human reasoning coupled with the semantics of personal construct psychology provides a basis for decision analysis that can be operationalized through computer algorithms to give tools of great practical importance. We hope this paper will stimulate further interest in the applications of these very powerful methodologies.

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