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FUZZY SETS AND DECISION ANALYSIS – A PERSPECTIVE

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The search for better tools for decision analysis is an unending one. Decision-making is a fundamental process involving information acquisition, modelling, simulation and action. It has philosophical, logical, psychological, sociological and engineering aspects. To have completed the process of decision analysis would be to have solved foundational problems in a wide range of basic disciplines that have been problematic for mankind throughout recorded history. It is most reasonable to be confident that our quest for better tools will never cease and that new techniques and new problems will go on forever being generated.

In this process of innovation there are natural cycles after which one can pause and review the types of change that have occurred. These are often not the *paradigm shifts* that Kuhn [5] describes in the natural sciences but rather *technological innovations* in which we have generated additional techniques that supplement rather than replace those already available. Thus, the term "better tools" must usually be qualified by the question, "better tools for what?". The papers in this book are concerned with *fuzzy sets* as a better tool for dealing with certain types of essential imprecision in decision analysis. This introduction gives some of the motivation behind the search for such improved tools in connection with imprecision.

The term *fuzzy* in the sense used here seems to have been first introduced in 1962 by Zadeh [9] in a paper concerned with the transition from circuit theory to system theory in which he called for a "mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions". This paper was followed three years later by a technical exposition of just such a mathematics now termed the theory of *fuzzy sets* (Zadeh [10]). A bibliography shows the number of papers on fuzzy sets climbing cumulatively from 2 in 1965 to 69 in 1970 and over 600 in 1975 (Gaines and Kohout [3]); a 1979 bibliography lists over 1500 articles (Kandel and Yager [4]). In 1978 the growth in the literature was such as to justify a journal devoted just to the one topic, and North-Holland commenced publication of *Fuzzy Sets and Systems*.

Now, some 20 years after Zadeh's original call for a new mathematics, fuzzy set theory has clearly become accepted in the literature and most authors do not feel the need to explain or justify their use of the techniques involved.

However, it has not always been so, and there was much early debate about the need for any alternatives whatsoever to probability theory. To some extent that debate still continues in many application areas and, in the context of this book, it is useful to go back to the original statement of Bellman and Zadeh in 1970 about the role of fuzzy sets in decision analysis:

> Much of the decision-making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. To deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory and, more particularly, the tools provided by decision theory, control theory and information theory. In so doing, we are tacitly accepting the premise that imprecision – whatever its nature – can be equated with randomness. This, in our view, is a questionable assumption. Specifically, our contention is that there is a need for differentiation between *randomness* and *fuzziness*, with the latter being a major source of imprecision in many decision processes. By fuzziness, we mean a type of imprecision which is associated with *fuzzy sets*, that is, classes in which there is no sharp transition from membership to nonmembership. For example, the class of *green objects* is a fuzzy set. So are the classes of objects characterized by such commonly used adjectives as large, small, significant, important, serious, simple, accurate, approximate, etc. Actually, in sharp contrast to the notion of a class or a set in mathematics, most of the classes in the real world do not have crisp boundaries which separate those objects which belong to a class from those which do not. In this connection, it is important to note that, in the discourse between humans, fuzzy statements such as "John is *several* inches taller than Jim," "x is *much larger* than y," "Corporation X has a *bright future*," "the stock market has suffered a *sharp decline*," convey information despite the imprecision of the italicized words (Bellman and Zadeh [1, pp. B141–B142]).

It is perhaps worth noting that this is a very precise statement – fuzzy set theory has never been an invitation to fuzzy thinking! It is also worth emphasizing that it is not a sweeping statement that in some sense probability theory itself is wrong – only that there are forms of uncertainty where it gives an inappropriate representation. There is also no implication that in general it is desirable to fuzzify our theories – that imprecision is intrinsically desirable. The point that was being made then, and bears repetition again, is that in the decision process certain forms of imprecision occur that are intrinsic to the problem and for which the probability calculus is inadequate. Bellman and Zadeh give a concise abstract classification of these forms of imprecision in terms of "classes in which there is no sharp transition from membership to non-membership".

There appear to be two possible and distinct objections that might be made to the rationale expressed in the quotation above. First, that the imprecision involved is not essential and does not require representation. Secondly, that probability theory already provides an adequate representation. Gaines has given an example of a planning situation where the role of imprecise statements as very accurate representations of information is apparent:

> unwarranted precision can itself be highly misleading since actions may be taken based on it – "we will deliver 7 parcels each weighing 15.2 Kilograms at the rear entrance of building

> 6A on 15th February at 9.03 p.m.", "we will deliver some heavy equipment to your site Saturday evening", and "see you with the goods over the weekend", may each refer to the same event but are clearly not interchangeable, i.e., each conveys an exact meaning that (presumably) properly represents what is to occur. If we prefer the precision of the first statement it is not for its own sake but because the tighter tolerances it implies on the actual situation allow us to plan ahead with greater accuracy and less use of resources. However, if the third statement really represents all that can be said it would be ridiculous to replace it with either of the previous ones. It would be equally ridiculous to say nothing. However, even the least precise of the three statements does provide a basis for planning and action. A key aspect of executive action is planning under uncertainty and normal language provides a means for imprecision to be clearly and exactly expressed (Gaines [2, p. 303]).

The conclusion that we have to make decisions given imprecise information that, however, is a very precise representation of the situation seems a very clear one in the context of realistic theories of decision analysis – this is how people operate in the real world and involves forms of information that any decision aid must also be able to represent adequately. Again it is worth noting that this statement is totally independent of any requirement for precision in the development of science – it does not in itself call a halt to attempts to model any process, animate or inanimate, as precisely as possible. As Karl Popper has noted in the context of the philosophy of science:

> both precision and certainty are false ideals. They are impossible to attain, and therefore dangerously misleading if they are uncritically accepted as guides. The quest for precision is analogous to the quest for certainty, and both should be abandoned. I do not suggest, of course, that an increase in the precision of, say, a prediction, or even a formulation, may not sometimes be highly desirable. What I do suggest is that it is always undesirable to make an effort to increase precision for its own sake – especially linguistic precision – since this usually leads to lack of clarity, and to a waste of time and effort on preliminaries which often turn to be useless, because they are bypassed by the real advance of the subject: one should never try to be more precise than the problem situation demands (Popper [7, p. 17]).

The second objection that may be raised to the need for fuzzy set theory in decision analysis is that we already have tools to deal with uncertainty and imprecision, notably the probability calculus. Here the counter-argument is slightly more difficult since there is always a greater problem to show that a mathematical system cannot cope with a problem rather than that we do not know how to use it to do so – indeed at a formal level there are close links between probability theory and fuzzy set theory that are discussed in this book. However, it is easy to give examples to show that our conventional interpretations of probability theory in terms of likelihoods or frequencies is not appropriate to the kinds of imprecision exemplified above. The term *green* defines a fuzy set of objects not because the colour of any one of them varies each time it is examined but because there is reasonable doubt about whether a borderline case belongs to the set or not. The parcel delivery example above gives three definitions of the nature of the event which are increasingly fuzzy

only in allowing the deliverer greater and greater freedom in the class of actions which satisfy his definition. What is defined is not the *probability* of an event occurring but the range of *possible* events that may occur.

Zadeh gives a simple example of this difference between probabilistic uncertainty and possibilistic imprecision by considering the statement: "Hans ate X eggs for breakfast." He notes:

> We may associate a possibility distribution with X by interpreting $F(u)$ as the degree of ease with which Hans can eat u eggs. We may also associate a probability distribution with X by interpreting $P(u)$ as the probability of Hans eating u eggs for breakfast. Assuming that we employ some explicit or implicit criterion for assessing the degree of ease with which Hans can eat u eggs for breakfast, the values of $F(u)$ and $P(u)$ might be as shown:

u	1	2	3	4	5	6	7	8
$F(u)$	1	1	1	1	0.8	0.6	0.4	0.2
$P(u)$	0.1	0.8	0.1	0	0	0	0	0

> We observe that, whereas the possibility that Hans may eat 3 eggs for breakfast is 1, the probability that he may do might be quite small, e.g. 0.1. Thus, a high degree of possibility does not imply a high degree of probability, nor does a low degree of probability imply a low degree of possibility. However, if an event is impossible, it is bound to be improbable (Zadeh [12, p. 8]).

In summary, a significant part of the application of fuzzy set theory in decision analysis arises through the interpretation of some forms of imprecise statement as placing a *possibilistic restriction* on the class of events which satisfy that statement. This restriction is then represented through a set with graded membership such that any event has a *degree of membership* in the set defining the extent to which it is consistent with the possibilistic restriction. From a technical point of view a theory of imprecision based on sets with graded membership is readily constructed. The natural operations on sets, such as the union and intersection, are also readily defined and White [8] has shown that the definitions used by Zadeh [10] led to a set theory in which the axiom of comprehension, that every predicate defines a set, is consistent and does not lead to the paradoxes of classical naive set theory. However, questions of a psychological nature clearly remain. For example, how do the inferences that people make in combining imprecise statements correspond to operations upon fuzzy sets? These questions have now been studied extensively through psychological experiments (Zimmermann and Zysno [14]).

Such links back to the human decision processes are significant also in reminding us that the origins of fuzzy set theory were through a desire to have a more adequate representation of these processes, and that the examples given in the early literature such as the Bellman and Zadeh quotation above are

linguistic. The term "several" is a fuzzy quantifier; the term "much larger" expresses a fuzzy relationship; the term "bright future" defines a fuzzy restriction. Human decision analysis is essentially linguistic and we tend to forget this when we undertake technical decision analysis based on mathematical systems. Zadeh [11,13] has emphasized the linguistic basis of decision analysis and presented the theory of fuzzy sets as a basis for representing the underlying processes of linguistic reasoning. Much of the literature is concerned with the linguistic analysis of decision processes and their representation through fuzzy sets. For example, one of the first successful applications of fuzzy sets to realistic decision problems was that of Mamdani [6], who showed that, in control of a simple dynamic plant, rules from human controllers in linguistic form could be directly interpreted by a computer algorithm to give an effective controller. This work has led to many practical applications which are reviewed by Mamdani in this book.

One of the important ways in which the theory of fuzzy sets may be applied to decision analysis relates to the use of fuzzy probabilities in the calculation of expected utilities. More specifically, a serious shortcoming of the conventional formulation of the principle of maximization of expected utility is that it is based on an assumption which is violated in most realistic settings.

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