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# FUZZY ENTAILMENT ANALYSIS

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## ABSTRACT

Personal construct technology can be used to form man-computer symbiotic systems of a truly dialectical nature. These enable the construct systems of individuals to be elicited interactively and give access to the resultant database for purposes of personal comprehension and decision making. A new method of analysis has been developed for such personal construct databases using a logical model of constructs as fuzzy predicates applying to elements. This may be used to derive the entailment structure of constructs and in this paper we present the analysis and compare it with the FOCUS clustering technique.

## INTRODUCTION

The computer analysis of repertory grids representing personal construct systems has become of great interest and wide application in recent years. Shaw (1980b) reviewed some of the general issues in psychological scaling with reference to repertory grids, including the problems of the use of scales by individuals and the interpretation of the meaning of such data by the psychologist. Conventional data analysis techniques for repertory grids are based on multivariate analyses that treat the data as vectors in a space to be rotated and clustered. We have suggested previously (Gaines & Shaw 1980) that the same data may instead be regarded as logical truth values of fuzzy predicates and analysed logically. In this paper we develop this argument further and illustrate its practical application through an example.

## GRID ANALYSIS BASED ON LOGICAL PREDICATES

Figure 1 is an example of a repertory grid elicited from a student, Arthur, about his views of different learning situations using the PEGASUS grid elicitation program (Shaw 1980a, p.65). Essentially Arthur has rated a number of elements representing learning situations, such as "lecture" and "tutorial", on a number of constructs describing the different learning situations, such as "like---dislike". The constructs used are bipolar and Arthur has rated the elements on a five-point scale with 1 meaning that the element is at the left-hand pole, e.g. "like", and 5 meaning that it is at the right-hand pole, e.g. "dislike".

Conventional analyses regard the rows in the grid as a vector of values giving the rating of each element in turn on the scale for a construct. From this point of view each construct becomes represented as a point in a multi-dimensional space whose dimension is the number of elements involved. The natural relation to examine between constructs is then the distance between them in this space. Two constructs which are zero distance apart are such that all elements are construed in the same way in relation to them and hence we might infer that they are being used in the same way - in some way they are equivalent constructs. For constructs which are not equivalent one may analyse the entire constellation in space to determine a set of axes such that the projection of each construct onto the first axis accounts for most of the distance between them, the projection on the second axis accounts for most of the remaining distance, and so on. This is a principal components analysis of the construct space (Slater 1977).

Shaw's (1980a) FOCUS algorithm is a distance-based cluster analysis that sorts the constructs into a linear order such that constructs closest together in the space are also closest together in the order. It has the advantage in presentation that the sorting is used only to re-present the original grid re-organized by the 'neighbourness' of constructs and elements. It is up to the user to construe meaning into the result and confirm this directly in terms of the original data. Figure 2 shows the grid of Figure 1 as processed by FOCUS: the constructs have been hierarchically clustered as (3,(((((6,7),2),4),1),5)) and the elements as (((9,6),(4,(3,(2,(1,5))))),(7,8)).

There is an alternative way of looking at the grid of Figure 1 which views it not as a set of vectors in a space but instead as an assignment of truth-values to logical predicates. We may take the right-hand pole of each construct in Figure 1 to be a logical predicate that may be applied to a learning situation and take the assignment of a value to a particular element in the grid to be the degree of truth of that predicate applied to that element. Conversely we may take the value assigned to be the degree of falsity of the predicate corresponding to the left-hand pole of the construct applied to that element. We normally require that an element be assigned to one, and only one pole, so that if the predicate corresponding to the left-hand pole is true for some element then that corresponding to the right-hand pole must be false, and vice versa. A repertory grid, such as that of Figure 1, is then the matrix of such truth values for the constructs and elements involved.

Kelly (1955) presented constructs as binary categories and based his own methodology for eliciting constructs on this. If elements are restricted to be definitely assigned to one or the other of the two poles of a construct then the truth values involved are just "true" and "false" and a repertory grid may be analysed using conventional 2-valued logic (Gaines & Shaw 1980). However, later workers found rating scale assignments necessary, such as those of Figure 1, and these generate an essentially multi-valued logic (Rescher 1969). We have found it convenient to relate the truth-values to those of Zadeh's (1965) fuzzy set theory and regard the truth values 1, 2, 3, 4 & 5 as being equivalent to fuzzy degrees of membership to "true" of 0.0, 0.2, 0.4, 0.8 & 1.0, respectively. It is then possible to extend techniques derived from a 2-valued logical analysis of repertory grids directly to the n-valued case by regarding the logical expressions as being extended to those of a fuzzy logic.

## FUZZY ENTAILMENT ANALYSIS OF REPERTORY GRIDS

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Entailment is an asymmetric relation not analysable through distance-based clustering techniques and the poles of two constructs may be quite distinct in

terms of equivalence yet closely related in terms of entailment. For example suppose that in construing people someone uses the two constructs

m:runs---doesn't run and n:energetic---passive, then we might well expect to find that LHPm entails LHPn but that LHPn does not entail LHPm, that is that being a runner entails being energetic but being energetic does not entail being a runner. If we analyse such a construct structure in terms of distance measures and hence of equivalence only then we shall not derive such asymmetrical relations between constructs even though they are meaningful and of practical interest.

It is straightforward to derive the entailment structure between the poles of constructs. We only have to check the truth of the four possible implications for all elements. Thus LHPm entails LHPn is checked by noting whether whenever an element is assigned to the left-hand pole of m it is also assigned to the left-hand pole of n. If so, then the entailment relation holds true, otherwise it is false. Clearly, as we noted above, it would also suffice to check that whenever an element is not assigned to the left-hand pole of n it is also not assigned to the left-hand pole of m. We call the program that performs this analysis ENTAIL (Entailment Nets Through Analysing Implicational Links).

In real-world data the entailments are themselves not necessarily clearly defined and it is necessary to make provision for graded entailment measures. One measure has been given in Shaw & Gaines (1980) where we introduce the predicate usually in the analysis performed by ENTAIL. This predicate is a quantifier similar in nature to the "for all" quantifier, but qualified to allow for some disconfirming instances so that it may be read as "for all but N cases" where N is some small number such as 1 or 2. Such a quantifier allows a natural grading of entailment in terms that are immediately meaningful to the originator of the grid: "when you say someone runs you always also say they are energetic" or "when you say someone runs you usually also say they are energetic". Use of the quantifier usually to give a graded analysis gives a structure similar to the connectivity levels coming from Atkin's Q-Analysis.

Figure 3 shows the entailments under the quantifier "usually" between the poles of the constructs derived by ENTAIL from the grid of Figure 1 drawn out as a directed graph. There are effectively two main sub-graphs which are mirror images of one another plus two isolated poles. One of the graphs shows the entailments for one set of poles, and the other the entailments for the opposite poles. Because of the essential bipolarity assumed in the elicitation of the grid the two graphs are essentially the same with the arrows and poles reversed in one relative to the other. There is a close correspondence between the construct clusters resulting and those of FOCUS.

## CONCLUSIONS

We have shown that viewing repertory grid data as a set of truth values in a multivalued logic leads to new forms of analysis. On a particular example the clusters produced by fuzzy ENTAIL analysis have been shown to be closely related to those of the FOCUS cluster analysis. This paper gives further evidence of the close links between Kelly's personal construct theory and Zadeh's development of fuzzy reasoning and the fruitful interplay between the two.

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| | | * 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| INVOLVEMENT | 1 | * 4 | 3 | 2 | 1 | 5 | 1 | 2 | 3 | 1 | REMOTENESS |
| FLEXIBLE | 2 | * 4 | 4 | 3 | 2 | 5 | 1 | 5 | 2 | 1 | RIGID |
| EQUIPMENT | 3 | * 4 | 5 | 5 | 1 | 1 | 3 | 2 | 1 | 5 | NO EQUIPMENT |
| SELF-ORGANISED | 4 | * 5 | 4 | 4 | 3 | 5 | 1 | 2 | 2 | 1 | STAFF-ORGANISED |
| SMALL GROUP | 5 | * 5 | 2 | 4 | 4 | 5 | 1 | 1 | 1 | 3 | LARGE GROUP |
| VARIABLE CONTENT | 6 | * 4 | 4 | 2 | 3 | 5 | 1 | 5 | 5 | 1 | SPECIFIC CONTENT |
| LIKE | 7 | * 5 | 4 | 2 | 2 | 5 | 1 | 5 | 5 | 1 | DISLIKE |

INFORMAL INTERACTION
VIDEO TAPE
PROGRAMMED TEXT
LIBRARY
FILM
PRACTICAL
SEMINAR
TUTORIAL
LECTURE

FIGURE 1 Arthur's Repertory Grid

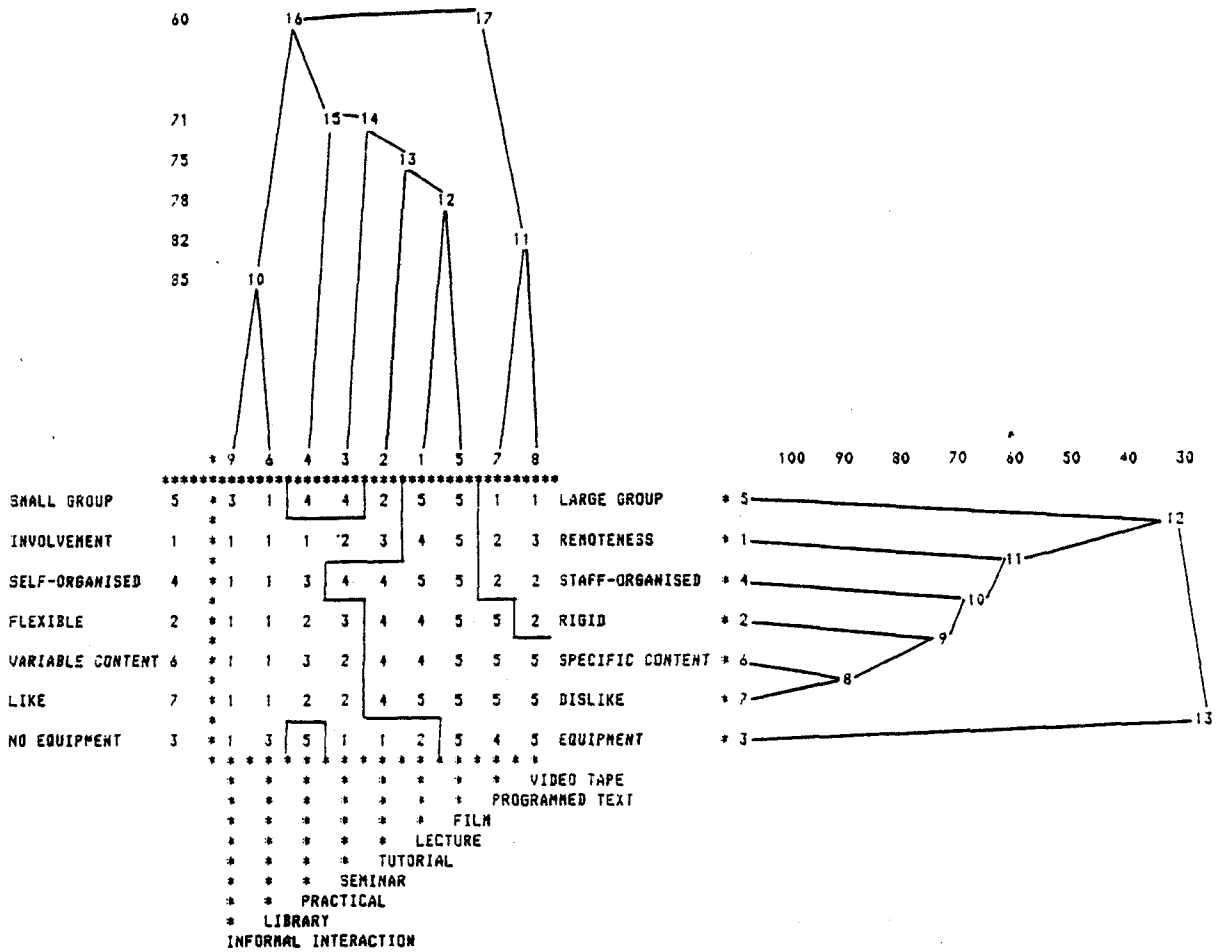


FIGURE 2 Cluster Analysis of Arthur's Grid by FOCUS

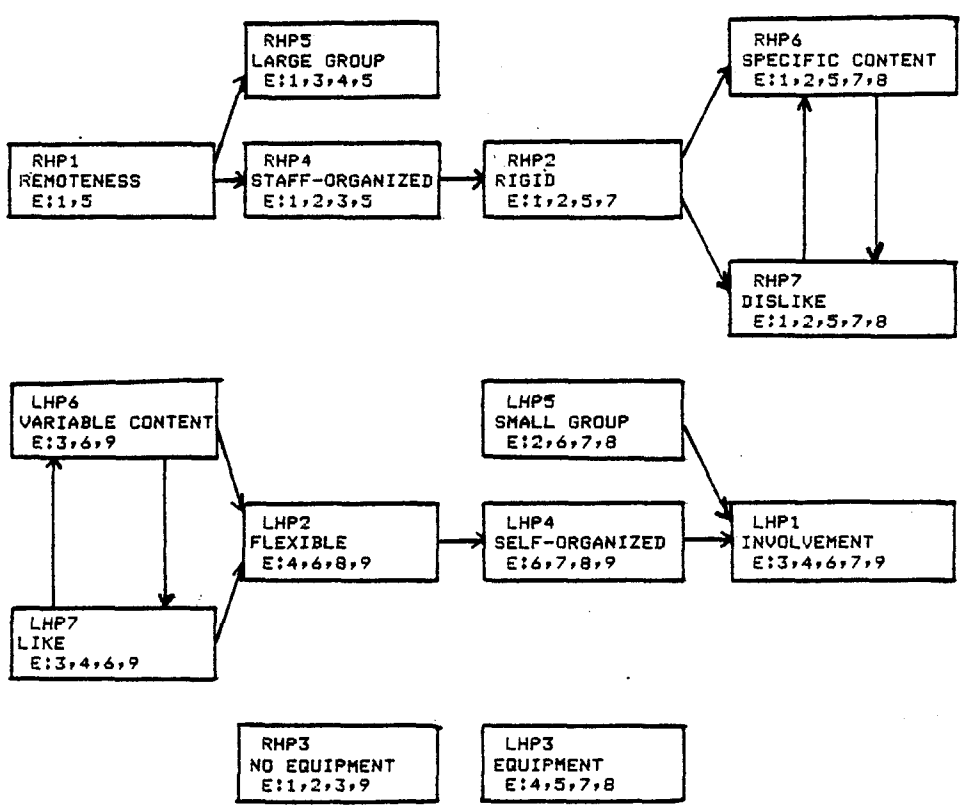


FIGURE 3 Entailment Analysis of Arthur's Grid by ENTAIL