FUZZY SEMANTICS FOR PERSONAL CONSTRUING

Mildred L.G. Shaw

Middlesex Polytechnic
Trent Park
Cockfosters
Herts. U.K.

Brian R. Gaines

Centre for Man-Computer Studies
92, Shakespeare Tower
Barbican
London EC2. U.K.

ABSTRACT

Conventional analyses of personal construct structures use some form of clustering technique to draw out similarities between constructs. However, constructs also often have natural precedence or entailment structures that a similarity analysis obscures. We have previously suggested a fuzzy semantics for personal constructs that leads to natural measures of entailment, and also takes account of the relevance of constructs. This paper presents these fuzzy semantics together with examples of the computer analysis of entailment structures elicited through the PEGASUS interactive construct elicitation system. The entailment structures derived from the grids elicited are compared with those derived by an alternative technique in order to provide some initial validation of the semantic model.

INTRODUCTION

Increasingly computers are being used as tools for aiding knowledge acquisition and decision making, either alone or in some form of man-machine symbiosis. It is possible to give a system-theoretic framework for much of the operation of such computer-based systems enabling suitable algorithms to be formally derived. However, most practical systems commence with some inbuilt vocabulary for the problem area for which they are designed and this itself is not subject to modification either by the computer or by the user. Some degree of pre-supposed terminology is common to all problem-solving environments even when automation is not involved. If we take a problem to an "expert" then we first of all have to express it in such a way that he can understand it. We may well find, however, that this task is itself the most difficult of all.

In this paper we argue first of all for the importance of this terminological level in any form of knowledge acquisition and decision making system, and show its significance in terms of Klir's epistemological hierarchy. We relate this to Kelly's work on personal construct systems and to studies of the computer-based elicitation of construct structures. The dialectical nature of constructs is then discussed in relation to system theory and systems of practical reasoning. Finally a new approach to the analysis of construct systems in terms of implicational operators in fuzzy reasoning is described and compared through an example with other forms of construct analysis.

LEVELS OF KNOWLEDGE

One of the most useful frameworks within which to discuss the operational aspects of the acquisition of knowledge is the "epistemological hierarchy" proposed by Klir (1976). The lowest level is one of source systems, effectively one of data terminology definition whereby the way in which any observed behaviour of a system about which knowledge is to be acquired is defined and agreed. The next level is one of data systems, effectively one of system observation whereby the actual behaviour of some system is described in terms of the agreed domain of discourse at level zero. Levels above these are concerned with models of the data and their inter-relationships.

The importance of considering these levels is that establishing any system for the acquisition of knowledge involves making a number of decisions at each level. From a formal point of view the decisions taken at each level are quite distinct from one another and they are also quite arbitrary. From a practical point of view, however, the decisions at different levels interact with one another and are certainly not arbitrary since they determine the very success of a specific knowledge acquisition system. If it is not explicitly realized that such decisions are being made and are necessary to any operational system then it is easy to become confused about the reasons for the success or failure of any particular system. It is also possible to generate fruitless debates about the merits of different approaches to knowledge acquisition systems which are based on the failure to distinguish between decisions made at different levels in the hierarchy.

THE SOURCE SYSTEM AND LANGUAGE

In particular, the very lowest level of the

source system is often unrecognized as a major determinant of system success or failure. The terminology used in describing data is tacitly accepted and its derivation is not taken to be a significant part of establishing the knowledge acquisition system. However, in the literature of both arts and sciences it will be found that descriptive terminology changes both organically and discontinuously as disciplines develop and undergo various forms of "revolution" (Kuhn 1962). The literature on knowledge acquisition is most often concerned with the upper levels of Klir's hierarchy, such as the relationship between a data system and the generative system that models it, rather than the language in which the data itself is described.

This neglect of the source system may well be a legacy of positivism where linguistic invariance was considered a prerequisite of any acceptable inductive system and hence the actual effects of data language variation in Klir's source system would not be relevant. However, the role of language in the results of science has long been recognized. For example, Cavallo and Klir (1978) note that Poincare ("All that the investigator manages to extract from a fact is the language in which it is expressed") and Wittgenstein ("The limits of my language mean the limits of my world") recognized the fundamental role of language at the base of knowledge, and Wittgenstein's concept of our playing "language games" now permeates much of the epistemological literature.

In recent years the hypothesis of linguistic relativity has been posed most strongly by Whorf:

> "We are thus introduced to a new principle of relativity which holds that all observers are not led by the same physical evidence to the same picture of the universe, unless their linguistic backgrounds are similar." (Whorf 1952 p.21)

Bertalanffy adopts this viewpoint in General System Theory:

> "There is nothing singular or particularly sacred about the system of physics...other symbolic systems...are equally legitimate although they are far from having the same degree of precision. And in other cultures...basically different kinds of 'science' may be possible which would represent other aspects of reality as well or even better than does our so-called scientific world picture." (Bertalanffy 1968 p.260)

PERSONAL CONSTRUCTS

One of the most detailed studies of the operational role and significance of the source system level on the acquisition of knowledge arises from the psycho-philosophical work of the clinical psychologist George Kelly. In his seminal work, The Psychology of Personal Constructs, he notes that:

> "Man looks at his world through transparent patterns or templets which he creates and then attempts to fit over the realities of which the world is composed." (Kelly 1955, pp.8-9)

Kelly continually emphasizes the epistemological role of these constructs in predicting and controlling the world and their ontological status as personal conjectures rather than reality-derived absolutes:

> "Constructs are used for predictions of things to come, and the world keeps rolling on and revealing these predictions to be either correct or misleading. This fact provides the basis for the revision of constructs and, eventually, of whole construct systems." (Kelly 1955 p.14)

Kelly himself developed an elaborate but fairly informal epistemology based on his constructive alternativism and its consequences stated as a fundamental postulate and a set of "corollaries". He also developed techniques for eliciting the personal construct structure of particular individuals and it is this aspect of his work which has been primarily developed by later workers, mainly in the context of clinical psychology, that is, of people with abnormal construct systems. In recent years applications of Kelly's ideas in management science has led to an interest in the personal construct systems of professional staff, and more recently interactive programs have been developed that elicit an individual's construct system within a definite context (Shaw 1979, Boxer 1979).

Shaw and Gaines (1979) have recently discussed such systems within the framework of general knowledge acquisition and the personal computer, and have given a fuzzy semantic model for the personal construct structures. This paper develops this work further and gives computational examples of the fuzzy semantic model applied to the analysis of the implication structure of data already analysed by other methods.

CONSTRUCTS AND FUZZY SETS

In the physical sciences the expected and preferred source system in which to represent data is quantitative. We use a source system of physical quantities and their precise measurement. However, the underlying constructs of physics have been derived and refined over a very long period and are themselves of a peculiar, and perhaps unique, nature. The existence of continuous and limitless scales for physical variables of length, time, mass, charge, and so on, is an important phenomenon that marks out the constructs involved as being different from those in many other sciences. However, the existence of refined measuring schemes for some constructs should not blind us to their close relationships to other constructs for which no such physical measurement exists. For example, Gaines compares and contrasts the concepts of 'tallness' and 'beauty':

> "the concept, the perception, of 'tallness' exists in a more primitive sense than does the measurement of 'height'... We are able to generate and follow arguments involving 'tallness' without having any concept of inches, centimetres, or any other metric scales... whilst

a 'scientific' analysis might conclude that there is a wide and ill-defined range of physical phenomena that combine in an extremely complex fashion to produce the subjective impression of beauty, in everyday reasoning it is as primitive a term as...tallness. We certainly do not distinguish between them in arguments such as:

> He likes girls that are tall and beautiful.
> Mary is not very tall but very beautiful.
> He will probably like Mary.

or

> This ladder is too tall to put in the kitchen.
> This vase is too beautiful to put in the kitchen."
> (Gaines 1976)

He suggest that fuzzy set theory provides a model of constructs at the source system level which is more primitive and generally applicable than the quantitative scales of the physical sciences. Under certain conditions measuring schemes may be available which preserve all the order relations of the fuzzy sets and obey additional axioms. Deriving these additional, higher-order constraints, is itself an important task of empirical science. However, one should not pre-suppose their existence and the general assumption at the source level can be only that there may be predicates relevant to the problem under consideration that define fuzzy sets in relation to the entities involved.

The links between the basic constructs used at the source level and fuzzy set theory have already been established. Goguen (1974) takes a formal axiomatic approach to the notion of a 'concept' in natural and artificial languages and shows within a very general category-theoretic framework that one obtains generalized fuzzy sets. Kelly (1955) presented constructs as binary categories and based his own methodology for eliciting constructs on this. However, later workers found the need for 'shades of grey' between the two poles of a construct and Kelly (1970) notes that this is consistent with his notion of a 'construct':

> "The construct, of itself, is the kind of contrast one perceives... while constructs do not represent or symbolize events, they do enable us to cope with events, which is a statement of a quite different order...They also enable us to put events into arrays or scales, if we wish." (Kelly 1970, pp.13-14)

At a fundamental level we can see Kelly's constructive alternativism as being based on the primitive operation that Spencer Brown calls "making a distinction". Brown commences his seminal work on Laws of Form by noting:

> "The theme of this book is that a universe comes into being when a space is severed or taken apart...By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance." (Spencer Brown 1969 p.v)

The interpolation within the basic bipolar distinction that has been made can be generated through logical operations that correspond to paradoxes of self-reference in classical logic. Varela (1975) shows how an essentially three-valued logical calculus arises from acceptance of a single self-referential form in Brown's calculus of distinctions. Gaines (1976) shows how such "primitive paradoxes" may be iterated to give an indefinite number of distinctions between the poles of that originally made, and hence how the fuzzy truth value of an arbitrary proposition may be derived through a Dedekind section.

FUZZY SET SEMANTICS FOR CONSTRUCTS

In our previous paper (Shaw & Gaines 1979) we propose the working hypothesis:

Hyp1: The basis underlying personal constructs is that a construct is a predicate defining a fuzzy set such that a construed element has a degree of membership to that set.

and note that there is no assumption that different people make use of the same construct labels or constructs, and that fuzzy sets in which the same elements have different degrees of membership are essentially different fuzzy sets and hence define different constructs. Thus the personal nature of constructs is inherently allowed for in this model. However, the assumption of defined "degrees of membership" is a strong one since it enables us to compare different fuzzy sets in a variety of ways that well-order them. These in themselves generate fuzzy sets, and hence possible constructs at a meta-level, e.g. the fuzzy set of constructs similar to construct A on some criterion. This is particularly important in the extension of interactive construct elicitation systems such as Shaw's PEGASUS because it enables feedback about the analysis of construct structures to be couched in the same terminology as was used in eliciting them.

We also noted that Hyp1 takes us only a small step towards a calculus of constructs because it says nothing about the way in which the fuzzy sets may be manipulated to give us fuzzy reasoning and hence a model underlying construct generation, reasoning with constructs, etc. There are a number of possible foundations of fuzzy reasoning (Gaines 1976) and we shall assume for purposes of this paper:

Hyp2: The fuzzy sets underlying constructs are manipulated according to an uncertainty logic (Gaines 1978).

There are many uncertainty logics and it will not be necessary in the following discussion to use more than a fragment common to most of them. In particular, we need the fairly weak assumption about the form of fuzzy implication that if A

implies B then the truth value of B is greater than or equal to that of A.

We shall adopt the convention in writing formula for fuzzy reasoning in this paper that predicates are in upper case letters, logical variables are in lower case letters, and that the concatenation of a predicate and a variable is both a logical formula and the degree of membership of that variable in the fuzzy set defined by the predicate, with any ambiguity resolved by context.

NEGATION, OPPOSITES AND RELEVANCE

One apparent problem with fuzzy sets in catching the nature of personal constructs is that their definition contains no reference to the bipolarity which Kelly felt to be a particularly important feature of personal constructs. Hypl may appear to lead to precisely those defects of formal logic that Kelly warns against:

> "Now conventional logic would say that black and white should be treated as separate concepts. Moreover, it would say that the opposite of black can only be stated as not black, and the opposite of white can only be stated as not white. Thus the person whose field we mentioned would have shoes which would be just as much not white as the time of day, and he would write on paper which would be just as not black as the distance to his office." (Kelly 1955 p.106)

Part of the problem that Kelly is discussing here is one of relevance. The standard predicate calculus fails to distinguish between "not" and "not relevant" - it is only in recent years that formal logics accounting for relevance have been established (Anderson and Belnap 1975). However, what he also brings out is the psychological role of the concept of opposite which has no logical counterpart - it is related to negation but not identical to it.

This introduction of the importance of modelling the role of opposites in human thinking is particularly interesting for the links it provides between Kelly's epistemology and those of many other philosophers. Emphasis of the dichotomous nature of human reasoning is common and ancient, for example the Pythagoreans used a table of opposites in analysing entities with ten constructs:

Limited	Unlimited
Odd	Even
Unity	Plurality
Right	Left
Male	Female
At rest	In motion
Straight	Curved
Light	Darkness
Good	Evil
Square	Oblong

but like virtually all later thinkers they do not justify the assertion that opposites are fundamental to reasoning. Kelly does not himself do so except by quotations like that above which point out by example the difference between the negation of a construct and an opposite to it.

Mao Tsetung (1937) in his essay "On Contradiction" emphasizes the essential interdependence of opposites:

> "no contradictory aspect can exist in isolation. Without its opposite aspect, each loses the condition for its existence...Without life, there would be no death; without death there would be no life. Without above there would be no below; without below there would be no above...It is so with all opposites; in given conditions, on the one hand they are opposed to each other, and on the other hand they are interconnected, interpenetrating, interpermeating and interdependent" (Mao 1937 p.61)

Mao also brings in the notion of relevance in defining opposites and uses the notions of contradiction yet identity amongst opposites in his exposition of Marxist epistemology which closely mirrors Kelly's constructive alternativism:

> "Marxists recognize that in the absolute and general process of development of the universe, the development of each particular process is relative, and that hence, in the endless flow of absolute truth, man's knowledge of a particular process at any given stage of development is only relative truth" (Mao 1937 p.12)

This line of reasoning can be traced back through Lenin (1914) to Hegel whose basic logic of thesis and antithesis leading to a synthesis is founded on what seems to be the most careful distinction between opposite and negation in the philosophical literature. Hegel distinguishes between negation as an absolute difference and opposition as essential difference and Bogomolov (1977) singles this out as the foundation of dialectical logic:

> "the investigation of the relation of two objects...begins with establishing the difference between them, expressed in the most general form, with their mutual negation (A and -A). To put it differently the second object acts initially as the simple negation of the first and is naturally expressed in logic by its indefinite negation...Describing this kind of development of the concept, Hegel saw in it the transition from absolute difference to essential difference (variety), and from this to opposition (antithesis), as one of the stages of the general path from identity through difference to contradiction." (Bogomolov 1977 p.137)

Thus we may see that Hegel's dialectics is crucially dependent on the transition from the concept of general negation to that of opposition. An opposite is some basis for there being negation, some reason for it, and it is the underlying construct to which this opposition is relevant that Hegel regards as the "synthesis" of the opposition between thesis and antithesis. The problem in developing an adequate 'dialectical logic' which would be a foundation for both Kelly's and Hegel's epistemologies (which are operationally remarkably

similar - the key difference is that Hegel had a concept of movement towards 'absolute truth' which Kelly does not) is that classical logic does not allow for relevance and the distinction between opposite and negation. In our previous paper we showed that fuzzy logic does have the mechanisms to allow for such a distinction.

OPPOSITES IN FUZZY LOGIC

Logics based on fuzzy set theory do allow us to make a distinction between "negation" and "opposite" and to relate this to "relevance", thus providing the tools that Kelly needed to fully explicate these key distinctions. First, it is worth noting that Zadeh in presenting fuzzy set models of verbal reasoning makes precisely the distinction between opposite and negation that Kelly makes: the fuzzy set representing "young" is quite distinct from the fuzzy set representing "not old" (e.g. Zadeh 1976 p.290). Thus, Kelly's requirement that this distinction be made can be satisfied - it is also worth noting that his argument for the intrinisic bipolarity of constructs is borne out by the extent to which such fuzzy predicates are presented in the literature as pairs of opposites.

If, however, we start with essentially bipolar constructs then we cannot treat relevance within a uniform framework - Kelly has to introduce it separarately in terms of constructs having a "range of convenience". However, by considering an element to have quite distinct degrees of membership to the two poles of a construct, i.e. to a construct and its "opposite" we can also capture the concept of relevance - i.e. a construct is irrelevant to an element if its degree of membership to both poles of the construct is zero. Kelly could have captured this in his binary model of constructs by noting, for example, that the construct good---bad is irrelevant to something which is both not-good and not-bad (it is clearly debatable still whether this explication of relevance captures all its psychological connotations - we would suggest not, only that it captures some key ones). Those who extended his bipolar notion to allow for multipoint rating scales also failed to allow for relevance when they made the scales a 1-dimensional interpolation between the two poles of a construct.

Clearly whilst the notion of "opposite" is not the same as that of "negation" it does have some relationship to it - the "opposite" predicate is constrained by similar but weaker constraints than the negated predicate. We would certainly require a construct and its opposite to be symmetric in their properties since the label of "opposite" could be applied to either relative to the other. Thus, if OPP PRED is the opposite of PRED, we would have the following substitution rule:

Sub1: A valid formula remains valid under the simultaneous uniform substitutions: OPP PRED $\rightarrow$ PRED and PRED $\rightarrow$ OPP PRED.

By using Sub1 in the equivalence, OPP PRED $\equiv$ OPP PRED, we obtain:

$$\forall\ x \quad \text{OPP OPP PRED } x \equiv x \qquad (1)$$

which emphasizes the similarity of OPP to negation, NOT. We would also expect symmetry with respect to negation so that the negation of an opposite is the opposite of the negation, i.e. a second substitution rule:

Sub2: A valid formula remains valid under the substitution anywhere of NEG OPP for OPP NEG, and vice versa.

In terms of the arguments above an opposite is something which implies the negation so that:

$$\forall\ x \quad \text{OPP PRED } x \supset \text{NOT PRED } x \qquad (2)$$

which gives us in terms of degrees of membership:

$$\text{OPP PRED } x \leq 1 - \text{PRED } x$$

or

$$\text{PRED } x + \text{OPP PRED } x \leq 1 \qquad (3)$$

whereas for negation, NOT, we have:

$$\text{PRED } x + \text{NOT PRED } x = 1 \qquad (4)$$

so that a negation is an opposite, but not necessarily vice versa.

We may now also define the relevance, REL, of a predicate to an element as the degree to which inequality (3) is an equality, i.e.:

$$\text{REL PRED } x \equiv (\text{NOT OPP PRED } x \supset \text{PRED } x) \qquad (5)$$

which in terms of degree of membership gives:

$$\text{REL PRED } x = \text{PRED } x + \text{NOT OPP PRED } x \qquad (6)$$

This also corresponds to the extent to which OPP and NOT are the same:

$$\text{REL PRED } x \equiv (\text{OPP PRED } x = \text{NOT PRED } x) \qquad (7)$$

These definitions show the close relationships between opposite, negation and relevance that Kelly hints at in his work but did not have the formal tools to define. The discussion of this section demonstrates that Hypl, although omitting specific mention of bipolarity, actually gives one adequate tools with which to explicate both the notions of "opposite" and "relevance" in personal construct theory. It further suggests that in the elicitation of personal constructs the bipolarity does have a significant role to play for, without information on the degree of membership to both poles of a construct, we cannot distinguish between its not applying and not being relevant. However, this also requires a change in the methodology of construct elicitation since current techniques, such as those used in PEGASUS (Shaw 1979), by forcing elements to be rated against constructs on a linear scale are automatically assuming relevance. This is not unreasonable given that both elements and constructs are elicited from the user to be mutually relevant to the context of a specifically selected topic. It suggests, however, that in a large-scale system with wide-ranging applications

such that uniform relevance cannot be taken for granted it would be appropriate to allow users to ascribe separate ratings to the two poles of a construct.

ANALYSIS OF CONSTRUCT STRUCTURES

The analysis of data from experiments on construct elicitation is a major task in its own right. One may regard it as a problem of attempting to determine the entailment structure for a class of constructs defined extensively by the degrees of membership of a set of elements to various fuzzy sets. There is also the converse problem of determining possible relationships between elements given their co-ordinates as members of a class of fuzzy sets.

We generally need first of all to define some similarity measures such as the degree of equivalence, EQUIV, between two constructs:

$$\text{EQUIV } A\ B = \forall x\ (A\ x \equiv B\ x) \qquad (8)$$

so that in terms of degree of membership:

$$\text{EQUIV } A\ B = 1 - \max_x |A\ x - B\ x| \qquad (9)$$

i.e. it is determined by the largest difference in the ratings of the same element on the two different constructs.

This is a reasonable measure of similarity but not a very robust one against errors in the data collected. If it were not possible to check unusual discrepancies by asking the user to re-rate the appropriate constructs then it might be felt appropriate to hedge the notion of equivalence by defining the similarity measure USUALLYEQUIV as:

$$\text{USUALLYEQUIV } A\ B \equiv (\text{Usually } x\ \ A\ x \equiv B\ x) \qquad (10)$$

where the quantifier 'Usually' is defined through a fuzzy set in which the 100% occurrence of the predicate has a degree of membership 1 and the 0% has 0 with intermediate frequencies having intermediate degrees of membership. The effect of this is to allow for some major variations with only slight loss of truth value - it is similar to the use of quartiles in statistics and the concept of a 'set of measure zero' in probability theory.

Equivalence between constructs is one important measure in allowing them to be clustered in terms of similarity. However, it is also of interest to investigate the implicational relationships between them, and we may define similar measures to (10) by replacing $A\ x \equiv B\ x$ with one of the four distinct implications possible: $A\ x \supset B\ x$, $B\ x \supset A\ x$, $A\ x \supset$ NOT $B\ x$, or $B\ x \supset$ NOT $A\ x$. This gives us a directed fuzzy graph of implications between constructs that may itself be clustered to determine the main entailment structure for the constructs.

EXAMPLE OF ANALYSIS

To illustrate the technique for construct structure analysis described in the previous section we have taken an example of a grid elicited using PEGASUS and analysed using FOCUS (Shaw 1979). This has been re-analysed by a program that calculates the degree of implication between two constructs. The data taken for analysis is somewhat incestuous since it was elicited from one of the authors as a set of constructs relating to her range of grid elicitation and analysis programs. However, this data was a particularly useful starting point for us as it is possible to see clearly whether any new results of the analysis made reasonable sense.

The elements used were the programs: (1) FOCUS - a grid analysis program which clusters elements and constructs; (2) SPACED - a version of FOCUS which blocks the printout according to the clusters; (3) PEGASUS - an interactive grid elicitation program; (4) PEGBANK - a version of PEGASUS which holds an expert bank of constructs against which elicited constructs are matched; (5) MIN-PEG - a version of PEGASUS without continuous feedback of results; (6) SOCIOGRIDS - an analysis program for a set of repertory grids; (7) CORE(1) - an interactive program which finds the intersection of two grids elicited from one person at different times; (8) CORE(2) - same as CORE(1) but the grids are from different people; (9) MINUS - finds the differences between two grids; (10) ARGUS - an interactive program which elicits six grids from one person from different perspectives; (11) FOCI - a version of FOCUS which gives an interpretation of the results; (12) PRE-PEG - a version of PEGASUS which uses a previously started grid.

Twelve constructs were elicited for this set of elements and these are shown down the sides of the focused grid in Figure 1. For example construct 12 is "clustering" versus "comparison". FOCUS has re-ordered the rows of constructs and the columns of elements to cluster together those which have similar patterns of ratings. For example element cluster 14 consists of 1 (FOCUS) and 11 (FOCI), and construct cluster 14 consists of 7 (conversation with self --- conversation with others) and 6 (one person involved --- more than one person). From this analysis it is easy to see which elements and constructs are likely to occur together.

The grid shown in Figure 1 was treated as if the figures in it were now degrees of membership to fuzzy predicates (with 1 zero membership and 5 unity membership). It was then analysed for implicational structure as described in the previous section. Figure 2 gives a table of the implications derived using the concept of "usually implies" already discussed. If one picks out of this table the implications true at the 4 or 5 levels then one obtains the directed graph of implications shown in Figure 2. Note that the equivalence between constructs 2, 3 & 10, and between 6, 7, & 9, shown in the FOCUS analysis of Figure 1 is also picked out in the implicational analysis. Some of the other linkages in Figure 2 have also been picked up by FOCUS, for example between (2,3,10) and 11 and 4. However, the unidirectional nature of these linkages is missing because the FOCUS analysis is looking at degrees of equivalence only, not of implication.

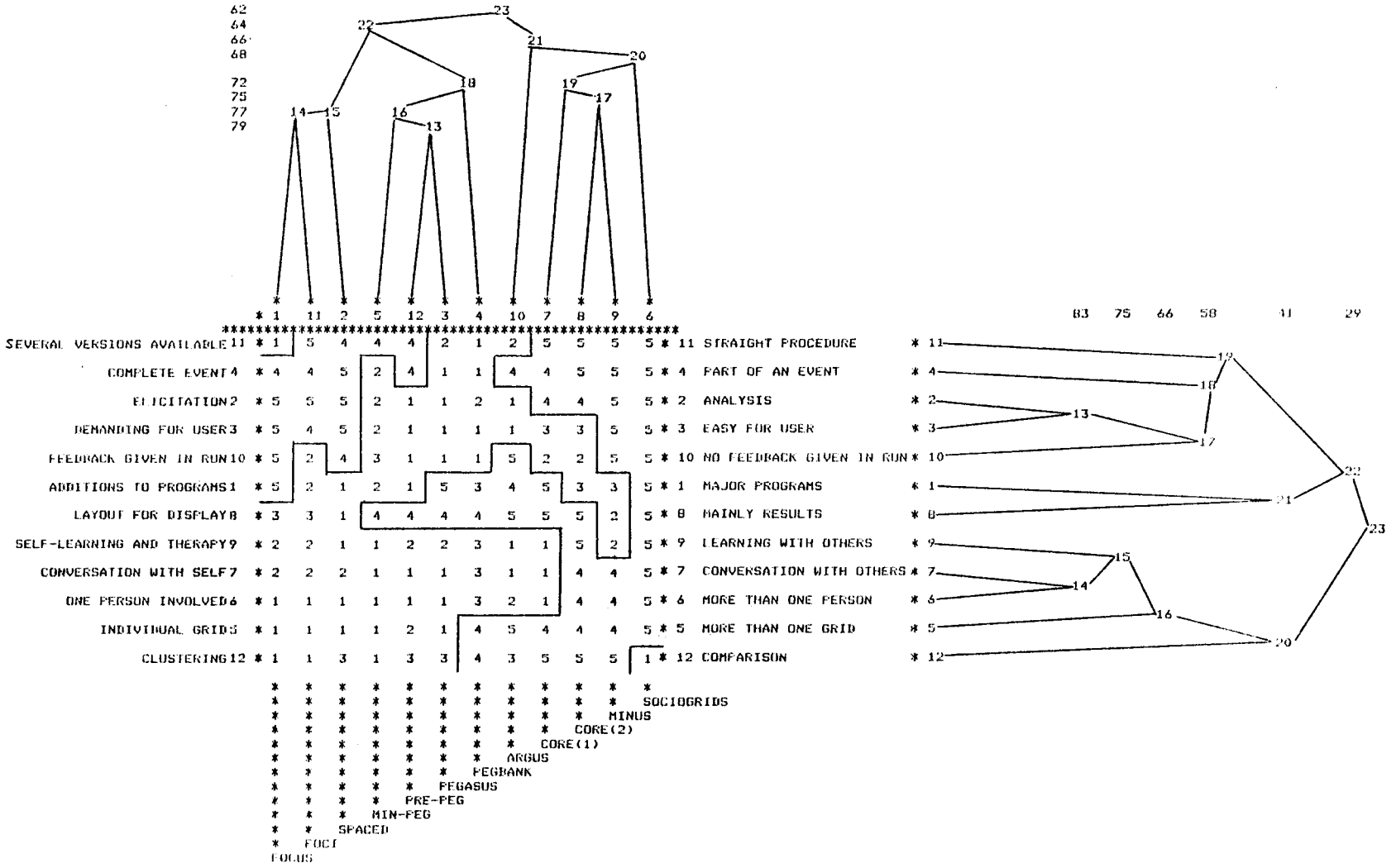
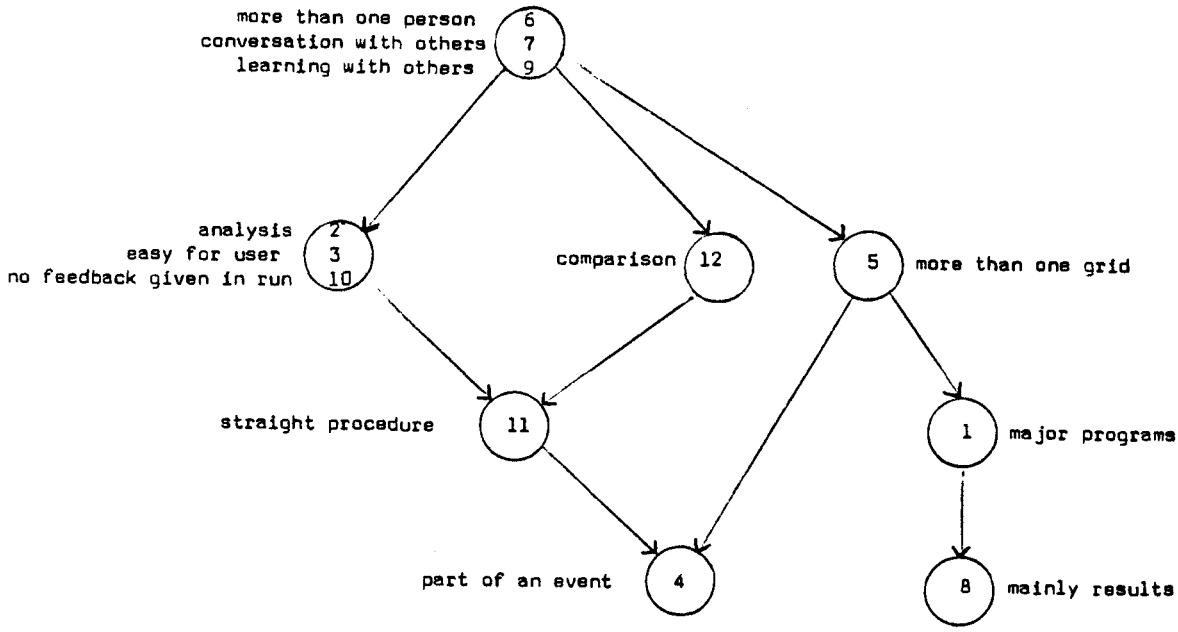


Figure 1. The focused grid showing constructs relating to Shaw's computer programs.



Implicational Analysis of Properties "Usually"

	*	1	2	3	4	5	6	7	8	9	10	11	12	
	*	*	*	*	*	*	*	*	*	*	*	*	*	*
B	1*	5	2	3	2	4	4	4	3	4	3	2	3	A
	2*	2	5	5	2	3	4	5	2	4	4	3	3	
	3*	2	4	5	2	2	4	4	2	3	4	3	3	
	4*	3	4	5	5	4	5	5	2	4	4	4	3	
	5*	1	1	1	2	5	5	4	2	4	2	2	3	
	6*	1	1	1	2	2	5	4	2	4	2	1	3	A ⊃ B
	7*	1	2	2	2	2	5	5	1	4	2	2	3	
	8*	4	2	2	2	5	5	4	5	5	2	2	3	
	9*	2	3	2	2	3	4	4	1	5	2	2	2	
	10*	3	3	4	2	3	3	3	2	3	5	2	2	
	11*	2	4	4	3	2	5	4	2	4	2	5	4	
	12*	1	1	1	2	3	3	4	2	4	1	1	5	

Figure 2. Table and graph of implications derived from 'usually implies'.

Like any technique for drawing conclusions from a limited set of empirical data the implicational analysis can only suggest linkages, not prove that they are there. An apparent implication may be an artefact of the data collected rather than an essential feature of the use of the constructs involved. One can test the derived implications by showing them to the subject and asking whether they are valid or not. This is consistent with the philosophy of personal construct theory in its clinical and managerial applications. It is also a consistent addition to PEGASUS to have it feed back implications and, if they are not accepted as valid, to ask for further examples of elements that break them. Conversely, if the user feels that an implication should hold that does not come out of the analysis then the program can list precisely those items of data entered that cause the required implication not to hold.

CONCLUSIONS

This paper is one further step towards our objective of providing a man-machine symbiotic system for epistemological and decision-making studies. The initial results of the empirical studies of fuzzy set semantics for the analysis of personal constructs are very promising, but clearly need further investigation before they can be accepted as providing an adequate model for human construct structures. Likewise the fuzzy semantic foundations for the notions of 'opposite' and 'relevance' also seem to provide interesting formal models for these significant higher-level constructs and would be worth deeper investigation in terms of the logics involved.

REFERENCES

Anderson, A.R. & Belnap, N.D.
1975 Entailment. Princeton University Press.

Bertalanffy, L.von
1968 General System Theory. Penguin Press, London.

Bogomolov, A.S.
1977 "Dialectical Contradiction and Its Solution. Concerning the Central Problem of Dialectical Logic." in Philosophy in the USSR. Problems of Dialectical Materialism. 134-156.

Boxer, P.J.
1979 "Reflective Analysis." Int. J. Man-Machine Studies 11: 547-584.

Cavallo, R. & Klir, G.J.
1978 "A Conceptual Foundation for Systems Problem Solving." Int. J. Systems Sci. 9: 219-236.

Gaines, B.R.
1976 "Foundations of Fuzzy Reasoning." Int. J. Man-Machine Studies 8: 623-668.

1978 "Fuzzy and Probability Uncertainty Logics." Information and Control 38: 154-169.

Goguen, J.A.
1974 "Concept Representation in Natural and Artificial Languages: Axioms, Extensions and Applications for Fuzzy Sets." Int. J. Man-Machine Studies 6: 513-561.

Kelly, G.A.
1955 The Psychology of Personal Constructs. Norton, New York.

1970 "A Brief Introduction to Personal Construct Theory." in Perspectives in Personal Construct Theory. Academic Press, London.

Klir, G.J.
1976 "Identification of generative structures in empirical data." Int. J. General Systems 3: 89-104.

Kuhn, T.S.
1962 The Structure of Scientific Revolutions. University of Chicago Press.

Lenin, V.I.
1914 "Conspectus of Hegel's Book The Science of Logic." Collected Works 38:85-238. Lawrence & Wishart, London.

Mao Tsetung.
1937 Five Essays on Philosophy. Foreign Languages Press, Peking.

Shaw, M.L.G.
1979 On Becoming a Personal Scientist. Academic Press, London.

Shaw, M.L.G. & Gaines, B.R.
1979 "Externalizing the Personal World: Computer Aids to Epistemology." Improving the Human Condition: Quality and Stability in Social Systems. Society for General Systems Research.

Spencer Brown, G.
1969 Laws of Form. George Allen & Unwin, London.

Varela, F.J.
1975 "A Calculus for Self-Reference." Int. J. General Systems 2: 5-24.

Whorf, B.L.
1952 Collected Papers on Metalinguistics. Foreign Service Institute, Department of State, Washington.

Zadeh, L.A.
1976 "A Fuzzy Algorithmic Approach to the Definition of Complex or Imprecise Concepts." Int. J. Man-Machine Studies 8: 249-291.