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INFORMATION-THEORETIC ANALYSIS OF CONSTRUCT SYSTEMS

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Abstract: A method is given for inductive inference of entailments from repertory grid data allowing combined fuzzy logical and probabilistic hypotheses. The method provides a measure based on expected surprise as a means of estimating the significance of a hypothesis through its reduction of uncertainty. The results of the analysis are fuzzy linguistic inferences that may be interpreted directly to prime expert system shells. The overall methodology provides rigorous mathematical foundations for knowledge engineering based on personal construct psychology techniques.

INTRODUCTION

The repertory grid is a well-established technique for probing the construct systems of individuals (Fransella & Bannister 1977). However, the analysis of repertory grid data in such a way as to make legitimate inferences about the structure present is a difficult task. This paper presents a new approach to this analysis which has a number of significant advantages. It has theoretical foundations that are both mathematically sound and intuitively acceptable. It leads to a logical analysis of construct systems in terms of equivalences and entailments that are easily understood and expressed. Each element of the analysis carries with it an information-theoretic measure of its legitimacy as an inductive inference from the data in the grid. This measure corresponds to a significance measure in standard statistics and is the appropriate equivalent given the non-parametric nature of repertory grid data.

The new analysis has been implemented in the computer program **ENTAIL II**, an extension of that described previously (Gaines & Shaw 1981). ENTAIL II has proved very succesful in its application to expert system development, and seems to generate only those rules which are meaningful to the expert. It is suggested that this new method of analysis overcomes many of the problems of previous techniques while not losing any of the meaningful inferences generated by them.

REPERTORY GRID ANALYSIS

Kelly's (1955) **repertory grid** is a way of representing personal constructs as a set of distinctions made about elements relevant to the problem domain. In clinical psychology this domain will often be personal relationships and the elements may be family members and friends. In the development of expert systems the elements will be key entities in the problem domain such as oil-well sites or business transactions. Repertory grids have been widely used: in clinical psychology (Shaw 1981, Shepherd & Watson 1982); to study processes of knowledge acquisition in education (Pope & Shaw 1981); and to study decision making by individuals and groups in management (Shaw 1980).

Figure 1 shows a repertory grid elicited from a manager on the topic of "personnel appraisal" (Shaw 1980). The **elements** he is construing are his subordinates and are shown as the rows of a matrix. The **constructs** he is using to make bipolar distinctions between these elements are shown as the columns of the matrix. A rating scale from one through five is being used to enable him to make more precise distinctions in situations where he does not wish to allocate an element definitely to one pole of a construct. A rating of 1 means at the left-hand pole (LHP), and of 5 at the right-hand pole (RHP).

It is conventional to analyse a repertory grid as if it were a set of vectors of numeric data representing the position of each element in the space of constructs. This leads naturally to distance-based cluster-analytic approaches such as principal components analysis, for example Slater's (1976) INGRID, and hierachical sorting methods, for example Shaw's (1980) FOCUS. However, while distance-based cluster analyses can portray two-sided relations between constructs, that construct A is similar to construct B, they cannot derive one-sided inferential relations, that construct A entails construct B. We have previously suggested a logical analysis of construct entailment based on an interpretation of repertory grids as fuzzy predicates (Shaw and Gaines 1979, 1980).

If element e has a rating of r on construct c where the range of ratings is 1 through n, then we say that e has a degree of membership of (r-1)/(n-1) to the right-hand pole (RHP) of construct c, and a degree of membership (n-r)/(n-1) to the left-hand pole (LHP) of construct c. We may regard each pole as representing a logical predicate that defines a fuzzy set and the repertory grid as a set of such fuzzy sets, one for each predicate. Following the conventions of Gaines (1976a) we shall use ordinary characters for fuzzy sets, boldface characters for fuzzy variables, and denote the degree of membership of a variable to a set by concatenating it to the right of the set. Thus, x is the fuzzy set defined by predicate x (a LHP or RHP of the repertory grid) and the degree of membership of element e to fuzzy set x is xe.

Title:ICI-P130
Elements 10, constructs 14, range 1-5
Purpose: staff appraisal

		*	1	2	3	4	5	6	7	8	9	10	*
intelligent	1	*	1	1	4	5	3	3	5	2	3	5	* 1 dim
willing	2	*	1	2	4	5	1	1	4	3	1	2	* 2 unwilling
new boy	3	*	1	2	3	5	4	4	4	1	4	3	* 3 old sweats
little supervision reqd	4	*	3	1	4	5	2	1	5	2	2	3	* 4 need supervision
motivated	5	*	1	1	4	5	2	2	5	3	3	2	* 5 less motivated
reliable	6	*	3	2	2	5	1	1	5	1	2	3	* 6 not so reliable
mild	7	*	3	4	5	2	2	3	1	5	4	5	* 7 abrasive
ideas men	8	*	1	1	5	4	2	3	1	3	4	4	* 8 staid
self starters	9	*	2	1	5	5	1	3	5	3	4	5	* 9 need a push
creative	10	*	1	1	5	5	2	3	4	3	4	5	* 10 non-creative
helpful	11	*	4	3	4	2	3	5	1	4	5	5	* 11 unhelpful
professional	12	*	1	2	3	3	2	1	5	2	4	4	* 12 less professional
overall rating high	13	*	2	1	3	4	1	2	5	2	3	4	* 13 overall rating low
messers	14	*	2	2	5	4	3	5	1	5	3	1	* 14 tidy
			*	*	*	*	*	*	*	*	*	S10	
			*	*	*	*	*	*	*	*	S9		
			*	*	*	*	*	*	*	S8			
			*	*	*	*	*	*	S7				
			*	*	*	*	*	S6					
			*	*	*	*	S5						
			*	*	*	S4							
			*	*	S3								
			*	S2									
			S1										

Figure 1 Repertory Grid from a Manager on Staff Appraisal

If we take u to be the predicate defined by LHP 9 (L9) in Figure 1 and v to be that defined by LHP 13 (L13) then the fuzzy sets defined by the grid are:

$$u = (0.75,1.00,0.00,0.00,1.00,0.50,1.00,0.50,0.25,0.00) \quad (1)$$

$$v = (0.75,1.00,0.50,0.25,1.00,0.75,0.00,0.75,0.50,0.25) \quad (2)$$

One distance-based measure of the similarity of two predicates, x and y, is:

$$\Box(x \equiv y) = \sum_{e} (xe - ye)^2/n \quad (3)$$

where n is the cardinality of the set of elements:

$$n = \sum_{e} 1 \quad (4)$$

This is the covariance measure that is used in the principal components analysis of repertory grids (Slater 1976). Another distance-based measure is:

$$\Box(x \equiv y) = \sum_{e} |xe - ye|/n \quad (5)$$

This is used by Shaw (1980) for hierarchical clustering and may be regarded as a fuzzy measure of the distance between two predicates using the sigma-count of a fuzzy set (Zadeh 1983).

We have used the symbol for logical necessity, $\Box$, in (3) and (5) to indicate that the measures defined may be regarded as estimators of the necessity of the predicates p and q being equivalent in a fuzzy modal logic. Both the measures derived have proved effective in distance-based clustering of repertory grid data. However, the semantics of both measures are difficult to define in terms of fuzzy logic and linguistic reasoning.

The measure of (3) has no known semantics in fuzzy logic and that of (5) involves complex iterated implications (Gaines 1983). We have suggested an alternative measure of equivalence involving only simple operations in fuzzy logic (Shaw & Gaines 1979, 1980, 1981, 1982):

$$\Box(x \equiv y) = \bigwedge_{e} (xe \equiv ye) \qquad (6)$$

which, if we treat the logical connectives as those of the Lukasiewicz L_1 form of fuzzy logic, becomes:

$$\Box(x \equiv y) = \mathrm{MIN}_{e} \min(1-xe+ye, 1-ye+xe) \qquad (7)$$

This logical formulation is particularly interesting because it extends from the symmetrical equivalence relation used in similarity clustering to the asymmetric entailment relation used for inference in ESs:

$$\Box(x \supset y) = \bigwedge_{e} (xe \supset ye) \qquad (8)$$

which for the Lukasiewicz L_1 form of fuzzy logic, becomes:

$$\Box(x \equiv y) = \mathrm{MIN}_{e} \min(1, 1-xe+ye) \qquad (9)$$

We may use the usual symbols for necessary implication, or entailment:

$$x \rightarrow y \equiv \Box(x \supset y) \qquad x \leftrightarrow y \equiv \Box(x \equiv y) \qquad (10)$$

The logical entailment relations of (6) through (10) have been incorporated in the repertory grid analysis program ENTAIL and shown to be similar in clustering to the conventional similarity-based programs while giving additional data about one-sided entailments (Gaines & Shaw 1981). Boose (1984) has used measures derived from these in his Expertise Transfer

System (ETS) and shown it to give rapid prototyping of expert systems for a wide range of applications.

However, ENTAIL as originally presented derives all entailments that are **consistent** with the data in the grid, that is any **possible** entailment that is not disconfirmed by the data. This presents two problems:

(a) that no estimate is given of the significance of these entailments, the degree to which the data confirms them;

(b) that there may be few entailments that are totally true yet large numbers that are partially true and the theory so far gives no way of selecting those that are reasonable.

These are partly problems of induction from finite data, partly problems of the material conditional which is confirmed in trivial cases ($p \supset q$ is true if p is false), and partly problems of the logical predicates defined so far not capturing all that may be reasonably said about the data.

In the next section we tackle the first two problems of estimating the significance of entailments. In the following section we enrich the set of predicates used to include probabilistic assertions. Together, these give a formal theory of the induction of entailments from repertory grid data.

SIGNIFICANCE ANALYSIS OF REPERTORY GRID DATA

The significance of an assertion may be measured in terms of the reduction of uncertainty that results from that assertion. If we consider the fuzzy set:

$$w = (0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00) \qquad (11)$$

then the assertions $w \rightarrow v$ and $u \rightarrow v$ both have truth value 1 from (9) yet for the first knowing we gives no reduction in the uncertainty of ve whereas for the second knowing ue gives a great deal of reduction.

We may measure this reduction in uncertainty by defining a measure of the uncertainty about an individual event and then deriving the average

uncertainty over all events. Gaines (1977) defines the **surprise** at an event to be minus the logarithm of the probability ascribed to that event before it occurred, and shows how surprise minimization may be used to formalize inductive inference. We may apply this to the fuzzy sets defining predicates by examining the distribution of degrees of membership, the probabilities that a given degree of membership will occur. The distribution, ∇x, of the fuzzy set x may itself be regarded as a fuzzy set of degrees of membership to these probabilities and written:

$$\nabla x = \{m/p(m): \exists \mathbf{e}, m=x\mathbf{e}\} \tag{12}$$

where p(m) is the probability (relative frequency) of the degree of membership m in the fuzzy set x and "/" is used as a separator as has become conventional for fuzzy sets. For the examples given we have:

$$\nabla u = \{1.00/0.3, 0.75/0.1, 0.50/0.2, 0.25/0.10, 0.00/0.30\} \tag{13}$$

$$\nabla v = \{1.00/0.2, 0.75/0.3, 0.50/0.2, 0.25/0.20, 0.00/0.10\} \tag{14}$$

$$\nabla w = \{0.00/1.00\} \tag{15}$$

The expected surprise for a distribution ∇x is:

$$a(x) = \sum_{\mathbf{e}} -\log_2(p(x\mathbf{e}))/n \tag{16}$$

which is just the Shannon entropy for the distribution.

We may use this formulation to define the conditional surprise as the expected surprise for the distribution given another distribution and a hypothesis about their relationship:

$$a(x|y \rightarrow x) = \sum_{\mathbf{e}} -\log_2(p(x\mathbf{e}|y\mathbf{e}, y \rightarrow x))/n \tag{17}$$

where $p(x\mathbf{e}|y\mathbf{e}, y \rightarrow x)$ is the probability of $x\mathbf{e}$ having its degree of membership

given the degree of membership of ye and the truth value of the assertion y→x. We may interpret this assertion as a constraint on the relation between xe and ye and this enables the distribution of xe to be restricted when the value of ye is given. For example, given that u→v=1, and given the first value of ue in (1) as 0.75 we can infer that the conditional distribution for the first value of ve in (2) is {1.00/0.4,0.75/0.6} which has a greatly reduced uncertainty from the unconditional distribution of (14). The conditional probability is p(ve|ue,u→v)=0.6 compared with the unconditional probability p(ve)=0.3.

We may now define the uncertainty reduction due to a hypothesis as the difference between the unconditional and conditional expected surprise given the hypothesis:

$$\rho(y \rightarrow x) = \alpha(x) - \alpha(x|y \rightarrow x) \qquad (18)$$

This uncertainty reduction can be used to rank the entailments induced by ENTAIL II. Clearly such trivial entailments as w→v have zero uncertainty reduction, as do such equally trivial entailments as v→v and v→w, even though all three have truth value 1. The entailment v→u is not trivial but its low truth value of 0.5 leads to it providing little constraint and hence a low value of uncertainty reduction.

Figure 2 is an ENTAIL II analysis of the grid of Figure 1 with the truth values and uncertainty reductions (termed "information") shown together with the entailments expressed linguistically. The column marked "Prob." will be discussed in the next section. It will be noted that ENTAIL II checks for equivalence separately from the check for one-way entailment. This is because the uncertainty reduction from the hypothesis of equivalence, x≡y, cannot be inferred from the separate uncertainty reductions of the logically equivalent pair of hypotheses, x→y and y→x, although it will be at least the sum of these. In order to rank-order the hypotheses the uncertainty reduction for one-sided entailment is doubled for comparison with that of two-sided equivalence.

Note how a hypothesis of high truth value such as R13→R9 can give less

Entail	Truth	Prob.	Information (Cutoff 35)	
L 9 → L13	100	100	59	self starters → overall rating high
L 1 → L 9	75	100	58	intelligent → self starters
L10 → L 8	100	100	57	creative → ideas men
[illegible] 9 → R 1	75	100	56	need a push → dim
L 1 → L10	75	100	52	intelligent → creative
R 8 → R10	100	100	52	staid → non-creative
R13 → R 6	75	100	52	overall rating low → not so reliable
L10 → L 9	75	100	51	creative → self starters
R10 → R 1	75	100	49	non-creative → dim
L 9 → L10	75	100	47	self starters → creative
L10 ≡ L 1	75	100	47	creative ≡ intelligent
R 1 ≡ R10	75	100	47	dim ≡ non-creative
R 9 ≡ R10	75	100	47	need a push ≡ non-creative
R13 → R 9	100	100	46	overall rating low → need a push
R 4 → R13	75	100	44	need supervision → overall rating low
R 5 → R 4	75	100	44	less motivated → need supervision
R 5 → R13	75	100	44	less motivated → overall rating low
L 1 → L 3	75	100	42	intelligent → new boy
L 6 → L13	75	100	42	reliable → overall rating high
L 4 ≡ L13	75	100	39	little supervision reqd ≡ overall rating high
L13 ≡ L 6	75	100	39	overall rating high ≡ reliable
R 1 → R 6	50	100	39	dim → not so reliable
R 6 ≡ R13	75	100	39	not so reliable ≡ overall rating low
R13 ≡ R 4	75	100	39	overall rating low ≡ need supervision
R12 → R13	75	100	38	less professional → overall rating low
R13 → R12	75	100	38	overall rating low → less professional
L 8 ≡ L10	100	70	37	ideas men ≡ creative
R 9 → R 4	50	100	37	need a push → need supervision
R 9 → R12	50	100	37	need a push → less professional
R 9 → R13	50	100	37	need a push → overall rating low
R10 ≡ R 8	100	70	37	non-creative ≡ staid
L 4 → L 5	75	100	36	little supervision reqd → motivated
L 4 → L 9	50	100	36	little supervision reqd → self starters
L12 → L 9	50	100	36	professional → self starters
L12 ≡ L13	75	100	36	professional ≡ overall rating high
L13 → L 5	75	100	36	overall rating high → motivated
L13 → L 9	50	100	36	overall rating high → self starters
R 2 → R 4	75	100	35	unwilling → need supervision
R 2 → R 5	75	100	35	unwilling → less motivated
R 2 → R13	75	100	35	unwilling → overall rating low
R 3 → R 1	75	100	35	old sweats → dim

Figure 2 Entailment Analysis of Repertory Grid on Staff Appraisal

information reduction than one of lower truth value such as L1→L9. Note also how a one-sided hypothesis, such as L1→L10, may give greater uncertainty reduction than its two-sided equivalent, L1≡L10 and hence have a higher ranking. Similarly, the two one-sided hypotheses, such as L9→L10 and L10→L9 can also occur in preference to the two-sided hypotheses, L9≡L10. The list of hypotheses in Figure 2 has been terminated when the uncertainty reduction falls below 35% for a two-sided hypothesis. It can be truncated at any level and the entailments plotted in graphical form. However, since approximate inference is involved the graph will not necessarily be transitive and it is important to preserve the intransitivity. It avoids the paradoxes of approximate reasoning such as that of Sorites (Gaines 1976a).

The entailments derived by ENTAIL II can be converted directly to rules that may be loaded into a standard ES shell as Boose (1984) has shown. The truth value of the rule codes into the truth value represention used by the shell. The rules may be checked by the expert for their content but their prior evaluation for uncertainty reduction makes it unlikely that spurious rules will be generated. The rules form a decision tree expressing the relationship between an expert's data and his conclusions, both personally expressed. For example, the data of Figure 1 may be regarded as that of an expert on staff appraisal and one may be concerned with deriving his "overall rating" (construct 13) from behavioral assessments such as "intelligent" and "creative". The ENTAIL II analysis of Figure 2 shows that L1, L4, L6, L9, L10 and L12 imply L13, that "intelligent, creative, reliable and professional self-starters requiring little supervision receive a high overall rating", whereas R2, R4, R5, R6, R9 and R12 imply R13, that "being unwilling, less motivated, not so reliable, less professional, needing supervision and needing a push leads to a low overall rating."

PROBABILISTIC HYPOTHESES

In our initial analysis of entailment we introduced the notion of entailments being true "usually" and interpreted this as "in all except one instance." The original version of ENTAIL generates entailments that are

usually true and these have proved useful with some data where there is a common pattern upset only by an isolated case. The concept of "usually" can be generalized and formalized within the framework already established and it leads to an interesting two-dimensional analysis of fuzzy data.

We quantify the notion of a hypothesis being true "usually" by ascribing a probability to the hypothesis measured by the relative frequency with which it is true. For example, using the data of (1) and (2), u→v has truth value 1 with probability 1; it is always absolutely true. However, v→u is more interesting because it has truth value 0.5 with probability 1, truth value 0.75 with probability 0.9 (fails in one case out of ten), and truth value 1.0 with probability 0.4 (fails in six cases out of ten). Thus evaluation of an entailment is now not a fuzzy truth value but rather a distribution as already defined, a fuzzy set of degrees of membership of probabilities:

$$v \rightarrow u = \{1.00/0.4, 0.75/0.9, 0.50/1.0\} \qquad (19)$$

This type of trade-off distribution between the truth value of an assertion and its probability of being true occurs in inductive inference with fuzzy data, for example in inferring structure from fuzzy sequential data (Gaines 1979). We can evaluate the possible trade-offs by using the uncertainty reduction analysis based on expected surprise already described. The calculation of the conditional probability of an event under a hypothesis takes into account probabilistic assertions by calculating the expected distribution as a mixture of the unconditional distribution with the probability that the hypothesis is false, and the conditional distribution with the probability that the hypothesis is true. It is interesting to note that the correct calculation has to be based on considerations of expected surprise not expected uncertainty; the notion of surprise is more fundamental than that of uncertainty.

As can be seen from Figure 2 the preferred hypothesis for L13→L9 is a truth value of 0.50 with probability 1. However, the hypothesis L8≡L10 is preferred as having a truth value of 1 with probability 0.70. The distribution is:

$$L8 \equiv L10 = \{1.00/0.7, 0.75/0.9, 0.25/1.0\} \quad (20)$$

and the greatest uncertainty reduction is achieved with a hypothesis that is false in 3 cases.

It is also of interest to note that the notion of "usually" with the probabilistic interpretation given here may be regarded as a generalization of Atkin's (1974) Q-Analysis for fuzzy data (Gaines 1976b). We have previously compared fuzzy Q-Analysis with other distance-based clustering techniques for repertory grids and shown that it generates similar clusters (Gaines & Shaw 1981). The formulation of entailment analysis given here subsumes Q-Analysis as a special case of clustering based on two-sided logical equivalence but with probabilistic hypotheses.

CONCLUSIONS

A method has been given for inductive inference of entailments from repertory grid data allowing combined fuzzy logical and probabilistic hypotheses. The method provides a measure based on expected surprise as a means of estimating the significance of a hypothesis through its reduction of uncertainty. The results of the analysis are fuzzy linguistic inferences that may be interpreted directly to prime expert system shells. The overall methodology provides rigorous mathematical foundations for knowledge engineering based on personal construct psychology techniques.

We note in passing that the mathematical techniques presented in this paper have wider relevance than to repertory grid data. They may be regarded as a general way of inducing and representing structure from fuzzy data giving the degree of membership of abstract entities to certain attributes. They may also be regarded as non-parametric techniques for the statistical analysis of such data. The fuzzy-truth-value versus probability-of-correctness trade-off seems a fundamental concept of inductive inference and data analysis. It has a heuristic relationship to distance measures that have proved useful for many purposes but do not have a natural

interpretation except for parametric data. It is suggested that more general application of the techniques described in this paper will prove valuable.

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