KELLY'S PSCHOLOGICAL GEOMETRY AS AN INTENSIONAL LOGIC OF KNOWLEDGE REPRESENTATION

Mildred L G Shaw & Brian R Gaines
Knowledge Science Institute, University of Calgary
Calgary, Alberta, Canada T2N 1N4
mildred@cpsc.ucalgary.ca & gaines@cpsc.ucalgary.ca

Abstract: Personal construct psychology (PCP) is a theory of individual and group psychological and social processes that has been used extensively in knowledge acquisition research to model the cognitive processes of human experts. The psychology has the advantage of taking a constructivist position appropriate to the modeling of specialist human knowledge but basing this on a positivist scientific position that characterizes human conceptual structures in axiomatic terms that translate directly to computational form. The repertory grid knowledge elicitation methodology is directly derived from personal construct psychology. In its original form, this methodology was based on the notion of dichotomous constructs and did not encompass the ordinal relations between them captured in semantic net elicitation. However, it was extended in successive tools developed for applied knowledge acquisition and tested in a wide variety of applications. This paper gives an overview of PCP and its expression as an intensional logic describing the cognitive processes of anticipatory agents.

PSYCHOLOGICAL GEOMETRY

Kelly's "fundamental postulate" for personal construct psychology was that:

> "A person's processes are psychologically channelized by the way in which he anticipates events." (Kelly, 1955, p.46)

This was stated as a postulate to emphasize that it was presented as a convenient viewpoint from which to understand human behavior, not imputed to an underlying physiological or psychological reality. The basis of Kelly's approach to psychotherapy was that this was also a convenient viewpoint from which someone could understand, and modify, their own behavior. He saw all people as "personal scientists" in anticipating the world, and attempted to develop techniques where this anticipatory modeling activity was reflexively applied to the self. His first corollary, the construction corollary, states:

> "A person anticipates events by construing their replications." (p.50)

This emphasis on the role in behavior of a view to the future is what distinguishes Kelly's approach to psychology. He saw man as driven by the need to cope with coming events in the world and all other aspects of behavior as deriving from this:

> "A person's processes, psychologically speaking, slip into the grooves which are cut out by the mechanisms he adopts for realizing his objectives." (p.49)

These grooves provide templets for construing events which he termed "personal constructs":

> "Man looks at his world through transparent templets which he creates and then attempts to fit over the realities of which the world is composed." (pp.8-9)
>
> "Constructs are used for predictions of things to come, and the world keeps on rolling on and revealing these predictions to be either correct or misleading. This fact provides the basis for the revision of constructs and, eventually, of whole construct systems." (p.14)

Kelly introduces the notion of a *psychological space* as a term for a region in which we may place and classify elements of our experience. It is important to note that he did not suppose this space to pre-exist as a world of such elements, but rather to come into being through a process of construction by which we create a space in which to place elements as we come to construe them. He sees us as creating dimensions in personal psychological space as a way of providing a coordinate system for our experience, and emphasizes that the topology of the space comes into existence as it is divided:

> "Our psychological geometry is a geometry of dichotomies rather than the geometry of areas envisioned by the classical logic of concepts, or the geometry of lines envisioned by classical mathematical geometries. Each of our dichotomies has both a differentiating and an integrating function. That is to say it is the generalized form of the differentiating and integrating act by which man intervenes in his world. By such an act he interposes a difference between incidents — incidents that would otherwise be imperceptible to him because they are infinitely homogeneous. But also, by such an intervening act, he ascribes integrity to incidents that are otherwise imperceptible because they are infinitesimally fragmented. In this kind of geometrically structured world there are no distances. Each axis of reference represents not a line or continuum, as in analytic geometry, but one, and only one, distinction. However, there are angles. These are represented by contingencies or overlapping frequencies of incidents. Moreover, these angles of relationship between personal constructs change with the context of incidents to which the constructs are applied. Thus our psychological space is a space without distance, and, as in the case of non-Euclidian geometries, the relationships between directions change with the context." (Kelly, 1969)

It is this emphasis on the space itself being created by a process of making distinctions rather than being defined by the elements distinguished that gives personal construct psychology its intensional nature:

> "the construct is a basis of making a distinction...not a class of objects, or an abstraction of a class, but a dichotomous reference axis" (Kelly, 1970)

Figure 1 shows the main features of Kelly's notion of psychological space. A construct is a dichotomous reference axis. It defines a family of planes orthogonal to it that divide the space:

> "To catch a glimpse of psychological space we may imagine a system of planes, each with two sides or aspects, slicing through a galaxy of events" (Kelly, 1970)

However, only part of the space that is divided is used in placing elements:

> "A construct is convenient for the anticipation of a finite range of events only. A personal construct system can hardly be said to have universal utility. Not everything that happens in the world can be projected upon all the dichotomies that make up a person's outlook. . . The geometry of the mind is never a complete system." (Kelly, 1970)

This division defines the dichotomous poles of the construct:

> "Each construct involves two poles, one at each end of its dichotomy. The elements associated at each pole are like each other with respect to the construct and are unlike the elements at the other pole" (Kelly, 1955)

Even though Kelly's geometry is not metrically defined, he has no problems in using the defined constructs to generate a metric as shown in Figure 2:

> "imagine a system of planes, each with two sides or aspects, slicing through a galaxy of events...If the set is moved into all possible positions it generates a paracartesian hyperspace with its relatively concrete scalar axes" (Kelly, 1970)

Kelly does not intend that the system described be seen to be at the verbal level:

> "I have probably given the impression that a construct is highly articulate and cognitive...but personal construct theory is no more a cognitive theory than it is an affective or conative one. There are grounds for distinction that operate in one's life that seem to elude verbal expression" (Kelly, 1955)

He sees no conflict between his psychology and a neurological perspective:

> "A person develops a physiological construct system...but that does not prevent us examining the person's private system from a psychological point of view" (Kelly, 1955)

He sees his psychological geometry as providing a framework for a variety of viewpoints:

> "For the present we do not need to ask how man performs this intervening act — whether with his brain, his stomach, or his glands. Nor do we need to concern ourselves just yet with the essence of the act — whether it is cognitive, conative, or affective. Finally we need not agree on what kind of substance fills the psychological space we have structured — whether the space is stuffed with physiological things, social things, or mental things. All these matters are, at most, no more than subsequent issues, and indeed, as I personally suspect, may prove to be no issues at all, after we have put our mathematics to work." (Kelly, 1969)

Kelly introduces anticipation within the geometry by attaching actions to regions of psychological space:

> "A young girl anticipates marriage...The predicted husband does not exist for her in the flesh, but simply as the intersect of a limited number of conceptual dimensions. One day a young man plumps himself down on this waiting intersect...she marries him" (Kelly, 1955, p.121)

It is possible to develop a complete theory of cognition, action, learning and intention with the geometry. However, it is more useful to first develop an intensional logic isomorphic to the geometry and then present Kelly's psychology in both the geometry and the logic.

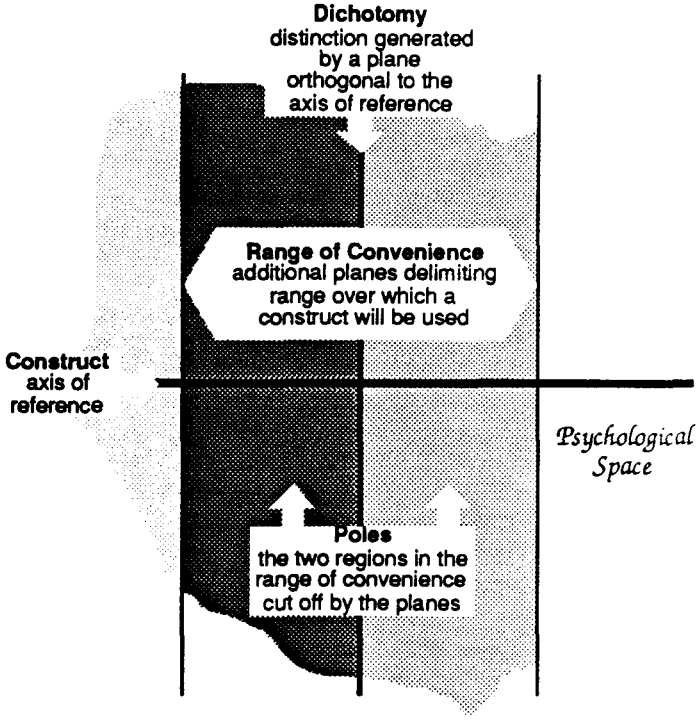

Figure 1 The geometry of psychological space

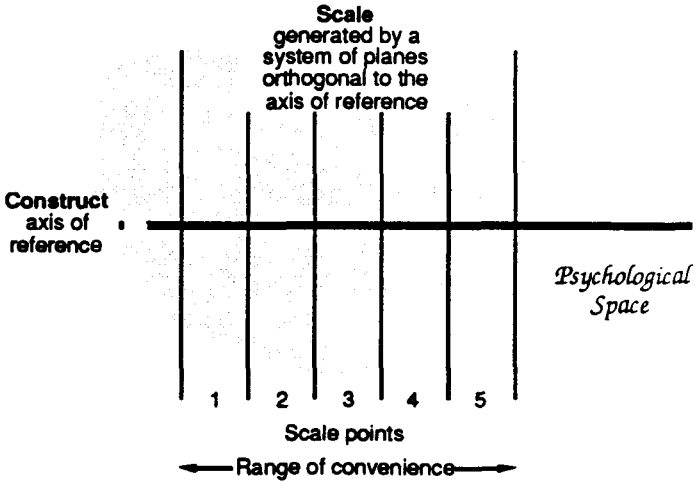

Figure 2 Scales in psychological space

CONSTRUCTS AND CONCEPTS

One obvious question about Kelly's use of the term construct was how it differs from the more conventional term "concept." He discusses this in the following terms:

> "We use the term *construct* in a manner which is somewhat parallel to the common usage of 'concept.' However, if one attempts to translate our *construct* into the more familiar term, 'concept,' he may find some confusion. We have included, as indeed some recent users of the term 'concept' have done, the more concretistic concepts which nineteenth-century psychologists would have insisted on calling 'percepts.' The notion of 'percept' has always carried the idea of its being a personal act—in that sense, our *construct* is in the tradition of 'percepts.' But we also see our *construct* as involving abstraction—in that sense our *construct* bears a resemblance to the traditional usage of 'concept.' ... Now when we assume that the construct is basically dichotomous, that it includes percepts, and that it is a better term for our purposes than the term 'concept,' we are not quarreling with those who would use it otherwise. Within some systems of logic the notion of contrast as something distinct from irrelevancy is not part of the assumptive structure. We, on the other hand are simply assuming that this is the way people do, in fact, think." (Kelly, 1955, p.70)

The dichotomous aspect of constructs is the most significant aspect of the difference between Kelly's constructs and current usage of the term, 'concept.' His *dichotomy corollary* states this:

> "A person's construction system is composed of a finite number of dichotomous constructs." (p.59)

and it is a consequence of the two-sided nature of a distinction represented in the geometry. The range of convenience captures the notion of relevancy and the distinction within it then generates a natural opposition. That people tend to conceptualize the world in terms of restricted sorts that are then dichotomized is a phenomenon identified in antiquity (Lloyd, 1966) and common across many cultures (Maybury-Lewis & Almagor, 1989). In a technical context, Kelly's construction is related to that of Zermelo-Fraenkel set theory in which the logical paradoxes of unrestricted comprehension are avoided by the restriction that conceptual definitions apply only to existing sub-sets, giving rise to a well-defined relative negation (Hatcher, 1982).

Since the term "concept" is used in psychology and in knowledge representation in somewhat different ways, it is useful to define it clearly in these contexts for the purposes of this paper. A psychological concept is defined to be that mental entity *imputed* to a distinction making agent as enabling it to make a particular distinction. Note that concepts are separated both from the distinctions they support and the entities they distinguish, and are not reified but seen as imputed to the agent. They are themselves distinctions made by an observer—possibly, a reflective observer. This is equivalent to Zadeh's (1964) introduction of the notion of the *state* of a system as an imputed parametrization accounting for a system's behavior. Concepts are state variables we impute to a knowledgeable agent. This definition also corresponds to Anglin's:

> "a concept is all of the knowledge possessed by an individual about a category of objects or events"—"Concepts mediate categorization but concepts are not the resultant categories." (Anglin, 1977)

A computational knowledge representation concept is defined to be that data structure used by a distinction making algorithm to enable it to make a particular distinction. No discussion of imputation is necessary because we are not dealing with identification of a black box from its behavior, but rather with the internal structure of an accessible open box. These differing interpretations correspond to the two sides of the Husserl-Frege debate (Mohanty, 1982): distinctions as mental phenomena underlying the interpretations the human mind places on experience, and distinctions as formal structures in their own right independent of their origins.

None of these additional connotations are necessary to the formal development of personal construct psychology. For the psychology, the geometry and the logic, distinctions are the only primitives. One might ask "what is distinguished?", but the answer will be "distinctions." This corresponds to deriving the two phenomena in Kelly's *construction corollary* from a single primitive, that we distinguish events in the undifferentiated stream of circumstance, and then we further distinguish among the distinguished events by construing.

FROM GEOMETRY TO LOGIC

It is very easy and highly instructive to move from Kelly's geometry to a corresponding intensional logic. We may take Kelly's notion of a distinction as primitive and see how distinctions may relate to each other in psychological space. If an element is placed within the region carved out by a distinction then we may say that the distinction is *asserted* to apply to the element. This corresponds to Zalta's (1983) distinction in his intensional logic of abstract objects that "abstract objects *encode* properties whereas existent objects cannot encode properties but do *exhibit* them." If one distinction carves out a region that contains that carved out by another then the first distinction may be said to *subsume* the second. If one distinction carves out a region that does not overlap that carved out by another then the first distinction may be said to be *disjoint* to the second. These relations are in themselves sufficient to define an intensional logic of distinctions in that more complex relations may be composed from them.

The subsumption and disjoint relations may be defined in an algebraic formalism by representing distinctions by bold lower case letters such that a distinction applied to another distinction is concatenated to the right of it. Then the definition above translates as one distinction will be said to *subsume* another if it can always be applied whenever the other can. It can be represented formally as:

"**b** subsumes **a**" $\quad \mathbf{a} \rightarrow \mathbf{b} \Leftrightarrow \vdash \mathbf{xa} \Rightarrow \vdash \mathbf{xb}$ (1)

That is, **b** subsumes **a**, if and only if whenever one asserts **xa** one also asserts **xb**. The definition is to be read intensionally in terms of a *commitment* to the way in which distinctions

will be made, such that if **a** is made then there is a commitment to **b** being made also. This is why the form $\forall x$ is avoided—the notion of all the distinctions to which **a** and **b** may be applied is not well-defined. However, if this extension is well-defined then the intensional formulation of (1) implies the extensional formulation in terms of $\forall x$. Marcus' (1962) non-standard, *substitutional* reading of the universal quantifier would be appropriate to the definition above, but the notation is avoided because the extensional reading is prevalent and significant in its own right.

Subsumption corresponds to increasing generality since the subsuming distinction can be applied to at least as many things as that subsumed. In (1) concept **a** is said to be *subordinate* to concept **b**, and **b** *superordinate* to **a**. Subsumption is an asymmetric, transitive relation, a partial pre-order, over distinctions, that supports the ordinal relations of Kelly's *organization corollary*.:

> "Each person characteristically evolves, for his convenience of anticipating events, a construction system embracing ordinal relationships between constructs." (p.56)

This notion of subsumption is adequate to capture Kelly's use of the term that one construct subsumes another, and also the use of the same term in knowledge representation, that one concept subsumes another. Subsumption between computational concepts corresponds to the "is-a" relation in knowledge representation schema. The interpretation of subsumption in terms of commitment above corresponds to the definitional form of the "is-a" relation. The computed form of "is-a" requires some further structures which are developed in the next Section when primitive and non-primitive concepts are differentiated.

The disjoint relation is definable in similar terms, that one distinction is disjoint with another in that one can never be applied whenever the other can. It can be represented formally as:

"a disjoint b" $\quad \mathbf{a}—\mathbf{b} \Leftrightarrow \vdash \mathbf{xa} \Rightarrow \neg \vdash \mathbf{xb}$ (2)

That is, **a** is disjoint with **b**, if and only if whenever one asserts **xa** one does not assert **xb**. The definition is again to be read intensionally in terms of a commitment to the way in which distinctions will be made, such that if a is made then there is a commitment to **b** not being made. Disjoint is a symmetric, intransitive relation over distinctions, and supports Kelly's *dichotomy corollary* and the definition of disjoint concepts in knowledge representation.

It is interesting to note that definition (2) is an asymmetric definition of what is clearly a symmetric relation. Logically, this is is possible because the reverse implication can be derived from (2), that is, if one asserts **xb** one cannot assert **xa** because that would imply $\neg \vdash \mathbf{xb}$. This derivation of symmetry from asymmetry may be logically simple, but it is not semantically trivial. In terms of knowledge representation it corresponds to the essential sequence of definitions: if we define **a** first we cannot define it to be disjoint with **b** because **b** is not yet defined. Psychologically, this asymmetry appears to be related to the empirical asymmetries Adams-Webber (1979) has observed in the use of the, apparently symmetric, poles of a construct.

The $\rightarrow$ and — relations are complementary in establishing four possible binary relations between distinctions, that $\mathbf{a} \rightarrow \mathbf{b}$, $\mathbf{b} \rightarrow \mathbf{a}$, $\mathbf{a}—\mathbf{b}$, or none of these. The two subsumption relations can hold together giving an equivalence relation on distinctions. The disjoint relation is incompatible with the subsumption relations, and is *inherited* through subsumption, that is:

$$\mathbf{a}—\mathbf{b} \text{ and } \mathbf{c} \rightarrow \mathbf{a} \Rightarrow \mathbf{c}—\mathbf{b} \qquad (3)$$

The logic defined so far is one of pure entailment, of strict or necessary implication, which forms the basis of a large family of intensional logics (Lemmon, Meredith, Meredith, Prior & Thomas, 1969). Before extending it further it is worth defining a visual language for the logic that parallels Kelly's psychological geometry in a more compact notation.

A VISUAL LANGUAGE FOR THE LOGIC

The arrow and line notion adopted in definitions (1) and (2) translates nicely to a graphical notation defining a *visual language* for the logic (Gaines, 1991b) as shown at the bottom of Figure 3. Kelly's 'construct' in psychological space is conveniently represented by a pair of disjoint concepts corresponding to the construct poles, both subsumed by a third corresponding to the range of convenience. This diagram is already a simple semantic network in the style of KL-ONE (Brachman & Schmolze, 1985) or KRS (Gaines, 1991a), but it has well-defined logical semantics as defined above, and also strong psychological foundations in personal construct psychology.

There is an analogy between the visual language and the representation of chemical structures as atoms and bonds. Distinctions are the atomic primitives in personal construct psychology, and further constructions may be seen as complex 'molecules' formed by distinctions joined through subsumption and disjoint 'bonds.' For example, Figure 3 shows how the geometry and the logical visual language may both be used to represent different foci of convenience and contexts:

> "A construct's range of convenience comprises all those things to which the user would find its application useful. A construct's focus of convenience comprises those particular things to which the user would find its application maximally useful. The context...is somewhat more restricted than the range of convenience...somewhat more extensive than the focus of convenience" (Kelly, 1955)

The distinctions, that cut out regions for 'person' and 'wine' each involve an additional dimension and are not adequately in a two-dimensional diagram. This is a problem for the geometry that is overcome in the visual language for the logic.

Multiple constructs in psychological space correspond to multiple axes of reference, and the plans representing their distinctions and ranges of convenience intersect to define regions of the space corresponding to conjunction, composition and multiple inheritance in the logic as shown in Figure 4.

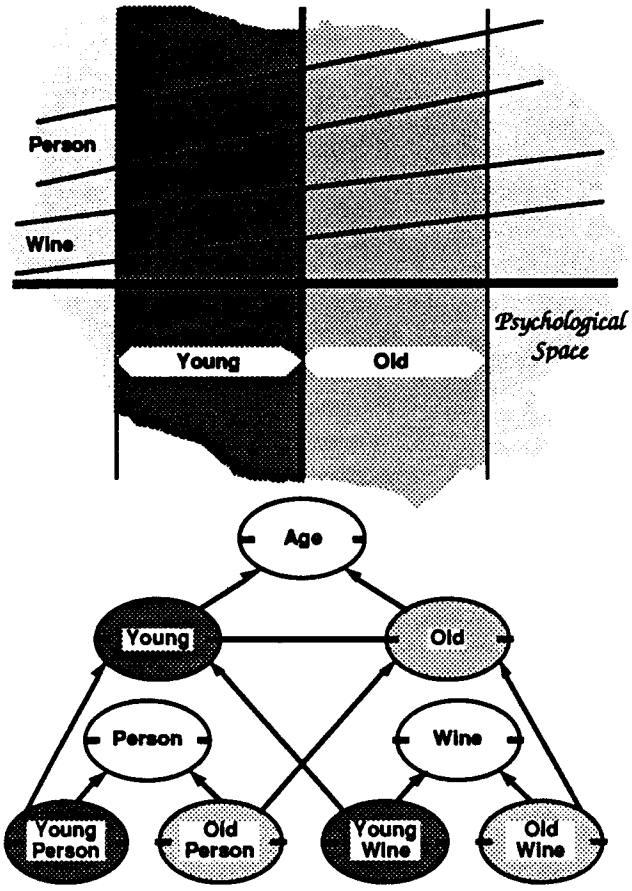

Figure 3 Logic of subsumed foci of convenience

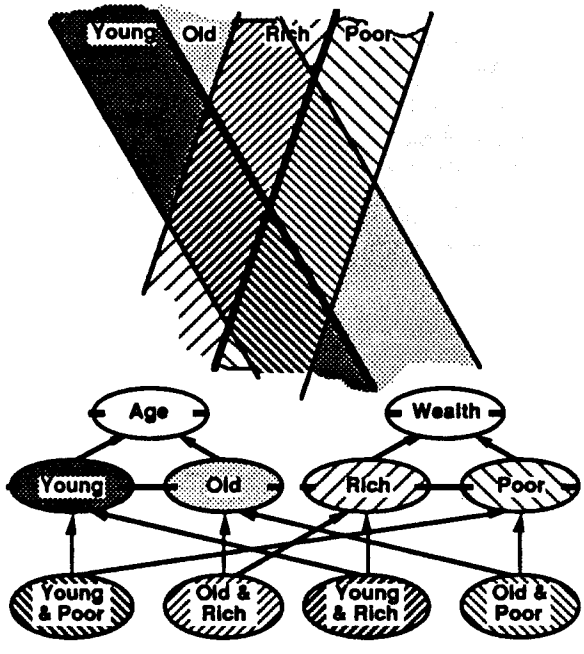

Figure 4 Logic of overlapping distinctions and their intersections

There is an important distinction to be made between the concepts defined by basic distinctions and those defined by intersections. The former are said to be *primitive concepts* and the latter non-primitive, or computed, concepts. In the visual language primitive concepts are distinguished by having a small internal horizontal line at their left and right edges. Primitive concepts are incompletely defined in that we have complete freedom of choice as to where to place an element relative to the planes defining their distinction and range of convenience. However, no such freedom exists for non-primitive concepts since they are defined as the intersection of primitive concepts. Logically, we have to *assert* that a primitive concept applies to an element, whereas we can either assert that a non-primitive applies or *recognize* that it applies through the previous assertion of the primitives that define it. In knowledge representation this recognition is termed *classification* (Borgida, Brachman, McGuinness & Resnick, 1989).

The definition of subsumption in (1) applies to non-primitive concepts, but it is no longer a matter of direct commitment but rather of derivation from the composition of commitments for concepts defining the intersection. In knowledge representation terms, the "is-a" relation for non-primitive concepts is computable rather than definable—the commitment to their definition in terms of their structure entails a commitment to a derived, rather than a defined, "is-a" relation. Confusion about these two forms of concept, and associated "is-a" relations, caused problems in early developments of semantic nets (Brachman, 1983).

Kelly's theory of anticipation is based on attaching significance to such recognizable intersections:

> "What one predicts is not a fully fleshed-out event, but simply the common intersect of a set of properties" (Kelly, 1955)

The logic remains intensional because there is no implication that elements have already been construed within the intersections. For example, in the marriage anticipation example given above, the predicted husband 'exists' only as the "intersect of a limited number of conceptual dimensions." The attachment of an anticipation to this intersect corresponds to a commitment to place an element that falls in this intersect in the region defined by the pole of some other construct also. In logic this is a *material implication* rather than an entailment in that it is not necessitated by the way in which the distinctions are defined but is instead an auxiliary commitment or *rule*.

Rules allow a cognitive system to be anticipatory in containing structures which from one set of distinctions made about an event will imply that others should be made leading to prediction or action. Rules play a similar role in computational systems in generating recommendations for decision or action. Overtly modeling the conceptual system of an expert as such a structure is a basis for emulating the expert's performance in a knowledge-based system.

As shown in Figure 5, Kelly's model of anticipation is represented in the visual language by an additional primitive, a rounded corner rectangle, representing material implication or a rule. This is not so readily represented in the geometry because it is a dynamic principle, "if you place an element in

this intersection then move it to this sub-intersection." A rule corresponds to an *attractor* in dynamics, that an element placed in a region of space is unstable and falls into the basin of an attractor (Abraham & Shaw, 1984). This can be represented in the geometry but the diagrams for activities of any reasonable complexity become very difficult to visualize and understand. It does, however, provide a nice link to the corresponding phenomena in the geometrical dynamics of neural networks underlying the phenomenon (Domany, Hemmen & Schulten, 1991).

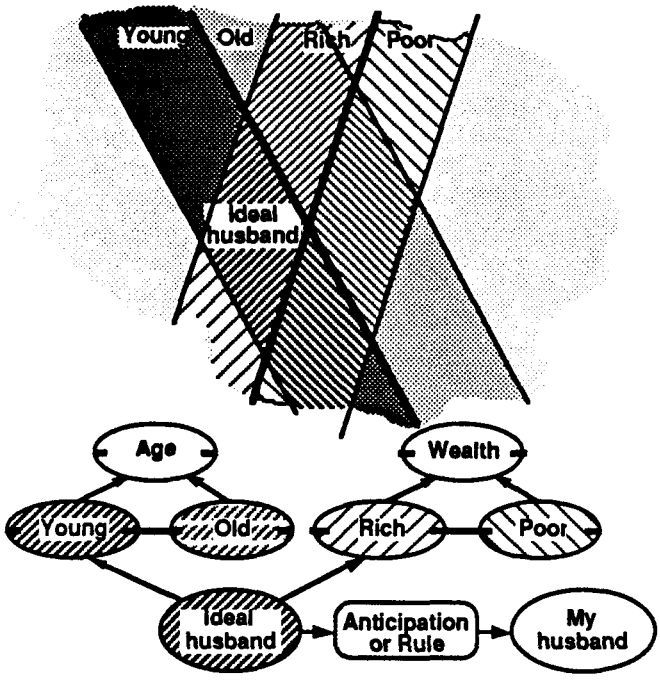

Figure 5 Logic of anticipation through rules

The abstraction of the geometry in the visual language of the logic corresponds to taking multiple cross sections of the geometrical representation from different perspectives and presenting them together. It is way of dealing with the visualization of a high-dimensional space.

ELEMENTS AND LOGICAL INDIVIDUALS

The discussion so far has focused on the conceptual architecture of psychological space rather than its application to the construing and anticipation of events. Concepts are distinctions as mental entities without any prescribed link to what is distinguished or represented. For Kelly representational links are established by construing elements of experience as being placed at points in psychological space. In logical terms his elements are *individuals* intended to rigidly designate something in a world of experience. An element or individual acts a *surrogate* in psychological or logical space for some thing in the represented world (Copeland & Khoshafian, 1986). In the visual language such surrogates are represented by a rectangle as shown in Figure 6 and termed *individuals*. Ferio has the properties of 'ideal husband' in figure 6 and hence the placement of a token representing his surrogate in psychological space would be unstable unless it was also in the region 'my husband'—or, in the logic, the rule shown would 'fire.'

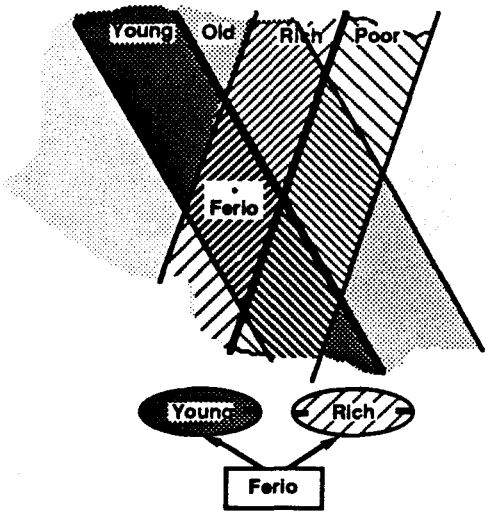

Figure 6 Representation of individuals

In terms of the primitives of distinctions, individuals are defined to be those distinctions which are initial in that they cannot be preceded by other distinctions. They are necessarily leaf nodes in graphs in the visual language. This corresponds to philosophical analyses such as those of Gracia (1983) who notes that traditionally individuals are characterized by five properties; they are entities which:

(a) lose their fundamental character if they are divided into parts;
(b) are distinct from all other entities, even those that are of their same type or kind;
(c) are part of a group type or class which has or can have several members;
(d) remain fundamentally the same through time and various changes;
(e) are the subject of predication but not predicable of other things.

Gracia argues against each of these in turn as being a necessary or sufficient feature to characterize individuals and suggests instead that it is *non-instantiability* that characterizes an individual; that is we cannot instantiate an individual. This corresponds to an initial distinction which cannot have another, an instance, to its left in the algebraic notation of (1).

Thus, while a distinction is the sole primitive, the difference between initial and non-initial distinctions is important. Individual concepts cannot be applied to other concepts and hence have no extensions or instances and cannot subsume one another. However, since individuals are concepts it is possible to have knowledge representation schemas which do not make the distinction explicitly. It is also possible to treat non-initial concepts as *abstract individuals* that represent the individuals that might be distinguished by them. When such an abstract individual has no extension, that is, there are not, or cannot be, any individuals that it distinguishes, then the paradoxes of sense and reference studied by Frege (1892), Meinong (1910) and Russell (1905), can arise. The "morning star, evening star" and "Scott, author of Waverley" equivalences are not problematic as they corresponding to two concepts subsuming the same individual. Non-existent objects such

as "the present King of France" are also not problematic because there is no dependence on their empty extensions. They are interesting because, like fictional objects (Woods, 1974), they mimic individuals in that they do not (rather than cannot) have another distinction to their left. This underlies Meinong's intuition that, in some sense, they correspond to real objects, but also leads to the confusion that was a problem in differentiating the definition of concepts from the assertion of those applicable to individuals (Woods, 1975).

RELATIONS AS CONSTRUCTS

In his applications of PCP, Kelly was primarily concerned with individually-centered construct systems. Hence he usually treats relations to others as attributes of one entity in the relation, that is if John is a parent of Jane, 'parent-of-Jane' is treated from John's viewpoint as an attribute of John. This is equivalent to the technique in combinatorial logic of treating multi-place functions as predicates through *Currying* (Hindley & Seldin, 1986), that is of combining one or more arguments with the function to define a new family of single argument functions. The equivalent construction in the geometry is to treat the relation 'John-parent-of-Jane' as itself an element in psychological space that acts as a place marker linking John to Jane by being distinguished in three ways: as a 'John-relation-', as a '-relation-Jane', and as a 'parent-child-relation.'

Figure 7 shows this and related constructions in the visual language of the logic using a knowledge acquisition tool that operationalizes the visual language. The Curried representation of 'John-parent-of-Jane' is shown at the top together with a set of superordinate concepts of increasing abstraction. Through these, for example, general properties of any child of John may be defined by an arrow out of the concept 'has-parent John', and similarly for properties of any parent of Jane by an arrow out of the concept 'has-child Jane.'

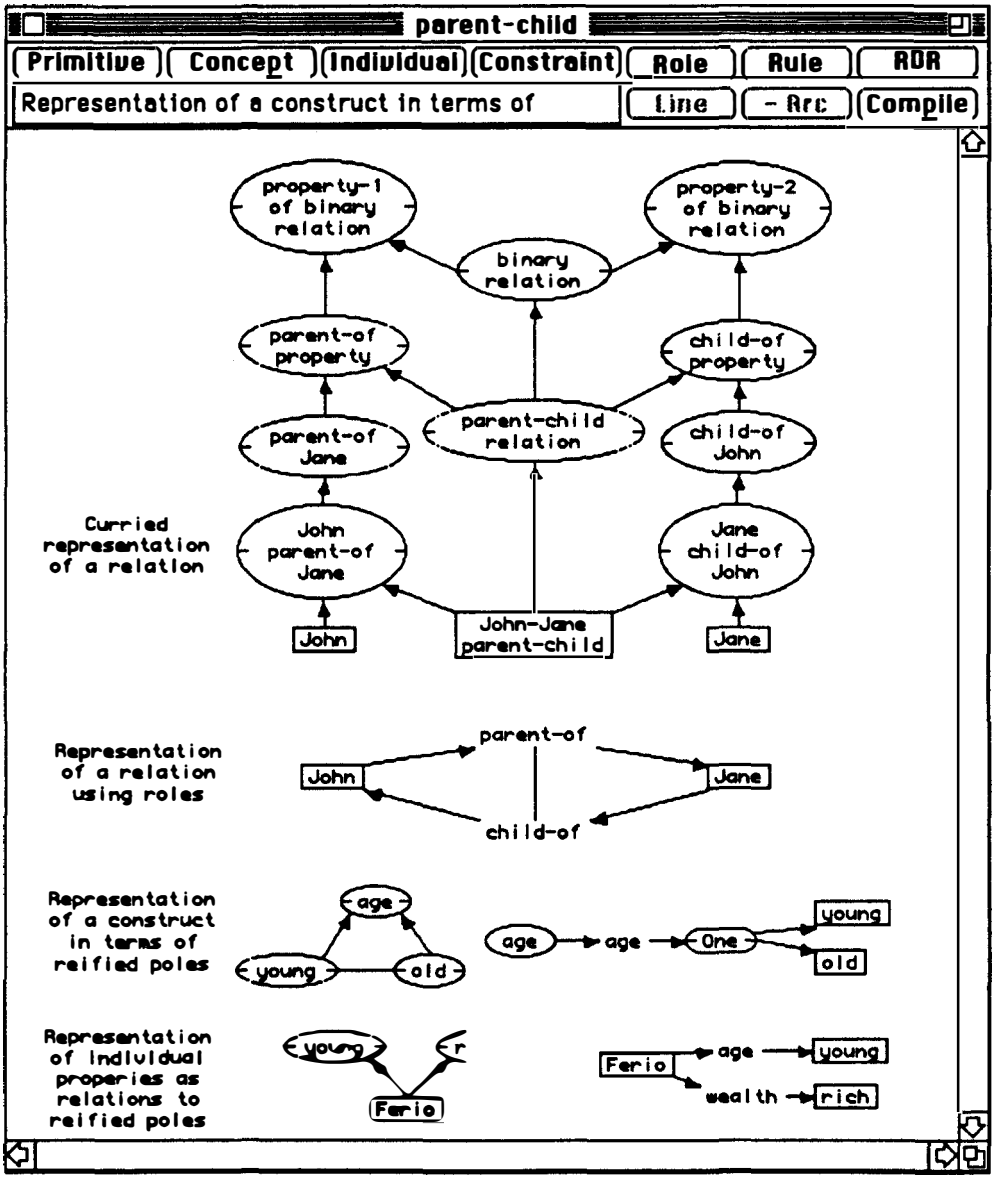

Figure 7 Representation of relations

In computational applications the representation of complex objects and situations as 'objective knowledge' not individually-centered requires relations to be simply represented and it is convenient to introduce a new primitive in the visual language specifically for relations. This is what is termed a "role" in KL-ONE as a generic term for relations and attributes, that also has connotations of individuals acquiring properties because they play a role in relation to another individual. A role in the visual language is shown by unboxed text as is apparent in the representation of the John-Jane relation just below the Curried form. A line between roles indicates that they are inverse to one another.

The availability of the role construction for relations allows properties to be expressed as attributes rather than as concept instantiation. One can *reify* the poles of a construct as if they were individuals and define a concept in terms of a relation to the reified poles. For example, near the bottom of Figure 7 is a bipolar construct, 'young—old' from Figures 4 and 5, and on its right is an equivalent representation in terms of reified poles. The new primitive introduced in the visual language is the constraint "One" meaning that the role is constrained to be filled by just one of the specified set of individuals.

There is a difference in the concept 'age' on the left of Figure 7 and that on the right. The left definition stands alone as a primitive whereas that on the right is actually defined in terms of a role with one of two possible values. One can say that the definition on the right has *internalized* the structure of the construct. This gives rise to a natural coordinate system for specifying the properties of individuals. At the bottom of Figure 7 is shown how the properties asserted of Ferio on the left may be represented as his relations to the reified poles of the corresponding constructs on the right—his values on the coordinates of 'age' and 'wealth.' The constructions on the right for 'age' and 'Ferio' in Figure 7 are visually and conceptually similar, but the first is definitional and the second is assertional. This is an example of the 'encoding-exhibiting' distinction already discussed.

The development of Kelly's geometry of psychological space with individuals (individual constants), concepts (predicates), subsumption (entailment), disjoint (relative negation), rules (material implications), and roles (relations), makes available a complete intensional logic of conceptual structures and their application. The visual language for the logic gives a formal semantics for semantic networks. The close relation between the psychological structures and notions and those of the intensional logic make it possible to build knowledge structures that are interpretable both psychologically and logically. This is the strength of personal construct psychology as a basis for knowledge acquisition tools.

THE REPERTORY GRID

Kelly (1955) introduces the "role repertory grid" as a means for investigating a person's conceptual structure relevant to inter-personal relations by having them classify a set of people significant to them in terms of elicited personal constructs. The 'repertory grid' methodology has evolved in the light of application experience and now has major differences from that described by Kelly. Shaw (1980) took advantage of the processing power and interactivity of computers to introduce on-line analysis and feedback to the person from whom the grid was being elicited. This form of the repertory grid has become a widely used and accepted technique for knowledge elicitation, and has been implemented as a major component of many computer-based knowledge acquisition systems (e.g. Gaines & Shaw 1987). Figure 8 shows the general form of a repertory grid and its relation to the conceptual structures already discussed. If one takes a particular concept somewhere in the lattice, and a set of individuals asserted to fall under that concept, then each distinction that may be made about individuals falling under that concept forms the rows of a matrix, the individuals form the columns, and the constraints applying to a particular individual relative to a particular distinction form the values in the matrix.

In simple applications of the repertory grid these constraints are taken to be the values of the individuals on the roles corresponding to the distinctions. However, it is apparent from Figure 8 that concepts subordinate to those defining the scope of the grid may also be used as if they were individuals, and these may be expected to have more general constraints than single values. Hence in extended repertory grid elicitation, such as that of AQUINAS (Boose & Bradshaw, 1987) the 'values' in the matrix can in themselves be complex constraints.

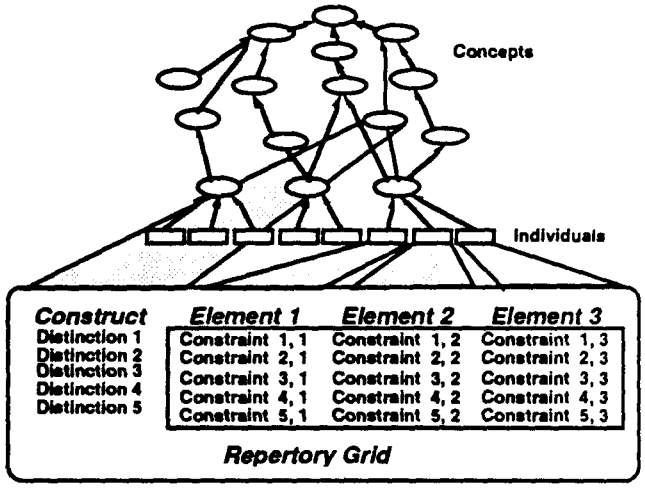

Figure 8 The repertory grid as a matrix of concepts, individuals and constraints

The psychological function of the repertory grid is to provide a technique for building the conceptual structure without direct elicitation of concepts and their structures and relationships. The assumption is that it may be easier for a person to provide exemplary individuals in the domain of interest, and then to state in fairly concrete terms how they would distinguish them in terms of properties relevant to the purpose of eliciting the grid. In terms of the intensional logic of the concept structure, the extensional specification of how concepts apply to individuals is clearly inadequate to fully specify the concept structure. However, the structure must be consistent with its model and hence it is possible through suitable analysis techniques to approximate the structure from the extensional data.

CONCLUSIONS

In conclusion, we suggest that personal construct psychology provides a very attractive foundation for knowledge acquisition methodologies and tools that have to bridge between human cognitive processes and computational knowledge representation. The many tools so far developed based on personal construct psychology encourage this conclusion. However, what has so far been developed taps only a small part of the potential and we may expect to see major new developments in the future. In particular, personal construct psychology offers the opportunity to integrate many different knowledge acquisition and knowledge representation approaches within a single principled framework.

ACKNOWLEDGEMENTS

Financial assistance for this work has been made available by the Natural Sciences and Engineering Research Council of Canada. We are grateful to many colleagues for discussions over the years that have influenced the research described in this paper, in particular John Boose, Jeff Bradshaw, Marc Linster, Alain Rappaport, Brian Woodward and other colleagues at the Knowledge Acquisition Workshops world-wide.

REFERENCES

Abraham, R.H. & Shaw, C.D. (1984) **Dynamics: The Geometry of Behavior.** Santa Cruz, California: Aerial Press.

Adams-Webber, J.R. (1979) **Personal Construct Theory: Concepts and Applications.** Chichester: Wiley.

Anglin, J.M. (1977) **Word, Object, and Conceptual Development.** Norton: New York.

Boose, J.H. & Bradshaw, J.M. (1987). Expertise transfer and complex problems: Using AQUINAS as a knowledge acquisition workbench for knowledge-based systems. **Int. Journal of Man-Machine Studies, 26,** 3-28.

Borgida, A., Brachman, R.J., McGuiness, D.L. & Resnick, L.A. (1989). CLASSIC: a structural data model for objects. Clifford, J., Lindsay, B. & Maier, D., Eds. **Proceedings of 1989 ACM SIGMOD International Conference on the Management of Data.** pp.58-67. New York: ACM Press.

Brachman, R.J. (1983). What IS-A is and isn't. **IEEE Computer, 16**(10) 30-36 (October).

Brachman, R.J. & Schmolze, J. (1985). An overview of the KL-ONE knowledge representation system. **Cognitive Science, 9**(2) 171-216 (April-June).

Copeland, G.P. & Khoshafian, S.N. (1986). Identity and versions for complex objects. MCC Technical Report DB-138-86. Austin, Texas: MCC.

Domany, E., Hemmen, J.L. & Schulten, K., Eds. (1991). **Models of Neural Networks.** Berlin: Springer.

Frege, G. (1892). On sense and reference. Geach, P. & Black, M., Eds. **Translations from the Philosophical Writings of Gottlieb Frege.** pp.56-78. Oxford: Basil Blackwell, 1970.

Gaines, B.R. (1991a) Empirical investigation of knowledge representation servers: design issues and applications experience with KRS. **AAAI Spring Symposium: Implemented Knowledge Representation and Reasoning Systems.** pp. 87-101. Stanford (March)—also **SIGART Bulletin 2**(3) 45-56.

Gaines, B.R. (1991b). An interactive visual language for term subsumption visual languages. **IJCAI'91: Proceedings of the Twelfth International Joint Conference on Artificial Intelligence.** pp.817-823. San Mateo, California: Morgan Kaufmann.

Gaines, B.R. & Shaw, M.L.G. (1987). Knowledge support systems. **ACM MCC-University Research Symposium.** Austin, Texas: MCC. 47-66.

Gracia, J.J.E. (1983) Individuals as instances. **Reviews of Metaphysics, 37**(1), 37-59 (September).

Hatcher, W.S. (1982). **The Logical Foundations of Mathematics** Oxford: Pergamon Press.

Hindley, J.R. & Seldon, J.P. (1986). **Introduction to Combinators and lamda-calculus.** Cambridge: Cambridge University Press.

Kelly, G.A. (1955). **The Psychology of Personal Constructs.** New York: Norton.

Kelly, G.A. (1969) A mathematical approach to psychology. Maher, B. Ed. **Clinical Psychology and Personality: The Selected Papers of George Kelly.** pp.94-113. New York: Wiley.

Kelly, G.A. (1970) A brief introduction to personal construct theory. Bannister, D. Ed. **Perspectives in Personal Construct Theory.** pp.1-29. London: Academic Press.

Lemmon, E.J., Meredith, C.A., Meredith, D., Prior, A.N. & Thomas, I. (1969). Calculi of pure strict implication. Davis, J.W., Hockney, D.J. & Wilson, W.K., Eds. **Philosophical Logic.** pp.215-250. Dordrecht, Holland: Reidel.

Lloyd, G.E.R. (1966) **Polarity and Analogy.** Cambridge: Cambridge University Press.

Marcus, R.B. (1962). Interpreting quantification. **Inquiry, 5,** 252-259.

Maybury-Lewis, D. & Almagor, U. (1989). (Eds) **The Attraction of Opposites.** Ann Arbor: University of Michigan Press.

Meinong, A. (1910). **On Assumptions.** Berkeley: University of California Press, 1983.

Mohanty, J.N. (1982). **Husserl and Frege.** Bloomington, Indiana: Indiana University Press.

Russell. B. (1905). On denoting. **Mind, 14,** 479-493.

Shaw, M.L.G. (1980). **On Becoming A Personal Scientist: Interactive Computer Elicitation of Personal Models Of The World.** London: Academic Press.

Woods, J. (1974). **The Logic of Fiction.** The Hague: Mouton.

Woods, W.A. (1975). What's in a link: foundations for semantic networks. Bobrow, D.G. & Collins, A.M. **Representation and Understanding: Studies in Cognitive Science.** pp.35-82. New York: Academic Press.

Zadeh, L.A. (1964) The concept of state in system theory. Mesarovic, M.D. (Ed.) **Views on General Systems Theory: Proceedings of the Second Systems Symposium at Case Institute of Technology.** New York: John Wiley.

Zalta, E.N. (1983). **Abstract Objects.** Dordrecht: Reidel.