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LOGICAL FOUNDATIONS FOR KNOWLEDGE REPRESENTATION IN INTELLIGENT SYSTEMS

Brian R. Gaines
Department of Computer Science, University of Calgary,
Alberta, Canada T2N 1N4

ABSTRACT. We have entered an era of intelligent system development in which fifth and sixth generation computing systems are expected to emulate and enhance natural intelligence. However, human knowledge processes are not well-understood and it is tempting to base the technology of intelligent systems on formal reasoning rather than human psychology. In particular, the attempt to base knowledge processing in fifth generation computing systems on logic programming suggests that we should examine the rationale behind standard logic. It was developed as a foundation for arithmetic and psychological considerations were ruled out as irrelevant. We should consider that there may be systems of logic more appropriate to knowledge processing. Logical and psychological system theories based on an axiom of unrestricted predication are plausible and attractive alternatives to standard logic. It is suggested that the design of logical inference processors should take such systems into account, and that investigation of the logical, psychological and systemic foundations of knowledge are fundamental to the development of intelligent systems.

KEYWORDS: Fifth generation, intelligent systems, logic, psychology, knowledge, representation, constructs, distinctions, uncertainty, probability, possibility, fuzzy.

Introduction: Knowledge and Psychology as Computing Technologies

The Japanese *fifth generation computing system* development project aims to:

> "provide knowledge processing systems" employing "the latest research results in VLSI technology, distributed processing, software engineering, knowledge engineering, artificial intelligence, and pattern information processing." (ICOT 1983)

Moto-oka (1982) notes:

> "In these systems, intelligence will be greatly improved to approach that of a human being, and, when compared with conventional systems, man-machine interface will become closer to the human system."

Intelligent knowledge processes have been peripheral considerations in the design of previous generations of computers, although their significance has grown (Gaines 1984), but they play such a key role in fifth generation systems that new questions of major importance arise, such as:

i) do we have an adequate theory of knowledge on which to base knowledge processing systems?
ii) do we have an adequate theory of human psychology on which to base human-computer interface design?

These are not just questions of general relevance to the next generation of computer systems but that, if the objectives of the fifth generation project are accepted, knowledge and psychology become key technologies on a par with vlsi and software engineering. Failure to provide adequate foundations and tools in these areas will imply failure of the main objective., although not necessarily failure to innovate and extend the state of the art.

This dependence is recognized and the question:

> "Are you saying that the design of the fifth-generation may be modeled by learning more about the human thinking process?"

has been answered:

> "Yes, we should have more research on human thinking processes, but we already have some basic structures. For more than 2000 years man has tried to find the basic operation of thinking and has established logic. The result is not necessarily sufficent; it's just the one that mankind found. At present we have only one solution—a system like predicate calculus. It is rather similar to the way man thinks. But we need more research. What, really, is going on in our brain? It's a rather difficult problem." (Fuchi, Sato & Miller 1984 pp.14-15)

This leads to a pragmatic approach to developing fifth generation computing systems, basing them on the best available foundations for knowledge and psychology, even if these are inadequate. Technology is often in advance of science and the pressure to develop the next generation of computers may be just what is needed to advance the sciences of knowledge and psychology. This approach leads to additional questions:

iii) how adequate is predicate calculus as a science of knowledge?
iv) how adequate is logic as a foundation for psychology?

The recently-announced *sixth generation computing system* development project in Japan proposes to tackle such questions through a research program involving inter-disciplinary interaction between computing science and *physiology, psychology, linguistics* and *logic* (STA 1985, Gaines 1986).

This paper addresses these questions at a constructive, operational level: what do we know about knowledge and psychology that might be relevant to intelligent systems; where might we expect problems in basing future computing systems on standard predicate calculus; what theories currently exist that might be used as a basis for fifth and sixth generation knowledge and psychology technologies?

The Divergence of Psychology and Logic

A philosophical dictionary definition of "knowledge" is that it is "apprehended truth" and that "certain knowledge is more than opinion, less than truth" (Runes 1960 p.161). Knowledge stands between opinion which is in the domain of psychology (the

study of mental phenomena) and truth which is the domain of logic (the study of truth preserving inferences); the underlying discipline is epistemology (the study of truth ascription). It would be reasonable to suppose that psychology and logic would be closely related aspects of a science of knowledge. Until recent times this was so, but in the late nineteenth century psychology and logic diverged and an attempt was made to set up a system of logic that stood alone, independent of any processes of human reasoning. The divergence stemmed from attempts to establish formal foundations for mathematics, particularly those of Frege (1884) and Husserl (1891).

The stimulus for Frege's work was to re-establish irrevocably the position that mathematics was a priori (independent of experience) and analytic (based on inferences in which the conclusion is part of the premise), in contradiction to Mill's (1843) position that arithmetic is not *a priori* but empirical and induced from experience, and Kant's (1781) position that it is *a priori* but synthetic and made known through experience. Husserl's work shows the arguments most strongly because, in response to Frege's (1894) critical review of his 1891 book, he became concerned with the role of psychology in logic and formulated the basis of pure logic clearly demarcated from processes of the mind (Husserl 1900). In his discussion and the distinctions he makes Husserl generates a model of relationships between logic, psychology and knowledge which underlies the development of modern epistemology (Mohanty 1982). Similarly, Frege generated a notion of pure logical inference that underlies the development of modern formal logic (Kneale and Kneale 1962).

These classical discussions may appear remote from modern computing and artificial intelligence. However, a recent paper by Lenat and Brown (1983) on *Why AM and Eurisko appear to work* puts forward the Kantian argument exactly. The problem is why AM succeeded in the first place and why it is so difficult to use the same paradigm to discover new heuristics. Their conclusion is that AM's success in making conjectures in basic mathematics was because set-and number-theoretic results are simply stated in LISP. The conjectures were to a large extent available *a priori* but were *synthetic* because AM needed its empirical investigations to trigger off their discovery. However, while the metarules of logic and arithmetic are embedded in LISP, those about discovery heuristics are not and could not be thus triggered. This not only demonstrates the Kantian argument but also suggests a line of attack that investigates the discovery heuristics as primitives in some operational calculus—a formal epistemology which Lenat (1982) has termed *heuretics*.

The Consequences of Anti-Psychologism in Logic

The anti-psychologistic basis of modern logic is natural in the light of the objective of formalizing arithmetic as a demonstration that it is *a priori* and synthetic. It rejects as irrelevant the questions:

1) How do we come to know arithmetic?

2) Why is arithmetic instantiated in real-world phenomena?

although a variety of answers to these questions are part of Frege's and Husserl's discussions. It also does not take into account the distinction between what is known and what might be inferred from this; Wittgenstein notes:

> "If I know an object I also know all its possible occurrences in states of affairs." (Wittgenstein 1921, 2.0123)

drawing attention to the gap between formal logic, where all consequences are "known", and human reasoning where they are not. It raises the questions:

3) What is the significance of not knowing all the logical consequences of a state of affairs?
4) How do we come to know particular consequences?

All of these questions are highly significant for knowledge-based systems yet outside the framework of standard formal logic. Questions 1 and 4 may be regarded as primarily within the domain of psychology and 2 within the domain of philosophy. The significance of question 3 was not realized until the notion of *computational complexity* was developed and formalized. The super-exponential combinatorial explosion of any consequence derivation system creates a boundary that is real and extremely "hard". The notion of a hard proof is not just psychological, a result of human frailty, but is systemic, a property of the problem for any derivation system.

The boundary created by computational complexity means that existence proofs relating to inference mean little in practical terms. It is not sufficient to know that solutions are eventually accessible; we need to know that they can be reached using reasonable resources. The knowledge representation and inference schema used must make the types of inference required easy. This is an extra-logical requirement but a critical one in practice. Hardware parallelism and speed increase cannot be used to overcome the problem because super-exponential growth overwhelms even large multiplicative factors. A danger in developing future computing systems is that any logic we build into the hardware may prove inappropriate to emulate human inference.

These considerations suggest that we should regard with some suspicion formal logics developed as foundations for mathematics and deliberately disregarding psychological considerations. It could be that mathematics epitomizes human intellectual achievement and standard predicate calculus captures all that is significant. Even this is suspect since we know of many problems with this calculus: in the foundations of set theory (Martin 1970); relevance of implication (Anderson & Belnap 1970), ontology of existence (Schock 1968); and extension beyond first order (Kneale & Kneale 1962).

However, while one may criticize standard predicate calculus for its known faults, it is not easy to suggest an alternative system for knowledge representation and inference. There are many studies of human reasoning and alternative logics but there has not been the intensive study of a coherent system for knowledge in general as there has been for that sub-set which is mathematics. Such a study has its place in the development of intelligent systems, and the remainder of this paper surveys some possible bases for such a system with strong logical and psychological foundations.

Psychological and Systemic Foundations

Kelly's *personal construct psychology* develops a complete theory of the normal and abnormal, which has strong systemic foundations (Gaines & Shaw 1981) and has been operationalized through computer programs (Shaw 1980). The person models the world using *personal constructs,* that is distinctions made about experience, which are the basis of knowledge acquisition:

> "Constructs are used for predictions of things to come, and the world keeps rolling on and revealing these predictions to be either correct or misleading. This fact provides a basis for the revision of constructs and, eventually, of whole construct systems." (Kelly 1955)

When Kelly developed his theory there was neither the mathematics to formalize it nor the computer technology to operationalize it. Despite this, his psychology has developed to become a major cognitive theory and to provide a variety of techniques for clinical, educational and managerial psychology (Shaw 1981) and the rapid prototyping of expert systems (Shaw & Gaines 1983, 1986b, Boose 1984). It has strong links with Brown's logical system in *Laws of Form* based on the notion underlying personal constructs of *making a distinction*:

> "The theme of this book is that a universe comes into being when a space is severed or taken apart...By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance." (Brown 1969)

Kelly's psychology and Brown's logic both take as primitive an axiom of unrestricted predication. In set-theoretic terms they take *comprehension* (Fraenkel, Bar-Hillel & Levy 1973) as *a priori* and derive all other concepts from this. This is not the standard approach, although some logicians have argued for it (Fitch 1952), and it is misleading to translate the terms into logic or set theory. They define a basis of knowledge which is prior to both and from which logic and set theory must be derived. The key questions are: to what does a construct apply; what is it that is distinguished? We fall into a trap if we have any answer based on prior knowledge. The proper answer is that what is distinguished is defined only by being distinguished; it requires an auxiliary ontological postulate to reify this by assuming some entity is thus defined.

Gaines (1980) suggests that what is distinguished is a *system,* and that this notion defines a system. To distinguish some entity is a necessary and sufficient criterion for its being a system and this is uniquely true for systems. This definition may appear too weak in that it does not involve the expected properties of a system. The Oxford English Dictionary notes that a system is "a group, set or aggregate of things, natural or artificial, forming a connected or complex whole." However, these are properties that derive from our definition: a set of things is treated as distinct from another and it is this which gives it coherence; it is also this which increases its complexity by giving it one more characteristic than it had before, that it has now been distinguished.

Distinctions Underlying Knowledge Processes and Their Logics

If we take the primitive process of making a distinction as that underlying our knowledge processes then it should be possible to analyze reasoning and its logical foundations in terms of the underlying distinctions involved. The essential concept of making a distinction is that some part of a world is separated out, or *marked.* At an abstract level, making a distinction is essentially a unipolar operation defining the marked region. However, the unmarked region is sometimes sufficiently well defined (if the world itself is well distinguished) for it to be treated as an opposite to the marked region and the distinction is then perceived as bipolar.

Figure 1 shows how some very basic distinctions establish the major domains of reasoning:

- Commence with the notion of the building material of a *complex universe* being some kind of *informal entropic chaos* having the property only that it is rich enough for any distinctions to be instantiated in it.
- Make the existential hypothesis that *there exist distinctions* to bring into being a sub-universe which is termed *anarchy* in that there are no constraints on the types of distinction allowed. This is termed *World 0* of *systems*, and since the distinctions made are only justified in terms of it being useful to make them, it is one of *utilitarian truth* and *pragmatic inference.* It may also be termed the world of *axiology* since the distinctions made are characterized by their presumed value.
- Use the ontological argument that *there exist necessary distinctions* and bring into being a sub-universe which is termed *actuality* in that the types of distinction allowed are those forced upon us by the physical world. This is termed *World 1* of *physical reality*, and since the distinctions made are justified in terms of it being necessary to make them, it is one of *correspondence truth* and *causal inference.* It may also be termed the world of *epistemology* since the distinctions made are characterized by their value in modeling the physical world.
- Use the ontological argument that *there exist distinction makers* and bring into being a sub-universe which is termed *agency* in that the types of distinction allowed are those over which we have choice. This is termed *World 2* of *subjective experience*, and since the distinctions made are justified in terms of our being willing to make them, it is one of *performative truth* and *conventionalist inference.* It may also be termed the world of *psychology* since the distinctions made are characterized by their being chosen by an agent.
- Use the ontological argument that *there exist independent distinctions* and bring into being a sub-universe which is termed *abstraction* in that the types of distinction allowed are considered apart from their origins. This is termed *World 3* of *statements in themselves*, and since the distinctions made are justified in terms of criteria determining that they belong together, it is one of *coherence truth* and *structuralist inference.* It may also be termed the world of *ontology* since the distinctions made are characterized by their being in their own right.

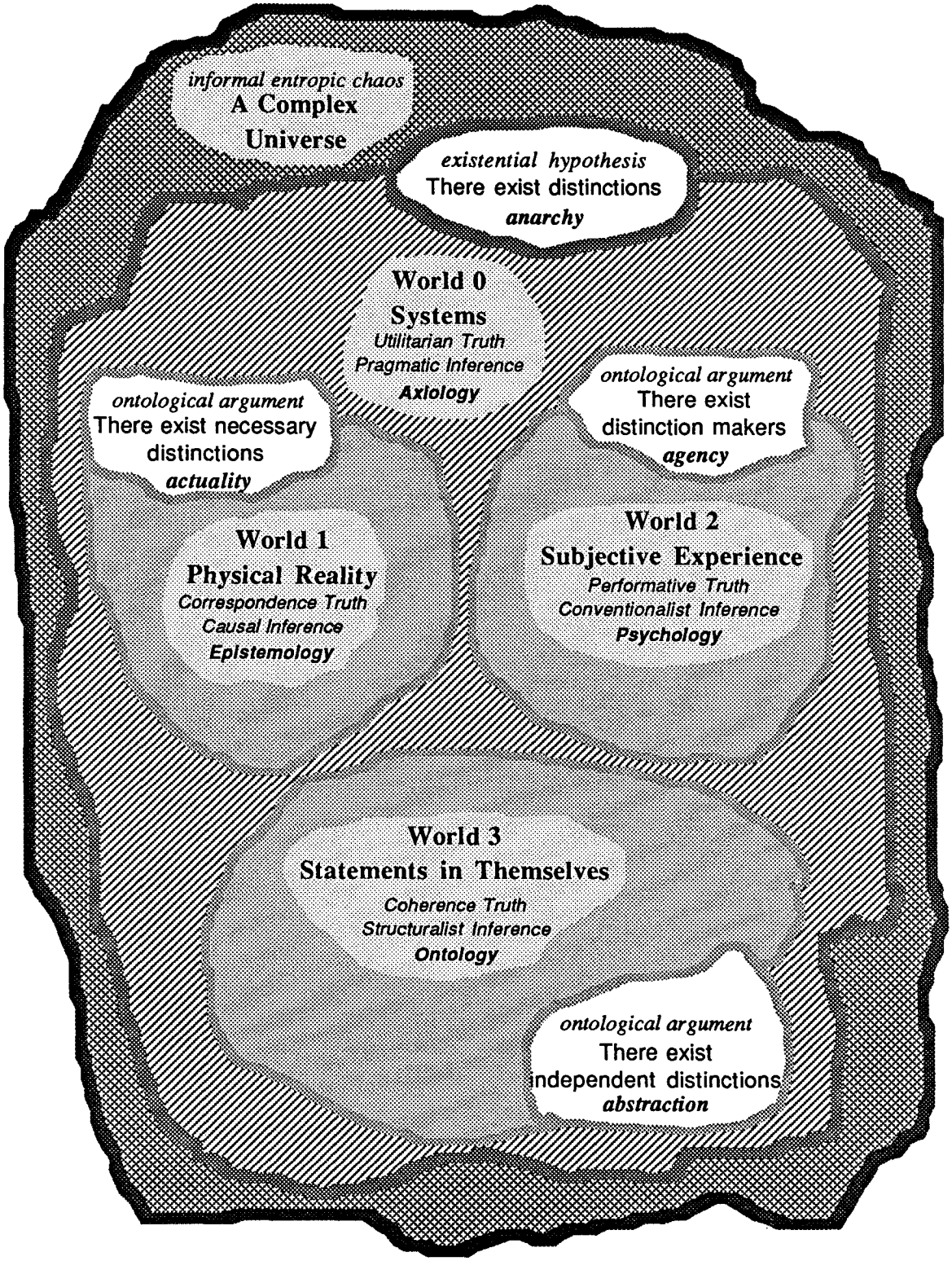

Fig.1 Worlds generated by basic distinctions

The worlds have been named to show that these primitive distinctions generate Popper's (1968) notion of *3 worlds* in the philosophy of scientific knowledge. They can also be shown to be the generative principles behind many fundamental theories such as: Pask's (1980) discourse universals in *conversation theory*; Fillmore's (1968) linguistic universals in *case grammars*; and Pope's (1984) *anthropological universals* (Gaines & Shaw 1984, 1985). Hierarchical application of the basic distinctions can be shown to generate complex technological frameworks such as Edwards and Mason's (1986) evaluation methodology for person-computer interaction (Gaines & Shaw 1986d). It is reasonable to suppose that these basic distinctions are the generators underlying any knowledge representation methodology.

A logic of knowledge systems

Figure 1 shows that the semantics of truth and inference in each world are different, but the generative process underlying the figure indicates that the logical foundations of truth and inference are universal, stemming from the underlying calculus of distinctions. Personal construct psychology and the calculus of distinctions both point to a knowledge representation system based on unrestricted predication as a primitive. The significance of such an axiom is that it forces us to develop system theories that do not run into the problems of this axiom in conventional mathematics. In set theory such predication leads to the inconsistency that Russell discovered in Frege's (1884) *Foundations of Arithmetic*, and requires the axiom of comprehension to be severely restricted by a complex set of rules. However, it has long been speculated that the inconsistency may be better ascribed to the law of contradiction in standard logic: that the same predicate cannot be both true and false about the same proposition. Maydole (1975) proved that most contenders for an alternative logic led to inconsistencies in set theory, and White (1979) showed that Lukasiewicz infinitely valued logic gives a consistent set theory.

This was a particularly important result in bringing theory and practice together because it was just this logic that Zadeh (1965) takes as a basis for fuzzy sets theory, and which has been used in many knowledge-based systems (Gupta, Kandel, Bandler & Kiszka 1985). The links from Brown's calculus of distinctions to Lukasiewicz logic have been established: Varela (1975) used the calculus to analyse self-referential systems using a construction based on Russell's paradox which gave a 3-valued logic; Gaines (1976) used a recursive form of this construction to generate the infinite valued Lukasiewicz logic; and Gaines (1983) shows that the Lukasiewicz axioms encompass modal, probability and fuzzy logics. The significance of this construction is that the axioms are taken first as defining a non-truth-functional *standard uncertainty logic*, fuzzy logic is then derived by adding the constraint that the logic be truth functional, and probability logic is derived by the alternative constraint that the law of contradiction apply. Figure 2 shows the relationships between the various logics of uncertainty that may be derived from the Lukasiewicz axioms for a system of distinctions underlying the constructs representing a person's knowledge.

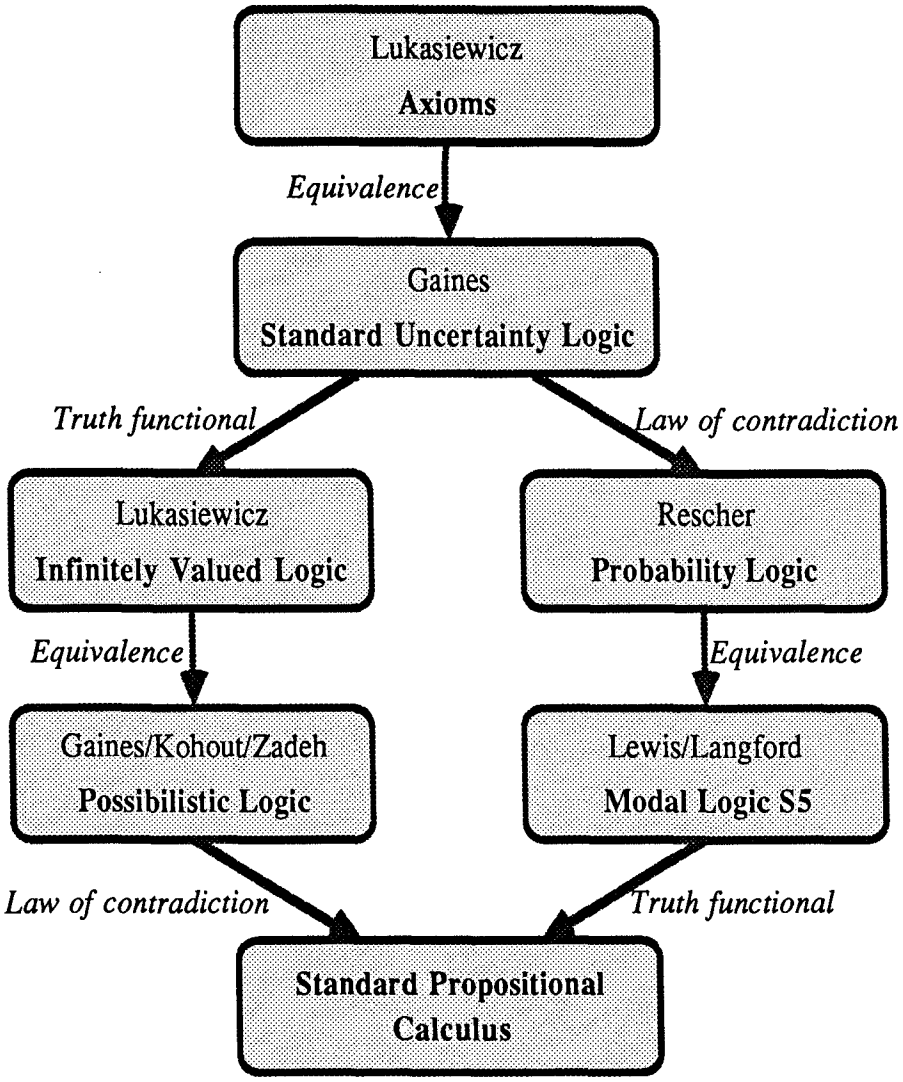

Fig.2 Logics of uncertainty generated by distinctions

The need for a system adequate to represent knowledge to encompass both imprecision and non-truth-functionality is obvious in retrospect. Most of the distinctions we make in everyday life do not have hard boundaries. The notion of a red rose is totally unambiguous and acceptable in human communication and reasoning. However, the set of red roses is not a standard set. It has boundary cases which are neither precisely red nor not-red. Yet we can not exclude these borderline cases from being considered as red roses. The laws of contradiction and the excluded middle do not apply. Non-truth-functionality arises because it is not possible to ascribe truth-values to the conjunction and disjunction of uncertain events without further information about their relationship. In probability theory this is well-known: if it is 0.5 certain you will phone today and 0.5 certain that I will be out of the office, what is the certainty that you will phone when I am out of the office? The answer can range from 0 if we have pre-arranged, to 0.5 if the two events are not causally connected, to 1.0 if I do not want to talk to you. It is possible to compute the constraints upon the result but not a single-valued result.

Probability logic is non-truth-functional, and Quinlan (1983) has shown that its constraint system may be used directly as a basis for improved inference in expert systems with uncertain rules and data. Giles (1985) has suggested that linear programming techniques be used as inference systems for generalized fuzzy logics. Cleary's (1986) implementation of interval arithmetic as an extension of Prolog may be used as an inference system for the standard uncertainty logic from which probability theory and fuzzy logic derive. Conventional fuzzy logic can be seen as resulting from the requirement to avoid the need for constraint methodologies by imposing truth-functionality. The common assumption of independence of events in probabilistic situations is an alternative way of forcing truth-functionality. Both these approaches are expedients whose utility depends on the problem domain.

Cheeseman (1985) has presented detailed arguments for probability theory being an adequate foundation for reasoning with uncertainty in artificial intelligence, encompassing such requirements as non-monotonic reasoning. His arguments carry over directly to the use of a more general standard uncertainty logic once it is noted that the rationale for probability theory he presents assumes that there are no borderline cases and that the law of the excluded middle holds. When it does not then the appropriate logic to which to apply Cheeseman's arguments is the more general standard uncertainty logic which subsumes probability logic.

Hierarchies of Distinctions in Knowledge Acquisition

Distinctions are not just static partitions of experience. They may be operations: actions in psychological terms; processes in computational terms. They form a natural hierarchy when part of an anticipatory modeling process which Klir (1976, 1984) has termed an *epistemological hierarchy*. This has a natural psychological interpretation when applied to the knowledge processes of the individual, and a natural cultural interpretation when applied to those of society (Shaw & Gaines 1986a).

A knowledge acquisition schema results from distinctions about distinctions at each level in the hierarchy. In prediction the key distinction is whether a level accounts for the information flowing through it, the *surprise* (Gaines 1977). The formal theory of knowledge acquisition is one in which models are selected at each level down the hierarchy to minimize the rate at which surprise is passing up the hierarchy. The criteria for model selection independent of the data are ones of *simplicity* or *complexity*: of two models which fit the data equally well choose the simplest. However, notions of simplicity/complexity are not well-defined nor intrinsic to the class of models. The ordering is arbitrary and in its most general form is just one of *preference*. Hence the general knowledge acquisition schema is one in which surprise flows up the hierarchy and preference flows down. In situations that are well-defined, such as determining the structure of a stochastic automaton from its behavior, such a modeling schema gives the correct results (Gaines 1977). Conversely, the success of the schema in stabilizing with regard to a given world defines the characteristics of that world. AM's "interest" (Davis & Lenat 1982) may be explicated as surprise generated when rules such as property inheritance are violated.

Thus the basic knowledge acquisition schema is one in which surprise flows up the hierarchy and preferences flow down. In primitive organisms only the lower levels of the hierarchy are developed, surprise is generated from experience and preference is genetically encoded. In higher organisms the modeling process generalizes both surprise and preference to cope with novel environments. Human life has developed the upper levels of the hierarchy and detached surprise from experience and preference from its genetic roots. Surprise can flow up from a level without flowing into it from below because the processes *at* that level have generated novelty. Preference can be generated at a high level detached from both experience and genetic origins and flow down to affect the relations of the organism to the world.

The hierarchy represents not only the knowledge processes of individuals but also those of societies. The human capability for an entire society to act as a distributed knowledge acquisition system is dependent on the role of communication processes in coordinating activity at a given level of the hierarchy across different people. This communication process whereby each person does not have to undertake all aspects of the inductive process but can share the results of such processing by others supports what is generally termed the *culture* of a society. People use language for much of this communication but they also have in common with other animals the capability to communicate cultural information without the use of language. Mimicry is an important mechanism for knowledge acquisition as is reinforcement through reward and punishment.

The human development of language enables coordination to take place in a rich and subtle fashion that greatly enhances, but does not replace, the more basic mechanisms in common with other species. It is particularly important at the upper levels of the hierarchy where direct expression is difficult. From an individual point of view, language is a way of by-passing the normal modeling procedures and interacting directly with the system at any level. In particular it can directly affect the preference system. Even when language cannot mediate the direct transmission of knowledge it may be used to achieve the same effect by the indirect support of other mechanisms, for example, one can describe a good learning environment, or a behavior in sufficient detail for mimicry. The evolutionary pressures in intelligent systems would be very strong in selecting genes giving the capability for a species to act as a single distributed individual, combining autonomy and cohesion through enhanced linguistic communication.

Summary and Conclusions

Within the scope of this paper its has been possible to outline briefly some foundational considerations for the logical representation of knowledge in intelligent systems. The abstract considerations discussed have been applied in other studies. For example, the model of knowledge processes described has been used to:

1. Develop algorithms for modeling the structure of a system from its behavior with strong formal foundations and practical applications (Gaines 1977);

2. Develop a model of the socio-economic trends in information technology that links the micro-structure of individual knowledge processes with the macro-structure of the computing industry, accounting for past foci of attention and forecasting future trends (Gaines & Shaw 1986c);
3 Develop a model of society as a distributed knowledge acquisition system in which the roles of various cultural and technological support systems are given clear meaning and operational definition, including classical media and expert systems (Shaw & Gaines 1986a);
4. Develop a model of the hierarchy of processes for knowledge transmission within society and use this to analyze the processes and problems of knowledge engineering for expert systems (Gaines & Shaw 1986b);
5. Develop interactive elicitation programs to map a person's constructs in an area of expertise, and algorithms to generate production rules from the elicited constructs, as a basis of knowledge engineering for expert systems (Shaw & Gaines 1986b, Gaines & Shaw 1986a);
6. Develop a model of the communication and knowledge processes in intelligent dialog systems and use this to analyze methodologies for system design and evaluation (Gaines & Shaw 1986d).

Understanding knowledge representation for intelligent systems, whether natural or artificial, requires a foundational analysis of the nature of knowledge, and the relevant neurological, psychological, sociological, anthropological, philosophical and logical processes, data and theories. We are dealing with a complex system with a long history of evolution and should not expect simplistic or early answers. We need to integrate a wide range of viewpoints, theories and methodologies into a coherent framework. Despite the immensity of the task, it seems that we have entered an era when a greatly increased understanding of intelligent systems may be possible.

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