Fourth International Congress on Personal Construct Psychology, 24-28th August, 1981, Brock University, St.Catherines, Ontario, Canada.

LOGICAL ANALYSIS OF PERSONAL CONSTRUCT SYSTEMS

Brian R. Gaines & Mildred L.G. Shaw

Centre for Man-Computer Studies,
92, Shakespeare Tower, Barbican, London EC2, UK.
&
Middlesex Polytechnic, Barnet, Herts

ABSTRACT

Both in our work on the technology of dialectical computer systems and on the fundamentals of human reasoning we have felt the need for an analysis of personal construct systems that goes beyond the distance-based grid analysis techniques commonly used. In his own practical development of a personal construct technology through the repertory grid and the extraction of "factors" from it Kelly treats constructs as if they gave a vector of measurements of the event rather than a logical representation of it. This approach seems to have been followed also by later workers on the analysis of the repertory grid through a variety of methods such as principal components analysis. In this paper we show that the analysis of construct systems as logical structures both encompasses many of the advantages of existing methods and also leads to interesting new directions of analysis.

In terms of a logical analysis Kelly's approach has common foundations with the later work of Spencer Brown in his Laws of Form where he uses the notion of a distinction. This paper outlines these foundations and shows how they lead to new techniques for analysing personal construct systems. Examples are given of the use of the program, ENTAIL, which gives a logical analysis of the asymmetric implication structure of constructs from data in a repertory grid.

INTRODUCTION

This paper presents part of an ongoing research programme concerned with the development of man-computer symbiotic systems for the amplification of human intelligence, creativity and similar capabilities necessary to cope with complex worlds of knowledge, decision and action. Our overall objective is to provide both formal and technological foundations for the role of man as a personal scientist, and for a group of people as a community of science, exploring worlds, forming hypotheses, using them as a basis for action and changing them through feedback from the worlds. To achieve this objective involves both practical work on actual man-computer systems applied to real problems and foundational work on the logics and systems theories underlying the acquisition of knowledge. In this paper we present salient aspects of the logical foundations of both the theory and practice.

In the quarter century since the publication of Kelly's (1955) first book there have been many developments that relate to it and form a basis for a fresh impetus. In philosophy the balance has been struck between the extremely personal epistemologies of the existentialists and the extremely impersonal epistemologies of the logical positivists. Attempts, such as those of Ayer (1936), to define precisely the one acceptable method of legitimating belief have died down, and Kelly's constructive alternativism has become fashionable in conventionalist and pluralist positions such as those of Gellner (1974) and Feyerabend (1975). In Kelly's work we can find incorporated both Kuhn's (1962) emphasis on the importance of the paradigm and hence the possibility of revolutionary changes in viewpoint, and Popper's (1972) emphasis on falsifiability as the prime test of meaningful belief. Modern philosophy has swung the focus of attention from science to the scientist, a viewpoint from which Kelly's work is central to the key issues.

Positivist science advanced as rapidly as it did because of its very close

links with formal logic. Kelly himself was probably influenced by this in his attempt to present his own position axiomatically as a "fundamental postulate" and a set of eleven "corollaries". However, the possibility of forming logical foundations for his theories, let alone axiomatizing them, was not within the realms of the mathematics open to him at the time. His founding of his epistemology on the process of making a bipolar distinction is a precursor to Brown's (1969) fundamental reformulation of propositional logic in his "calculus of distinctions". His concept of a construct applied to elements and having a range of convenience requires a modal logic incorporating notions of relevance, and the theory underlying these was only formalized during the mid 1960's (Snyder 1971, Anderson & Belnap 1975). The formalization of modal logic has been very fruitful in establishing semantic foundations for natural language (Cresswell 1973), and its basis in the concept of possible worlds (Lewis 1973, Bradley & Swartz 1979) seems very close to the model that Kelly needed for the dynamics of construct formation and modification. A related development in recent years has been that of multivalued logical foundations for set theory such as Zadeh's (1976) fuzzy logic, and the application of this also to modelling human semantic processes (Gaines 1976) has much in common with Kelly's approach.

The development of Kelly's work to date has tended to neglect the epistemological foundations and concentrate upon the clinical applications (Slater 1976) and its integration into the mainstream of work on personal psychology (Bannister & Fransella 1971, Hogan 1976). Because of the experimental nature of much of this work the analysis of Kelly's repertory grids through computer programs has itself become a significant line of development (Slater 1977, Shaw 1980a). The on-line application of computers to operationalize Kelly's construct theory and to reflect to an individual his role as a personal scientist adds a new dimension to the work.

This paper deals with the analysis of the grid rather than its elicitation

but it is worth emphasizing that our prime motivation for the form of logical analysis developed here was to extend techniques for the interactive elicitation of grids through feedback of the analysis. We have been aiming to develop a conversational, dialectical, system of computer programs for the self-reflective study of one's role as a personal scientist. With this in mind it has been important to develop forms of analysis that can support a conversation by commenting upon its contents without introducing new constructs beyond those the user already employs. It is this which has led us to a logical analysis in which the meta-language used for comments on construct structures is essentially the same as the object-language in which the information defining these structures is given.

LOGICAL ANALYSIS OF THE REPERTORY GRID

In his repertory grid technique Kelly attempted to get people to make their construct usage explicit by defining distinctions between elements, i.e. by assigning each element under consideration to one pole or the other of a construct. In essence Kelly placed the same fundamental emphasis as did Brown in his seminal work, Laws of Form, on the human, creative operation of "making a distinction":

> "The theme of this book is that a universe comes into being when a space is severed or taken apart... By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance." (Brown 1969 p.v)

Brown used this notion as the foundation for the development of a formal logical system that avoided some of the paradoxical consequences of classical logics, and the validity of his approach has been demonstrated by the recent work of Kohout & Pinkava (1980) embedding it in non-classical multivalued logics and that of White (1979) demonstrating that these logics give a consistent set theory allowing unrestricted predication. Slater has noted the "connections between personal construct theory, logic and probability theory" and states that:

> "The typical proposition in personal construct theory, E may be construed as C, paraphrases the typical proposition of Aristotelian logic, S is P, i.e. subject is predicate. It is difficult to conceive of any proposition that can be stated in one of these forms and not the other." (Slater 1977 p.34)

However, despite these logical connotations the commonly used forms of repertory grid analysis take the grid as a matrix of numbers and use distance-based numerical analysis and clustering techniques to derive an underlying structure

(Shaw 1980b, Shaw & Gaines 1981b). In this section we demonstrate that a completely logical analysis of the repertory grid is possible.

Figure 1 is a repertory grid from Leach (1981) which was completed by a man whose wife had recently died of cancer. The elements he uses are shown as columns, and the constructs as rows. A 1 indicates assignment of an element to the left-hand pole of the construct and a 2 to the right-hand pole. The values in the grid may be viewed in two ways: as numerical values; and as logical values. For example in principal components analysis we view the grid as a set of vectors in a space. However we may also view it as an assignment of truth-values to logical predicates. We may take the left-hand pole of each construct in Figure 1 to be a logical predicate that may be applied to a person and take the assignment of the value 1 to a particular element in the grid to mean that the predicate is true for that element. Conversely we may take the value of 2 assigned to an element for a construct to mean that the predicate represented by the left-hand pole of that construct is false for that element. It is convenient to use the abbreviation LHPm for the predicate that corresponds to the left-hand pole of construct m. Thus LHP5 is the predicate for the left-hand pole of construct 5. If we then also adopt the convention that En stands for the n'th element then the notation LHPm En may be used to denote the truth value of the predicate corresponding to the left-hand pole of construct m when applied to the logical constant corresponding to the n'th element. A repertory grid, such as that of Figure 1, is then the matrix of such truth values for the m constructs and n elements involved.

Because of the inverse relation usually assumed between assignments to the opposite poles of a construct in a conventional repertory grid, the predicate corresponding to the right-hand pole is logically related to that corresponding to the left-hand pole. We normally require that an element be assigned to one, and only one pole, so that if LHP E is true then RHP E must be false, and vice versa. Hence, LHP E is essentially the logical negation of RHP E. For the

current discussion we shall accept that this relation exists as a constraint between the two predicates corresponding to the two poles. However, it is not an essential one for the theory and we discuss later the possibility of relaxing it and the consequences of doing so. For this reason we shall carry out most of the discussion in terms of the left-hand poles and associated predicates primarily, noting occasionally the corresponding phenomena for right-hand poles.

First let us examine the previous relation of equivalence between constructs in logical terms. We can define two logical propositions to be equal if their truth-values are the same, and this also corresponds to their numerical truth-values being equal, e.g.

$$\underset{\sim}{\text{LHPm}}\ E = \underset{\sim}{\text{LHPn}}\ E \qquad (1)$$

We can define two logical propositions involving the same free variable as being equivalent if they are equal for all values of that free variable, e.g.

$$\forall E \quad \underset{\sim}{\text{LHPm}}\ E = \underset{\sim}{\text{LHPn}}\ E \qquad (2)$$

and it is then convenient to drop the variable and write:

$$\underset{\sim}{\text{LHPm}} \equiv \underset{\sim}{\text{LHPn}} \qquad (3)$$

Now this equivalence between the poles of constructs clearly coincides with our previously discussed equivalence in terms of distance. If two propositions are logically equivalent in this way then the vectors of truth-values against elements are the same and hence they are zero distance apart. The converse may also be shown for any proper distance measures.

However, in terms of logical relations equality is only one of many possible relations. There are six binary logical operators between propositions that establish relations between them. Two of these relations are symmetrical and correspond to the two propositions being equal, or to one being equal to the negation of the other. This corresponds to the reversal or reflection of constructs discussed above. The other four operators are forms of <u>implication</u> between propositions, that one proposition being true implies that the other is also true. The four forms arise because of the possibilities of negation, that

one being true implies the other is not and so on. They may all be derived from the one operator, $\supset$, where:

$$\underset{\sim}{\text{LHPm}}\ E \supset \underset{\sim}{\text{LHPn}}\ E \qquad (4)$$

means that the assignment of element E to the left-hand pole of construct m implies that it is also assigned to the left-hand pole of construct n.

In contrast to the equality relation, the implication relation is asymmetric. If we assert the implication given in (4) then we are only constraining the truth-value of LHPn E if LHPm E is true. If this is not so and element E is not assigned to the left-hand pole of construct m then we are saying nothing about its assignment to the left-hand pole of construct n. This contrasts to the equality relation asserted in (1) where the proposition LHPm E being false also leads to LHPn E being false in order to satisfy the equality.

One important property of the implication relation is its transitivity. From the way in which we have defined it we can see that if, as well as (4), we have:

$$\underset{\sim}{\text{LHPn}}\ E \supset \underset{\sim}{\text{LHPo}}\ E \qquad (5)$$

then we can derive:

$$\underset{\sim}{\text{LHPm}}\ E \supset \underset{\sim}{\text{LHPo}}\ E \qquad (6)$$

This is the normal transitivity of an implication relation in a logical calculus.

Asserting mutual implication between two propositions allows us to derive their equality. Thus adding the converse asymmetric assertion:

$$\underset{\sim}{\text{LHPn}}\ E \supset \underset{\sim}{\text{LHPm}}\ E \qquad (7)$$

to that of (4) does enable us to derive (1). From this we can see that the relation of implication is a weaker one than that of equality but closely related to it in that if we know the four implication relations between two propositons we may infer the two equivalence relations between them. These results from elementary propositional logic show that it is of interest to

consider the implication relation in repertory grid analysis since the equality and equivalence relations normally analysed may be derived from it but not vice versa.

In the same way that we moved from the relation of equality between individual propositions in (1) to that of universal equivalence between them in (2), we may say that one proposition involving a free variable entails another proposition involving the same variable if it has an implication relation with it for all values of the free variable, e.g.:

$$\forall\ E \quad \underset{\sim}{\text{LHPm}}\ E \supset \underset{\sim}{\text{LHPn}}\ E \qquad (8)$$

and it is then convenient to drop the variable and write:

$$\underset{\sim}{\text{LHPm}} \rightarrow \underset{\sim}{\text{LHPn}} \qquad (9)$$

We will read this as "the left hand pole of construct m entails the left hand pole of construct n". Clearly entailment, being derived from implication, is also asymmetric, and mutual entailment gives us equivalence in the same way as mutual implication gives us equality. Thus adding the converse entailment to (9):

$$\underset{\sim}{\text{LHPn}} \rightarrow \underset{\sim}{\text{LHPm}} \qquad (10)$$

to (9) itself allows us to derive the equivalence of (3). Note similarly that the entailment relation is transitive like the implication relation so that from (9) and:

$$\underset{\sim}{\text{LHPn}} \rightarrow \underset{\sim}{\text{LHPo}} \qquad (11)$$

we may derive:

$$\underset{\sim}{\text{LHPm}} \rightarrow \underset{\sim}{\text{LHPo}} \qquad (12)$$

We have linked the discussion of this section to personal construct theory. However we note that most of our definitions come direct from classical logic and are independent of personal construct theory. The formal mechanisms for defining entailment are rather more complex than those used here because the logic of entailment is concerned to avoid certain paradoxical results (Anderson & Belnap 1975). The nature of these paradoxes does have some interest in

personal construct theory because they are to do with relevance in entailment - does one proposition entail another in a relevant way or just through an artefact of the logical calculus? Similar but deeper questions arise when we consider the derivation of entailment from repertory grid data - is one construct relevant to another in the way in which it entails it or is the derived relation a fortuitous one? We consider such questions later.

It is also worth noting that our definitions of equivalence and entailment are also related to those in modal logics (Snyder 1971). We can regard (2) and (5) as being definitions of necessary equality and necessary implication in a quantification model of a modal logic. In the context of personal constructs we can see this best by noting that two verbal interpretations of (5) are acceptable: "when you assign an element to the left-hand pole of construct m you always also assign it to the left-hand pole of construct m", or "when you assign an element to the left-hand pole of construct n you necessarily assign it to the left-hand pole of construct m". These links may be formalized through a possible worlds (Bradley & Swartz 1979) model of modal expressions by noting that each element provides a possible world for construing. Entailments according to our definition then become logical implications that are true for all possible worlds currently under consideration. This is a useful and evocative viewpoint because it links personal construct theory with the linguistic semantics of counterfactuals and presuppositions (Lewis 1973) which is very relevant to Kelly's concept of constructs being "used for predictions of things to come". It also provides useful technical links into the formal mechanisms for treating the topological structure (Lemmon 1966) of possible worlds and its role in logic and semantics which seem equally applicable to personal construct theory.

To conclude the rather abstract discussion of this section and lead into the more concrete operational implementation of the next it is worth considering a specific example of what we mean by entailment, its asymmetry, and the derivation of equivalence from entailment but not vice versa. The poles of two

constructs may be quite distinct in terms of equivalence yet closely related in terms of entailment. For example suppose that in construing people someone uses the two constructs m:runs---doesn't run and n:energetic---passive, then we might well expect to find that LHPm entails LHPn but that LHPn does not entail LHPm, that is that being a runner entails being energetic but being energetic does not entail being a runner. If we analyse such a construct structure in terms of distance measures and hence of equivalence only then we shall not derive such asymmetrical relations between constructs even though they are meaningful and of practical interest.

ENTAIL - DERIVING ENTAILMENTS BETWEEN POLES OF CONSTRUCTS

It is simple to derive the entailment structure between the poles of constructs. We only have to check the truth of the four possible implications for all elements. Thus LHPm entails LHPn is checked by noting whether whenever an element is assigned to the left-hand pole of m it is also assigned to the left-hand pole of n. If so, then the entailment relation holds true, otherwise it is false. Clearly, as we noted above, it would also suffice to check that whenever an element is not assigned to the left-hand pole of n it is also not assigned to the left-hand pole of m. We call the program that performs this analysis ENTAIL (Entailment Nets Through Analysing Implicational Links). Note that the inference from a particular set of elements that one pole of construct m entails one pole of construct n is an inductive one if we assume that it applies to other elements in addition to those used in its derivation.

Figure 2 shows the matrix of entailment between the poles of the constructs derived by ENTAIL from the grid of Figure 1 drawn out as a directed graph. There are effectively two main sub-graphs which are mirror images of one another: one linking RHP3, LHP7, RHP6, LHP4, LHP5, LHP1 AND LHP2; and the other linking RHP2, RHP1, RHP4, RHP5, RHP7, LHP6 AND LHP3. One of the sub-graphs shows the entailments for one set of poles, and the other the entailments for the opposite poles. The two sub-graphs are linked together by entailments between the poles of constructs 1 and 2. Note the "reversal" of constructs 3 and 6 in that RHP3 and RHP6 appear in the sub-graph of the left-hand poles of the other constructs and vice versa. Because of the essential bipolarity assumed in the elicitation of the grid the two graphs are essentially the same with the arrows and poles reversed in one relative to the other. Later we discuss extensions to the form of grids which would result in such pairs of graphs not necessarily having such a simple relation.

Note that we have taken advantage of the transitivity of the entailment

		*	1	2	3	4	5	6	7	8	
		**********									
ADMINISTERING COMFORT	1	*	2	1	1	1	1	1	1	1	NOT
LIFE DEAR TO ME - "HAPPY"	2	*	1	1	2	1	1	1	1	1	NOT SO MUCH
SERIOUS APPROACH/INVOLVEMENT	3	*	1	1	1	1	2	2	1	1	MORE FLAMBOYANT
CONCERNED BUT NOT DESPONDENT	4	*	2	2	2	1	1	1	1	2	DESPONDENT
MORE ABLE TO KEEP COMFORTABLE	5	*	2	2	2	1	1	1	1	2	HELPLESS, NOT HAVING MEDICAL KNOWLEDGE
APPRECIATION OF LIFE DUE TO CIRCUMSTANCES	6	*	1	1	1	1	2	2	2	1	NOT SO MUCH
CONTINUING WITH LIFE AS BEFORE	7	*	2	2	2	1	1	1	2	2	WISER, HUMBLE - REACHED THIS POINT
		*	*	*	*	*	*	*	*	*	
			*	*	*	*	*	*	*	*	CLOSE FRIEND
			*	*	*	*	*	*	DISTRICT NURSE		
			*	*	*	*	*	HOSPITAL NURSE			
			*	*	*	*	HOSPITAL DOCTOR				
			*	*	*	GP					
			*	*	SELF NOW						
			*	SELF BEFORE							
			PATIENT								

Figure 1 Raw Grid from Leach (1981)

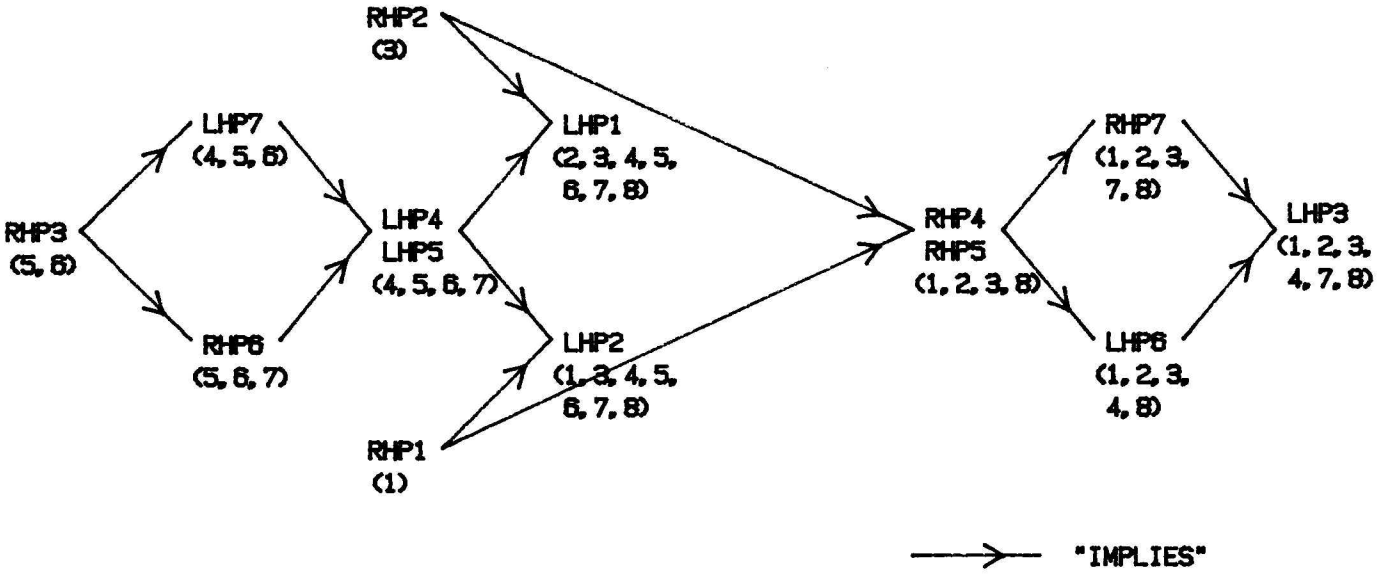


Figure 2 ENTAIL Analysis of Grid from Leach (1981)

relation not to draw in all the arrows strictly necessary. Thus we have not drawn arrows from RHP3 to LHP4, LHP5, LHP1 and LHP2 because there is an arrow from RHP3 to LHP7 and then one from LHP7 to LHP4 & LPH5, and LHP4 to LHP1 & LHP2. We can see from the figure that RHP3 entails LHP4, LHP5, LHP1 and LHP2 by tracing through the graph. Note the equivalence between LHP4 and LHP5 which is essentially mutual entailment.

EXTENDING ENTAILMENT TO RATING SCALES

Kelly (1955) presented constructs as binary categories and based his own methodology for eliciting constructs on this. However, other workers found the need for 'shades of grey' between the two poles of a construct and in a later work Kelly notes that this is consistent with his notion of a construct:

> "The construct, of itself, is the kind of contrast one perceives... while constructs do not represent or symbolize events, they do enable us to cope with events, which is a statement of a quite different order...They also enable us to put events into arrays or scales, if we wish." (Kelly 1970, pp.13-14)

It is common in many practical applications of repertory grids to use an N-point scale with 1 being an assignmment to the left-hand pole and N being an assignment to the right-hand pole, and intermediate numbers representing some form of 'intermediate' assignment. N is usually odd, 5 or 7, to allow a 'neutral' mid-point to the scale.

Methods of grid analysis based on distance measures apply readily to rating scale data if the scales themselves are assumed to form the axes of some co-ordinate system. Grid analysis based on logical predicates has been shown by Shaw & Gaines (1979) to extend naturally to rating scale data if the scales are assumed to represent truth values in a multi-valued logic (Rescher, 1969). In recent years such logics have been used by Zadeh (1965) to develop a theory of fuzzy sets that closely parallels that of classical set theory but allows for 'shades of grey' in set membership. He considers the elements of a set to take intermediate values of membership to the set and calls these values degrees of membership to the set. Since Zadeh's original study there has been a rapid growth in the literature on fuzzy sets and their application to system theory, medical diagnosis, psychological modelling, linguistics, and so on (Gaines & Kohout 1977).

One important feature of Zadeh's work in the context of grid analysis has

been its emphasis on the linguistic nature of human reasoning and the use of fuzzy set theory to model the use of hedges such as very and rather in human reasoning (Gaines 1976). This is similar to the interpretation of the points on a rating scale in terms of such hedges as very, slightly and quite used in semantic differential techniques (Osgood, Suci & Tannenbaum 1957). Thus there are natural verbal interpretations of the rating scale when values are input and these may also be applied to the equivalent values resulting from the ENTAIL analysis. One may say that there is a "quite strong" or a "very strong" entailment from one pole to another. Our requirement that the terminology and concepts of the analysis be those of the data thus continue to be satisfied in the extension to multivalued logics.

An approach to the grading of entailment has been given in Shaw & Gaines (1980) where we introduce the predicate usually in the analysis performed by ENTAIL. This predicate is a quantifier similar in nature to the "for all" quantifier, but qualified to allow for some disconfirming instances so that it may be read as "for all but N cases" where N is some small number such as 1 or 2. Such a quantifier allows a natural grading of entailment in terms that are immediately meaningful to the originator of the grid: "when you say someone runs you always also say they are energetic" or "when you say someone runs you usually also say they are energetic".

Figure 3 is a grid from Pope & Keen (1981) in which books have been assigned to the poles of constructs using a 5-point scale. ENTAIL derives the structure shown in Figure 4 for the perfect entailments in this grid, that, for example, "written as literature" or "complex" or "memorable" entail "worth the money". Figure 5 is derived for entailments under the predicate "usually" allowing one small deviation from perfect entailment. It subsumes the structure of Figure 4 and puts forward a number of other possible entailments. Clearly these are only conjectures and form the basis for a conversation. For example the entailment from RHP5 to RHP10, that "not relevant" entails "worth the

			1	2	3	4	5	6	7	8	
TOTALLY COMMERCIALISED	1	*	5	1	2	2	3	1	2	4	WRITTEN AS LITERATURE
SENSATIONAL	2	*	4	2	2	2	3	1	4	4	PLAIN
EXTERNAL CONFLICT ONLY	3	*	5	3	3	3	2	1	4	3	MUCH CONFLICT INTERNALLY
FLAT	4	*	5	3	3	4	2	1	3	3	HAD SOMETHING
RELEVANT TO TODAY	5	*	3	1	1	3	2	2	2	4	NOT RELEVANT
SIMPLE	6	*	4	1	3	2	3	1	4	3	COMPLEX
ENJOYABLE	7	*	3	3	2	2	5	3	2	5	NOT ENJOYABLE
COMPULSIVE	8	*	4	2	2	2	5	1	3	5	NOT COMPULSIVE
FORGET IT	9	*	4	1	3	3	2	1	4	2	MEMORABLE
POOR VALUE	10	*	5	1	4	4	3	1	5	4	WORTH THE MONEY
		*	*	*	*	*	*	*	*	*	
			*	*	*	*	*	*	*	*	MEMOIRS OF A FOX HUNTING MAN
			*	*	*	*	*	*	CATCH 22		
			*	*	*	*	*	THE DEEP			
			*	*	*	*	HIGH WIND IN JAMAICA				
			*	*	*	THE ODESSA FILE					
			*	*	THE DOGS OF WAR						
			*	THE MONEY CHANGERS							
			ST JOAN								

Figure 3 Raw Grid from Pope & Keen (1981)

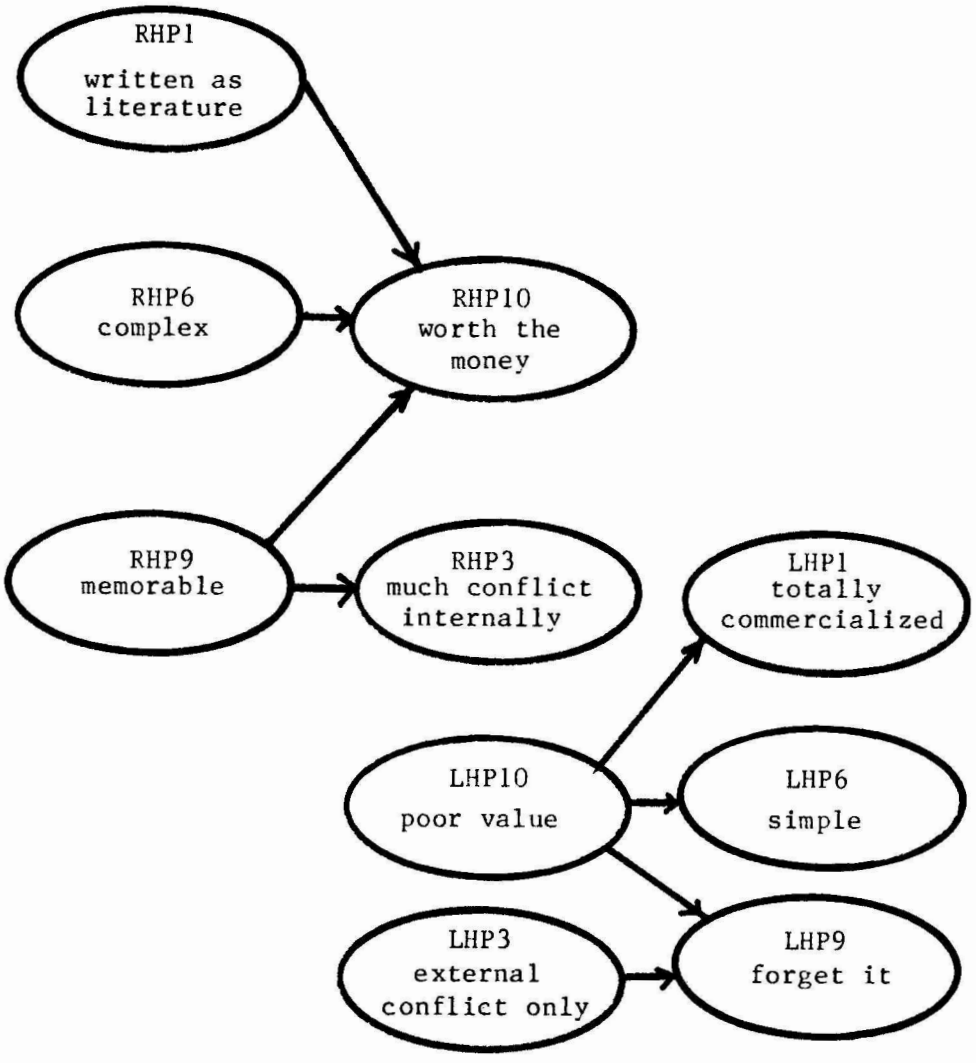


Figure 4 ENTAIL Analysis of Grid from Pope & Keen (1981)

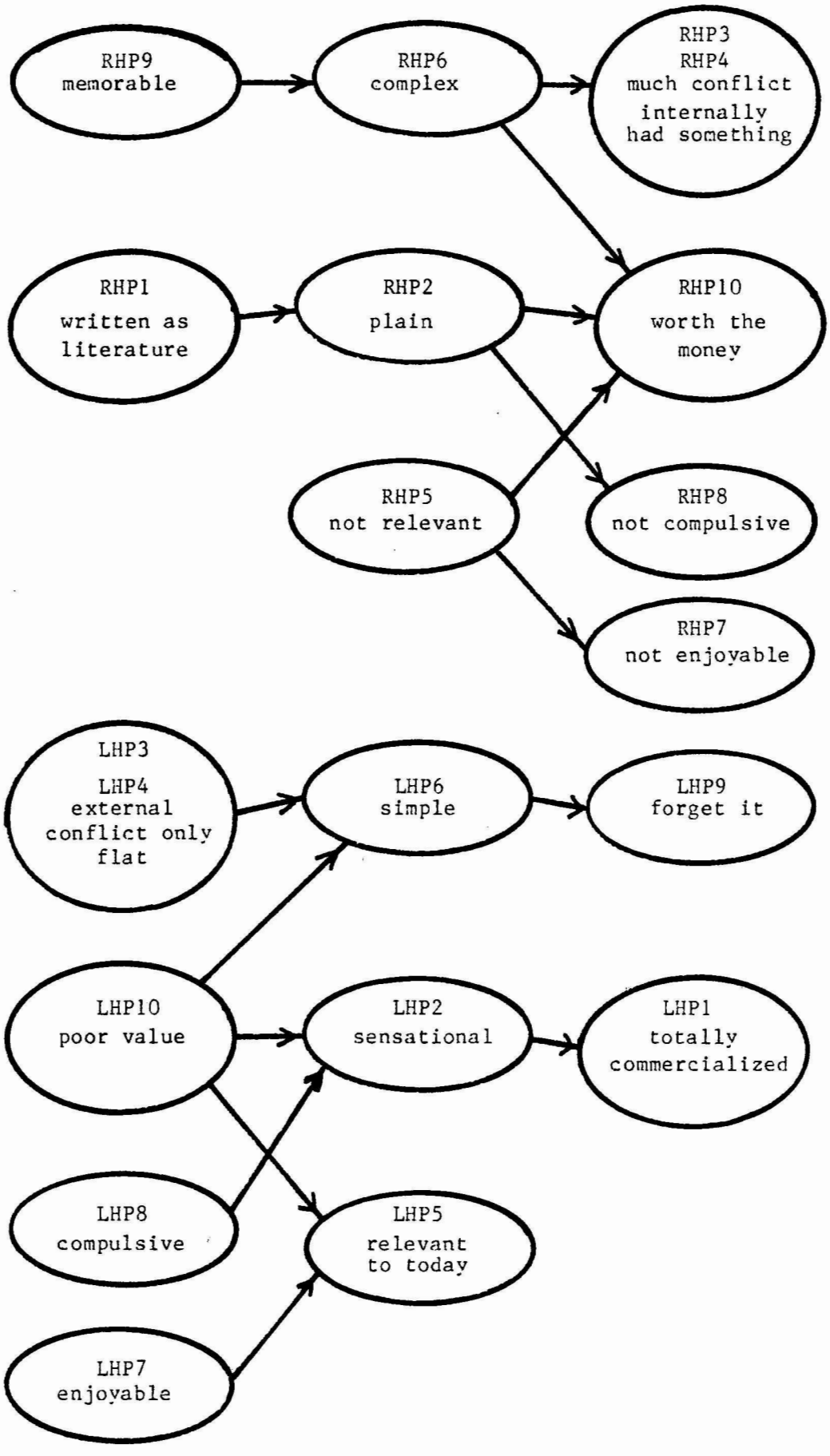


Figure 5 ENTAIL Analysis "Usually" of Grid from Pope & Keen (1981)

money", might be suspect and lead to the question "Can you think of a book which is not relevant and poor value?".

CONCLUSIONS

We have presented in this paper an approach to the analysis of repertory grids which is based on the classical predicate calculus, extended to multivalued logics in the case of rating scale data. There are many possible directions of further development arising out of this approach. One obvious methodological question is the relationship of the ENTAIL analysis to that of widely used programs such as INGRID and FOCUS, and this is discussed in the companion paper (Shaw & Gaines 1981c). We have emphasized the significance of the logical analysis for conversational programs and this is now the focus of our practical developments. For example a program such as PEGASUS (Shaw 1980a) should now be able to elicit new elements by queries such as:

"When you say a person runs you also say they are energetic. Can you give an example of someone who runs who is passive?"

It should also be possible for the user of PEGASUS to query the program as to why certain entailments do not hold, for example:

"Why not energetic implies runs?

Because you said that John is energetic and that John doesn't run."

There are also many theoretical issues that arise out of the logical analysis. For example, the directed graph of entailment is reminiscent of the type of structure that we get when considering Kelly's concepts of "superordination" and "subordination" between constructs. A theoretical analysis suggests that one construct is superordinate to another if it has a greater range of convenience and is related to the other construct through entailment. We have discussed elsewhere (Gaines & Shaw 1981) the relation of the ENTAIL analysis to that of Hinkle's (1965) "laddering" which derives the construct hierarchy directly by asking "why" questions to go up it and "how" questions to go down it:

"Why do you run?

To be energetic.

How are you energetic?

Through running."

We can infer from the first that running entails being energetic, and from the second being energetic is entailed by running. Thus, from a logical point of view the indirect elicitation of implicit entailments through ENTAIL and the direct elicitation of the construct hierarchy through laddering should correspond and it would be interesting to study this both theoretically and empirically.

There are also situations that we have mentioned in this paper where the assumed logical interrelationship between the predicates corresponding to the two poles of a construct does not hold and where the ENTAIL analysis should not necessarily produce symmetric sub-graphs. Landfield (1976) gives an example of a grid elicited from a client in which the responses "N" for neither and "?" for undecided was allowed. This may be analysed by ENTAIL by assuming that "N" corresponds to both the LHP and RHP predicates being false and "?" corresponds to them both being true. Such a logic corresponds in its truth functions to one analysed by Belnap (1976) for database systems (Gaines 1979) and leads to an entailment structure not necessarily symmetric between the poles. We have also recently developed a model for the logical extension of the "personal scientist" analysis to that of many people interacting in a "community of science" (Shaw & Gaines 1981a).

These are only some examples of the wide range of questions that arise out of the logical approach to grid analysis. We would like to suggest in conclusion that developments in mathematical, linguistic and philosophical logic in recent years cited in the introduction, notably those in relevance and multivalued logics, make available a set of theoretical and practical tools not available some twenty five years ago. It is now feasible to go back to the foundations of Kelly's work and develop his system of corollaries on a formal basis that

captures the full semantic richness of his concepts and is also in operational form.

REFERENCES

Anderson, A.R. & Belnap, N.D.

1975 Entailment. Princeton University Press, New Jersey.

Ayer, F.J.

1936 Language, Truth and Logic. Victor Gollancz, London.

Bannister, D. & Fransella, F.

1971 Inquiring Man. Penguin, London.

Belnap, N.D.

1976 "How a computer should think." Contemporary Aspects of Philosophy. (G.Ryle ed.) Oriel Press, Stockfield, 30-56.

Bradley, R. & Swartz, N.

1979 Possible Worlds. Basil Blackwell, Oxford.

Brown, G.S.

1969 Laws of Form. George Allen & Unwin, London.

Creswell, M.J.

1973 Logics and Languages. Methuen, London.

Feyerabend, P.

1975 Against Method. NLB, London.

Gaines, B.R.

1976 "Foundations of fuzzy reasoning." International Journal Man-Machine Studies 8: 623-668.

1979 "Logical foundations for database systems." International Journal Man-Machine Studies 11: 481-500.

Gaines, B.R. & L.J.Kohout

1977 "The fuzzy decade: a bibliography of fuzzy systems and closely related topics." International Journal Man-Machine Studies 9: 1-68.

Gaines, B.R. & Shaw, M.L.G.

1981 "New directions in the analysis and interactive elicitation of personal construct systems." Personal Construct Technology. (M.L.G.Shaw ed.) Academic Press, London.

Gellner, E.

1974 Legitimation of Belief. Cambridge University Press, Cambridge.

Hinkle, D.N.

1965 The change of personal constructs from the viewpoint of a theory of implications. PhD Thesis, Ohio State University.

Hogan, R.

1976 Personality Theory: The Personological Tradition. Prentice-Hall, New Jersey.

Kelly, G.A.

1955 The Psychology of Personal Constructs. Norton, New York.

1970 "A brief introduction to personal construct theory." Perspectives in Personal Construct Theory. Academic Press, London.

Kohout, L.J. & Pinkava, V.

1980 "The algebraic structure of the Spencer Brown and Varela calculi." International Journal of General Systems 6: 155-171.

Kuhn, T.S.

1962 The Structure of Scientific Revolutions. University of Chicago Press.

Landfield, A.

1976 "A personal construct approach to suicidal behaviour." Dimensions of Intrapersonal Space: Volume 1. (P.Slater ed.) John Wiley, London, 93-107.

Leach, C.

1981 "Direct analysis of a repertory grid." Personal Construct Technology. (M.L.G.Shaw ed.) Academic Press, London.

Lemmon, E.J.

1966 "Algebraic semantics for modal logics." Journal of Symbolic Logic 31: 46-65, 191-218.

Lewis, D.

1973 Counterfactuals. Basil Blackwell, Oxford.

Osgood, C.E., Suci, G.J. & Tannenbaum, P.H.

1957 The Measurement of Meaning. University of Illinois Press, Urbana.

Pope, M. & Keen, T.

1981 Personal Construct Psychology and Education. Academic Press, London.

Popper, K.R.

1972 Objective Knowledge. Clarendon Press, Oxford.

Rescher, N.

1969 Many-Valued Logic. McGraw-Hill, New York.

Shaw, M.L.G.

1980a On Becoming a Personal Scientist. Academic Press, London.

1980b "The analysis of a repertory grid." British Journal of Medical Psychology, 53, 117-126.

Shaw, M.L.G. & Gaines, B.R.

1979 "Externalizing the personal world: computer aids to epistemology." Improving the Human Condition: Quality and Stability in Social Systems. Society for General Systems Research, Kentucky, 136-145.

1980 "Fuzzy semantics for personal construing." Systems Science and Science. Society for General Systems Research, Kentucky, 146-154.

1981a "The personal scientist in the community of science." General Systems Research and Design: Precursors and Futures. Society for General Systems Research, Kentucky, 59-68.

1981b "Recent advances in the analysis of a repertory grid." British Journal of Medical Psychology, (to appear).

1981c "Comparison of techniques for analysing personal construct systems." Fourth International Congress on Personal Construct Psychology. Brock University, Canada.

Slater, P. (ed.)

1976 Dimensions of Intrapersonal Space: Volume 1. John Wiley, London.

1977 Dimensions of Intrapersonal Space: Volume 2. John Wiley, London.

Snyder, D.P.

1971 Modal Logic. Van Nostrand Reinhold, New York.

White, R.B.

1979 "The consistency of the axiom of comprehension in the infinite-valued predicate logic of Lukasiewicz." Journal of Philosophical Logic 8: 509-534.

Zadeh, L.A.

1965 "Fuzzy sets." Information and Control 8: 338-353.

1976 "A fuzzy algorithmic approach to the definition of complex or imprecise concepts." International Journal Man-Machine Studies 8: 249-291.