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LOGICAL FOUNDATIONS OF EXPERT SYSTEMS

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The application of fifth generation computer-based expert systems will be a major trend during the next decade. However, the knowledge technology underlying such systems is still largely pragmatic and lacks coherent foundations. In this paper we illustrate the operation and application of expert systems, and survey the foundations of expertise from both psychological and system theoretic points of view and propose an overall model of expertise. We relate this to the underlying knowledge structures and show how these are based on a calculus of distinctions that generates a variety of logical systems.

Fifth Generation Computers and Expert Systems

The primary objectives of the Fifth Generation Computer System (FGCS) development program initiated by the Japanese [39] are two-fold:

a) to provide knowledge processing rather than information processing;
b) to offer natural human-computer interfaces.

These objectives have been accepted as realistic because of the successful development of **expert systems** (ES) [46] that encode human knowledge and cognitive skills for the computer and make them accessible to others.

The first four generations of computers were **information processors.** The transition between each generation has involved the introduction of new technologies [19]. The first generation (1948-55) used vacuum tubes and delay lines and this **hardware** became transistors and core stores in the second generation (1956-63). The transition to the third generation (1964-71) was dominated by the availability of **software,** operating systems and compilers. The transition to the fourth generation (1972-79) was one of **person-computer interaction** in which interactive usage and personal computing became common. The transition to the fifth generation (1980-87) is now seen to be one of **knowledge processing** with the development of ES.

Discontinuities in Development

The hardware, software, peopleware, knowledgeware, sequence of transitions may be presented as the continuous evolution of an industry. However, the step to FGCS involves a transition that is different in kind and requires a more radical reappraisal than any before it. In the development of any major industry there are periods of growth which consolidate existing technologies and extend them to a widening range of applications. During these periods conceptual frameworks, design methodologies and implementation techniques become well-established, systematized and widely known and accepted. However, there are also phases of discontinuity in which new technologies open up new areas of application that are qualitatively different from the old and require new modes of thinking. The conceptual frameworks, design methodologies and implementation techniques after the discontinuity differ radically from those before it. During the transition there is confusion as the older modes of thinking are applied to the new situation and the newer modes of thinking have to establish their credibility as preferred alternatives to those with a track record of past success.

Information technology has now come into a phase of discontinuity where the new modes of thinking are necessary. We have been through a phase of consolidation of techniques and expansion of applications in which computing hardware costs have dropped to very low levels and an immense range of systems and applications software has been made available. Information processing is now available in all those markets that may require it with few impediments of cost or availability of hardware and software. Even the limitation of the shortage of trained computer professionals is fast disappearing as a high proportion of students in all disciplines graduate with computing training and computers themselves become simpler to program. However, just as we are beginning to celebrate the arrival of the **information age** [6], we are coming to realize that many of the predictions for that age are dependent on processing **knowledge** rather than information.

Information is not Knowledge

It is not easy to give a simple and widely acceptable definition of the difference between knowledge and information, and this will be discussed in more detail later. However, an example will serve to illustrate some essential distinctions. **Encyclopaedia Britannica** contains about 300M characters and a scientific book of 350 pages about 1M characters. With the availability of low-cost 140Mbyte Winchester discs and text compression techniques it is feasible to have the encyclopaedia or some 300 books on-line to a personal computer for about $5,000 dollars hardware cost. The advent of low-cost optical discs will allow a scientist to have 2Gbytes on line for this cost in a year or two: say, a general encyclopaedia, a specialist one for his subject area, some 2,000 books and 30,000 papers. For most university scientists this would represent a complete information environment for the needs of their personal interaction with their discipline in research, teaching and consultancy. If the numbers are too small then project them a further three years into the future for another tenfold increase in capacity.

We have passed, or are passing, the threshold of personal information availability requirements in terms of cost of storage. Speed of access is also no longer a problem since a fully inverted keyword index to the material can be provided at the cost of doubling the storage. The texts themselves will be made available at a price comparable to those of books once publishers can see a market of sufficient size, and prices should decline since the production, distribution and inventory costs of optical discs are much lower than those of books. Compared with previous media we can see that computer-based information technology has major advantages. The scientist has in a compact system a personal library that may be accessed easily through a complete index. It couples simply into his word-processor making referencing and quoting other work very easy.

However, apart from ease of storage and access what advantages does our computer-based system have over a library? All it does is mimic the previous medium of books and journals at a lower cost, in a smaller space and with greater speed of access. These are significant advances but they do not represent a qualitative change. The burden of using the information, recognizing patterns in it, linking it together to support an argument, searching it for solutions to a problem, all of these still reside with the scientist. It is the user who creates knowledge from the information.

It would be reasonable to suppose that the additional features necessary to make the system itself a knowledge processor could be developed using the same conceptual frameworks, design methodologies and implementation techniques that led to the present generation of information processors. However, there are some fundamental obstacles to this: conceptually, that we do not understand the processes of inference that a person uses in recognizing patterns, generating arguments and solving problems; and, technically, that

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these processes seem to involve computations of a complexity many orders of magnitude beyond those possible with current computers. In terms of our text-storage example, the information needs to come out of a passive storage medium into an active processing medium that is accessing substantial parts of it simultaneously through an indexing framework that is itself active and conceptual rather than textual.

The conflict between old and new in a phase of discontinuity forces a resolution, or synthesis, in which the fundamentals common to both are re-examined. There are presuppositions to the old solutions that are no longer valid. They need to be made overt if the new solutions are to be formalized and understood.

Paradigm Shifts in Information Technology

The previous section introduces the notion that in developing knowledge systems the computer industry is making a change in technology that is discontinuous with that preceeding it. There has been a **paradigm shift** [29] that shows up as a change in the conceptual framework for all aspects of computing. However, such discontinuities do not occur for no reason, as causes of disruption, rather they are the effects of deeper underlying causes such as social needs. There is much talk about the impact of information technology on society. However we can also see information technology as generated by the needs of an increasingly over-crowded, complex, inter-dependent world with limited resources.

Complexity Reduction

The German sociologist Luhmann in his work on **Trust and Power** encapsulates the dynamics of society by postulating **complexity-reduction** as the fundamental motivation for all our social institutions:

"The world is overwhelmingly complex for every kind of real system... Its possibilities exceed those to which the system has the capacity to respond. A system locates itself in a selectively constituted 'environment' and will disintegrate in the case of disjunction between environment and 'world'. Human beings, however, and they alone, are conscious of the world's complexity and therefore of the possibility of selecting their environment -- something which poses fundamental questions of self-preservation. Man has the capacity to comprehend the world, can see alternatives, possibilities, can realize his own ignorance, and can perceive himself as one who must make decisions." [31 p.6]

Luhmann's model seems to underly De Bono's optimism about the role of computers in **Future Positive:**

"By great good fortune, and just in time, we have to hand a device that can rescue us from the mass of complexity. That device is the computer. The computer will be to the organisation revolution what steam power was to the industrial revolution. The computer can extend our organizing power in the same way as steam extended muscle power..... Of course we have to ensure that the result is more human rather than less human. Similarly we have to use the computer to reduce complexity rather than to increase complexity, by making it possible to cope with increased complexity." [5 pp.18-19]

Industrial Technology Imposes Simplicity

The paradigm for industrial technology in the past has been to create Luhmann's simplified environment for man by dealing with the complexity of the world through removing it. The man-made environment is simple and regular and capable of being modeled easily. Where it interacts with nature, as in agriculture, the natural world is simplified by saturating it with variety-reducing mechanisms, such as weed-killers and pesticides. The technology of the past has made simplistic scientific models true by constraining natural phenomena to fit the models. Industrial technology has made much of the world simple, homogenous and comprehensible. It has minimized the knowledge necessary to operate in a complex world by simplifying the world. This has been an effective way to operate but not an efficient one in the preservation of non-renewable resources.

De Bono's optimism suggests that information technology is capable of extending our organizing capabilities to cope with a more complex world. This would be a reversal of the objectives of previous industrial technology and hence a major paradigm shift. Computers can be used to control the world and make it simple or they may be used to aid our interaction with a complex world. The second alternative is becoming important to us as the consequences of industrialization in destroying natural resources are realized [41]. It is becoming important to accept and preserve the variety of the natural world and use it to extract sufficient resources for society's needs whilst husbanding long-term resources.

People Cope with Residual Complexity

The major paradigm shift is thus from imposing simplicity on the world to coping with its complexity. A secondary paradigm shift results from this in that the previous system for coping with residual complexity has been people. When technological simplification breaks down people are used to solve the remaining problems. Automatic machines need human operators to cope with broken tools, misplaced parts and other contingencies outside the well-defined environment for which the machine has been designed. Some industrial processes, such as pig-iron smelters and lime-kilns, are not amenable to automatic control but require human operators. The shift in our conceptual frameworks has been to realize that we may be able to automate such processes not by modeling the complex world but instead by modeling the way in which people cope with it. Expert systems mimic specific problem-solving processes of people without analysing the underlying basis for the success of these processes.

ES represent a major shift in the paradigms of science and technology because they allow control without explicit understanding. The classical control paradigm is to develop a model of the plant and then to use this model to calculate an optimal control strategy. This works where the plant can be engineered to have a simple model amenable to analytic techniques. Even in this case there is no guarantee that an understandable plant is an efficient one in terms of yield. The paradigm increasingly falters as the plant involves natural processes not engineered for simplicity in modeling. It is symptomatic of our lack of respect for human capabilities that we failed to notice for so long that people could operate systems that they did not fully understand, and that this capability could be transferred to machines by modeling the operator rather than the plant.

Information and Knowledge

We can now bring together the discussion of the last two sections to give a succint account of the relation between information and knowledge in FGCS.

(1) The multi-Gbyte information system described in the previous section has the capacity to encode a substantial part of the complexity of the world, sufficient say to contain that information used by a scientist during his lifetime. However, it only stores and indexes that information. It cannot process it as knowledge to recognize patterns, support arguments and solve problems.

(2) Expert systems encode some basic aspects of people's capabilities to solve problems and support arguments in very limited contexts, that is involving a small amount of information. They are primitive knowledge processors in low information environments.

(3) Fifth generation developments will enhance the capabilities of expert systems and exploit them in commercial situations where knowledge processing is highly significant and the information is low.

(4) Once the immediate potential of simple expert systems has been realized there will be a shift towards integrating the low-cost, high-capacity information storage and processing capabilities of fourth generation computing systems with the

knowledge processing of FGCS. This will involve new hardware capable of actively modeling large information structures and new software capable of processing knowledge on a large scale.

With such **sixth generation** systems the paradigm shifts will be complete. We shall be able to represent the complexity of the world as information and process it as knowledge. To complete this shift we need to go back to fundamentals and understand the nature of knowledge. In particular, we need to understand the basis of the technological paradigm, why it has operated sucessfully, and why it has to change at the next stage. We also need to understand the basis of human knowledge processing and reverse the logic of the past that where people do not conform to our normative models of scientific reasoning that it is they who are in error.

The remainder of this paper examines some of these fundamental issues and attempts to link them to information and knowledge processing.

Expert Systems

One of the first expert system developments was that of Mamdani and Assilian in 1974 [32] that took human knowledge expressed verbally and encoded it directly as computer programs which acted on the environment. This work was undertaken as part of a study of machine learning in process control and the system being controlled was a small steam engine. The verbal rules were of the form:

If the pressure error is positive and big and the change in pressure error is not negative medium or big
Then make the heat change negative and big.

Fig. 1 Rule from fuzzy logic controller

Priming the learning machine by such rules was first introduced because learning took too long. It was expected that the rules would act like verbal instructions to a human operator and aid the machine in setting up an initial approximate control strategy. However, it was found that rules such as this may be interpreted algorithmically to produce computer control programs that are more effective than those derived from classical control theory. Additionally, the learning machine then proceeded only to learn less effective strategies. Hence interest switched to the process whereby human expression of verbal rules that appear vague can lead to highly effective control strategies, that is to the development of expert systems.

The rules on which a particular expert system is based may be seen as ways of representing knowledge. This is not a new concept as such rules have been used in the past quite independently of computer systems. Figure 2 is an excerpt from a training manual for lime kiln operators [42]. It seems that the encoding of expertise into a set of fairly vague but highly applicable rules is a standard way of making it available to others.

1. When the burning zone temperature is drastically low:
   (a) reduce kiln speed;
   (b) reduce fuel.
2. When the burning zone temperature is slightly low:
   (a) increase I.D. fan speed;
   (b) increase fuel rate.

Fig. 2 Rules from a lime kiln operator's manual

In parallel with the controller development, other expert systems were being developed for completely different domains. MYCIN [56] is a medical diagnosis expert system which aids a clinician to act as a consultant on infectious diseases. It uses rules of the form:

RULE50
If 1) the infection is primary-bacteremia, and
   2) the site of the culture is one of the sterile sites, and
   3) the suspected portal of entry of the organism is the gastro-intestinal tract
Then there is suggestive evidence (.7) that the identity of the organism is bacteroides.

Fig. 3 A MYCIN rule

Meta-Knowledge

The rules of Figs. 1 through 3 are expressions of inferences from data about the world to conclusions about it, a basic level of knowledge. It was rapidly discovered, however, that similar rules could express inferences at a higher level about the nature of these basic inferences. MYCIN worked well when the rules were complete and correct, but was difficult to set up the rules initially. It was also difficult to trace the rules causing errors in the deductions of MYCIN if the clinician feels that something has gone wrong. TEIRESIAS [4] was added as an auxiliary system with expertise about MYCIN. It has the facility to explain MYCIN's decisions, and help the clinician amend the rules when they lead to incorrect conclusions. TEIRESIAS uses a similar rule-based approach to reasoning as does MYCIN but the rules are now rules about the forms of rules and the use of rules. Figure 4 shows one of these **metarules.**

METARULE003:
If 1) there are rules which do not mention the current goal in their premise
   2) there are rules which mention the current goal in their premise
Then it is definite that the former should be done before the latter.

Fig. 4 A TEIRESIAS metarule

Mamdani, Procyk and Baaklini [33] found that learning could be introduced effectively in the steam engine controller through metarules that expressed the way in which the basic rules should be changed as a result of performance feedback. The learning level of their controller operator on rules of the form:

If time is small and error is negative big
Then desired change is big.
If time is big and error is positive zero
Then desired change is zero.

Fig. 5 Rules from fuzzy logic learning controller

Thus the expression of knowledge as inference rules generalizes from basic factual knowledge to that about the nature of knowledge itself and enables changes to be made in ES through discussion and learning. The present generation of ES has already gone beyond being a static recording of the knowledge of experts to being a technique for developing and extending expertise and acquiring knowledge.

Inducing Patterns in Information

The highest level of generality is shown in the rules of Lenat's AM [4] which searches for "interesting" conjectures in mathematics. What is remarkable about AM is the very notion of what it is doing. This is the highest level of scientific creation, looking for the patterns underlying knowledge. The concept of something being interesting seems peculiarly human and certainly too vague to form the basis of a computer program. However, AM is not only able to generate profound conjectures in mathematics but also meaningful ways of arriving at them. It is judged by the quality of its musings as it considers what it is doing.

Figure 6 shows two of the few hundred rules used by AM. Most of them are sufficiently general to apply to other situations. In an organizational context, for example, Rule 19 indicates that a committee which has decision-making powers that do not apply to its sub-committees below a certain size has

an interesting property (that of requiring a quorum). Its sub-committees also have an interesting property (that of being quorate).

19. If concept C possesses some very interesting property lacked by one of its specializations S,
Then both C and S become slightly more interesting.
38. If there are no known examples for the interesting concept X,
Then consider spending some time looking for such examples.

Fig. 6 Two rules of high generality from AM

AM has been criticized for some of the short-cuts taken in its implementation which cast doubts on the conclusions that may be drawn about the power and generality of the mechanisms used [48]. Lenat and Brown [30] have analysed its success in mathematics and failure elsewhere and suggest that AM had easy access to much of what it appeared to discover through the natural expression of certain mathemtical conjectures in Lisp. However, the program serves to illustrate that even very high level knowledge processing may be programmed using current techniques provided the space of information being processed is very small.

Imprecision in Expert Systems

It will be noted that the controller rules in Figs. 1 and 5 are imprecise, using terms such as "medium" and "big." Similarly the MYCIN rule of Fig. 2 notes that there is "suggestive evidence" for its inference rather than that it is true. Mamdani and Assilian were able to encode these imprecise rules and make inference from them using Zadeh's [60] fuzzy sets theory and its application to decision processes [61]. Shortliffe uses a similar method in MYCIN where the ".7" in the rule is a truth value in a multivalued logic, and Lenat uses such a measure of "interest" in AM. Our scientific training has taught us to regard imprecision as undesirable and it was not easy for the pioneers of expert systems to determine that it was a necessary component of any system adequate to encode human knowledge. However, a major weakness of past information processing systems, such as databases, has been that they are not able to cope with the vague expression of information in everyday life [17].

To see the significance of imprecise information consider a personnel database which keeps track of an employee's age. Current database systems require the age in years and a value "young" cannot be stored. Yet this information is adequate to exclude the employee in an enquiry such as "list all employees within two years of pensionable age." This is an example where we are imprecise because we lack information. It is still possible to argue that precision in itself is always a virtue. A reply to a request for a delivery date that says "soon" would be rated less satisfactory than one that says "in 3 days time"; the latter is more "businesslike."

However, there are other examples where the imprecision is intrinsic to the situation and being precise would falsely represent it. For example:

1) we will deliver 7 parcels each weighing 15.2 Kilograms at the rear entrance of building 6A on 15th February at 9.03 p.m.;
2) we will deliver some heavy equipment to your site Saturday evening;
3) see you with the goods over the weekend;

may each refer to the same event but are clearly not equivalent; each conveys an exact meaning that (presumably) properly represents what is to occur. If we prefer the precision of the first statement it is not for its own sake but because the tighter tolerances it implies on the actual situation allow us to plan ahead with greater accuracy and less use of resources. However, if the third statement really represents all that can be said it would be ridiculous to replace it with either of the previous ones. It would be equally ridiculous to say nothing, however, since even the least precise of the three statements does provide a basis for planning and action. A key aspect of executive action is planning under uncertainty and normal language provides a means for imprecision to be clearly and exactly expressed.

In retrieving information from a database the requirement for artificial precision is at least irritating and at worst highly misleading. For example the request:

a) list the young salesmen who have a good selling record for household goods in the north;

is perfectly comprehensible to a person. Translating it into:

b) list salesmen under 25 years old who have sold more than $20,000 of goods in the categories.... to shops in the regions....;

generates unnecessary work and makes no allowances for the whole spectrum of tradeoffs possible. It will not list the salesman of 26 who has made a real killing, or the one of 19 who sold $19,000, etc. The second request is more precise but the first represents far more accurately the actual **meaning** of the retrieval required.

Information and Knowledge Processing

The examples in this section illlustrate how expert systems may be seen as bridging the gap between information processing and knowledge processing. In the first four generations of computers we could encode only that part of human knowledge concerned with facts, arithmetic and precise logical deduction. This did not enable us to emulate a person's capability to reason with imprecise, possibly contradictory, information that requires knowledge of context, social norms, and the expectations of others, and the ability to use rules which are heuristic in that they are generally useful but may sometimes be wrong. Fifth generation systems using knowledge processing are providing facilities to emulate the capabilities of a skilled colleague and advisor. However, they are capable of operating within only small domains of information and their operation is heuristic without adequate formal foundations.

Knowledge and Psychology as FGCS Technologies

The dependence of FGCS developments on increased understanding of knowledge and human reasoning has been recognized by those concerned with the Japanese program and the question:

"Are you saying that the design of the fifth-generation may be modeled by learning more about the human thinking process?"

has been answered:

"Yes, we should have more research on human thinking processes, but we already have some basic structures. For more than 2000 years man has tried to find the basic operation of thinking and has established logic. The result is not necessarily sufficent; it's just the one that mankind found. At present we have only one solution - a system like predicate calculus. It is rather similar to the way man thinks. But we need more research. What, really, is going on in our brain? It's a rather difficult problem." [12]

This leads to a pragmatic approach to developing FGCS, basing them on the best available foundations for knowledge and psychology, even if these are inadequate. Technology is often in advance of science and the pressure to develop FGCS may be just what is needed to advance the sciences of knowledge and psychology. This approach leads to questions:

i) how adequate is predicate calculus as a science of knowledge?
iv) how adequate is logic as a foundation for psychology?

The Divergence of Psychology and Logic

A philosophical dictionary definition of "knowledge" is that it is "apprehended truth" and that "certain knowledge is more than opinion, less than truth" [49 p.161]. Knowledge stands between opinion which is in the domain of psychology (the study of mental phenomena) and truth which is the domain of logic (the study of truth preserving inferences); the underlying discipline is epistemology (the study of

truth ascription). It would be reasonable to suppose that psychology and logic would be closely related aspects of a science of knowledge. Until recent times this was so, but in the late nineteenth century psychology and logic diverged and an attempt was made to set up a system of logic that stood alone, independent of any processes of human reasoning. The divergence stemmed from attempts to establish formal foundations for mathematics, particularly those of Frege [10] and Husserl [23].

The stimulus for Frege's work was to re-establish irrevocably the position that mathematics was a priori (independent of experience) and analytic (based on inferences in which the conclusion is part of the premise), in contradiction to Mill's [37] position that arithmetic is not **a priori** but empirical and induced from experience, and Kant's [25] position that it is **a priori** but synthetic and made known through experience. Husserl's work shows the arguments most strongly because, in response to Frege's [11] critical review of his 1891 book, he became concerned with the role of psychology in logic and formulated the basis of pure logic clearly demarcated from processes of the mind [24]. In his discussion and the distinctions he makes Husserl generates a model of relationships between logic, psychology and knowledge which underlies the development of modern epistemology [38]. Similarly, Frege generated a notion of pure logical inference that underlies the development of modern formal logic [28].

These classical discussions may appear remote from modern computing and artificial intelligence (AI). However, Lenat and Brown's paper [30] on **Why AM and Eurisko appear to work** puts forward the Kantian argument exactly. The problem is why AM succeeded in the first place and why it is so difficult to use the same paradigm to discover new heuristics. Their conclusion is that AM's success in making conjectures in basic mathematics was because set- and number-theoretic results are simply stated in LISP. The conjectures were to a large extent available **a priori** but were **synthetic** because AM needed its empirical investigations to trigger off their discovery. However, while the metarules of logic and arithmetic are embedded in LISP, those about discovery heuristics are not and could not be thus triggered.

This analysis of AM not only demonstrates the Kantian argument but also suggests a line of attack that investigates the discovery heuristics as primitives in some operational calculus -- a formal epistemology. Many of the distinctions developed in philosophy in relation to reasoning and the operation of the mind have technological implications when applied to AI.

The Consequences of Anti-Psychologism in Logic

The anti-psychologistic basis of modern logic is natural in the light of the objective of formalizing arithmetic as a demonstration that it is **a priori** and synthetic. It rejects as irrelevant the questions:

1) How do we come to know arithmetic?
2) Why is arithmetic instantiated in real-world phenomena?

although a variety of answers to these questions are part of Frege's and Husserl's discussions. It also does not take into account the distinction between what is known and what might be inferred from this; Wittgenstein notes:

"If I know an object I also know all its possible occurences in states of affairs." [59 2.0123]

drawing attention to the gap between formal logic, where all consequences are "known", and human reasoning where they are not. It raises the questions:

3) What is the significance of not knowing all the logical consequences of a state of affairs?
4) How do we come to know particular consequences?

All of these questions are highly significant for knowledge-based systems yet outside the framework of standard formal logic. Questions 1 and 4 may be regarded as primarily within the domain of psychology and question 2 within the domain of philosophy. The significance of question 3 was not realized until the notion of **computational complexity** was developed and formalized. The super-exponential combinatorial explosion of any consequence derivation system creates a boundary that is real and extremely "hard"; the notion of a hard proof is not just psychological, a result of human frailty, but is systemic, a property of the problem for any derivation system.

The boundary created by computational complexity means that existence proofs relating to inference mean little in practical terms. It is not sufficient to know that solutions are eventually accessible; we need to know they can be reached using reasonable resources. The knowledge representation and inference schema used must make the types of inference required easy. This is an extra-logical requirement but a critical one in practice. Hardware parallelism and speed increase cannot be used to overcome the problem because super-exponential growth overwhelms even large multiplicative factors. A danger in developing FGCS is that any logic we build into the hardware may prove inappropriate to emulate human inference.

These considerations suggest that we should regard with some suspicion formal logics developed as foundations for mathematics and deliberately disregarding psychological considerations. It could be that mathematics epitomizes human intellectual achievement and standard predicate calculus (SPC) captures all that is significant. Even this is suspect since we know of many problems with SPC: in the foundations of set theory [35]; relevance of implication [1], ontology of existence [51]; and extension beyond first order [28].

However, while one may criticize SPC for its known faults, it is not easy to suggest an alternative system for knowledge representation and inference. There are many studies of human reasoning and alternative logics but there has not been the intensive study of a coherent system for knowledge in general as there has been for that sub-set which is mathematics. Such a study has its place in the development of FGCS, and in the remainder of this paper we survey some possible bases for such a system with strong logical and psychological foundations.

Psychological and Systemic Foundations

Kelly's **Personal Construct Psychology** (PCP) develops a complete theory of the normal and abnormal, which has strong systemic foundations [20] and has been operationalized through computer programs [53]. PCP models a person as using **personal constructs** as distinctions made about experience:

"transparent templets which he creates and then attempts to fit over the realities of which the world is composed"

and emphasizes the use of constructs for knowledge acquisition:

"Constructs are used for predictions of things to come, and the world keeps rolling on and revealing these predictions to be either correct or misleading. This fact provides a basis for the revision of constructs and, eventually, of whole construct systems." [26]

When Kelly developed his theory there was neither the mathematics to formalize it nor the computer technology to operationalize it. Despite this, PCP has developed to become a major psychological theory and to provide a variety of techniques used in clinical, educational and managerial psychology [54, 34]. PCP has strong links with Brown's logical system in **Laws of Form** based on the notion underlying personal constructs of **making a distinction:**

"The theme of this book is that a universe comes into being when a space is severed or taken apart...By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience

follow inexorably from the original act of severance." [2]

Logics Based on Unrestricted Predication

Kelly's psychology and Brown's logic both take as primitive an axiom of unrestricted predication. In set-theoretic terms they take **comprehension** [9] as **a priori** and derive all other concepts from this. This is not the standard and familiar approach, although logicians such as Fitch [8] have argued for it, and this makes it misleading to translate the Kelly/Brown discussions into logic and set theory; they are defining a basis of knowledge which is prior to both and from which logic and set theory must be derived. The key questions are: to what does a construct apply; what is it that is distinguished? We fall into a trap if we have any answer based on prior knowledge. The proper answer is that what is distinguished is defined only by being distinguished; it requires an auxiliary ontological postulate to reify this by assuming some entity is thus defined.

We have suggested [16] that what is distinguished is a **system** and that this notion defines a system. To distinguish some entity as being a system is a necessary and sufficient criterion for its being a system and this is uniquely true for systems. This definition may appear too weak in that it does not involve the expected properties of a system. The Oxford English Dictionary notes that a system is "a group, set or aggregate of things, natural or artificial, forming a connected or complex whole." However, these are properties that derive from our definition: a set of things is treated as distinct from another and it is this which gives it coherence; it is also this which increases its complexity by giving it one more characteristic than it had before, that it has now been distinguished.

A logic of knowledge systems

Kelly's personal construct psychology and Brown's calculus of distinctions both point to a knowledge representation system based on unrestricted predication as a primitive. The significance of such an axiom is that it forces us to develop system theories that do not run into the problems of this axiom in conventional mathematics. In set theory such predication leads to the inconsistency that Russell discovered in Frege's [10] **Foundations of Arithmetic,** and requires the axiom of comprehension to be severely restricted by a complex set of rules. However, it has long been speculated that the inconsistency may be better ascribed to the law of contradiction in standard logic: that the same predicate cannot be both true and false about the same proposition. Maydole [36] proved that most contenders for an alternative logic led to inconsistencies in set theory, and White [58] showed that Lukasiewicz infinitely valued logic gives a consistent set theory.

This was a particularly important result in bringing theory and practice together because it was just this logic that Zadeh [60] takes as a basis for fuzzy sets theory, Mamdani and Assilian [32] used in their expert system for linguistic control and Shortliffe [56] discusses as a formal basis for MYCIN. The links from Brown's calculus of distinctions to Lukasiewicz logic have been established: Varela [57] used the calculus to analyse self-referential systems using a construction based on Russell's paradox which gave a 3-valued logic; Gaines [14] used a recursive form of this construction to generate the infinite valued Lukasiewicz logic; and Gaines [18] shows that the Lukasiewicz axioms encompass modal, probability and fuzzy logics.

The significance of this construction is that the axioms are taken first as defining a non-truth-functional **standard uncertainty logic,** fuzzy logic is then derived by adding the constraint that the logic be truth functional, and probability logic is derived by the alternative constraint that the law of contradiction apply. Figure 7 shows the relationships between the various logics of uncertainty that may be derived from the Lukasiewicz axioms for a system of distinctions underlying the constructs representing a person's knowledge.

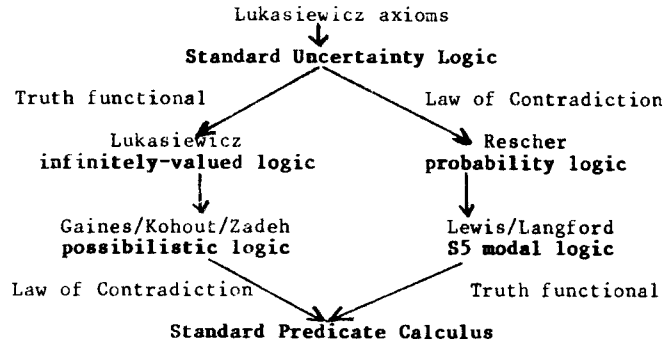

Fig. 7 Logics of uncertainty generated from calculus of distinctions

Imprecision and Non-Truth-Functionality

The need for a system adequate to represent knowledge to encompass both imprecision and non-truth-functionality is obvious in retrospect. Most of the distinctions we make in everyday life do not have hard boundaries. The notion of a red rose is totally unambiguous and acceptable in human communication and reasoning. However, the set of red roses is not a standard set. It has boundary cases which are neither precisely red nor not-red. Yet we can not exclude these borderline cases from being considered as red roses. The laws of contradiction and the excluded middle do not apply.

Non-truth-functionality arises because it is not possible to ascribe truth-values to the conjunction and disjunction of uncertain events without further information about their relationship. In probability theory this is well-known: if it is 50% certain you will phone today and 50% certain that I will be out of the office, what is the certainty that you will phone when I am out of the office? The answer can range from 0% if we have pre-arranged, to 50% if the two events are not causally connected, to 100% if I do not want to talk to you. It is possible to compute the constraints upon the result but not a single-valued result. Quinlan [44] has shown that the constraint systems of such non-truth-functional logics may be used as a basis for improved inference in expert systems with uncertain rules and data. Fuzzy logic can be seen as resulting from the requirement to avoid the calculation of such constraints by imposing truth-functionality. The common assumption of independence of events in probabilistic situations is an alternative way of forcing truth-functionality. Both these approaches are expedients whose utility depends on the problem domain.

An Epistemological Hierarchy

Distinctions are not just static partitions of experience. They may be operations: actions in psychological terms; processes in computational terms. This shows in Goguen's [21] category-theoretic formulation of fuzzy sets, Ralescu's [45] application of these to a general theory of modeling, and Goguen and Meseguer's [22] development of **OBJ,** a category-theoretic programming language. The role of distinctions at the base level of all models is evident in Klir's [27] hierarchy of modeling shown in Fig. 8. The loop from events through distinctions up through the modeling hierarchy and then down again to predictions and actions characterizes what Shaw [53] has termed the **personal scientist.** Note that the upper levels of modeling are totally dependent on the system of distinctions, or personal constructs, used to express experience through the source system. Klir developed this hierarchy for work on symbolic modeling systems and Gaines [15] has shown that it leads to general knowledge acquisition algorithms.

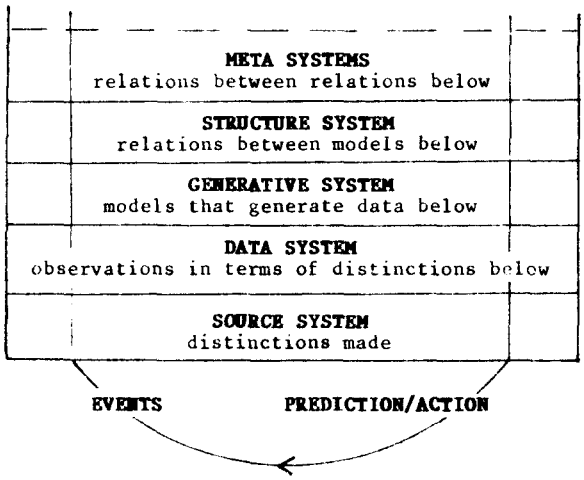

Fig. 8 Epistemological hierarchy of a personal scientist

This hierarchy does not introduce any additional primitives beyond that of making a distinction. The levels of the hierarchy are the results of distinctions that we make. Thus the source system is distinguished as those distinctions that the particular personal scientist makes; it is a distinction about distinctions defining the construct system of an individual. The data system is distinguished as those distinctions that have been made about a particular event; again a distinction about distinctions defining an event. The generative system is distinguished as a set of distinctions that also defines an event; these are model-generated rather than event-generated. It is the match between the model-generated and event-generated distinctions that determines the degree of approximation of the model to the world; this is a distinction about distinctions among distinctions that defines goodness of fit.

A modeling schema results from distinctions about distinctions at each level in the hierarchy. In prediction the key distinction is whether a level accounts for the information flowing through it and we may term this distinction one of **surprise** [15,52]. Surprise goes in opposition to the degree of membership of a predicted event to an actual event and the expected surprise is a form of entropy. If the degree of membership is m, then taking surprise to be 1-m leads to the expected surprise being a pre-Shannon entropy function, whereas taking it to be $-\log(_2)m$ leads to it being the familiar Shannon entropy [13]. Surprise at the lowest level of the hierarchy corresponds to distinctions being inadequate to capture events; surprise at the next level to inadequate variety to experience events; at the next to inadequate approximation to predict events; at the next to inadequate simplicity to explain events; and at the next to inadequate comprehensiveness to account for events. AM's "interest" [4] may be explicated as surprise generated when rules such as property inheritance are violated.

A Knowledge Hierarchy

The systemic hierarchy based on a calculus of distinctions has an analog based on personal constructs [20] shown in Fig. 9. The lowest level is one of **constructs**, distinctions made in interacting with the world. The next level is one of **experiences**, events which happen to us, and we make happen, in terms of the distinctions already made. Levels above these are **hypotheses** which are rationalizations of experience, **analogies** between these rationalizations and **transcendencies** which are preconceptions underlying rationality. Interaction with the world is, therefore, mediated through the construct system to produce experience which is modeled through the higher levels and leads to predictions, decisions and actions again mediated through the construct system.

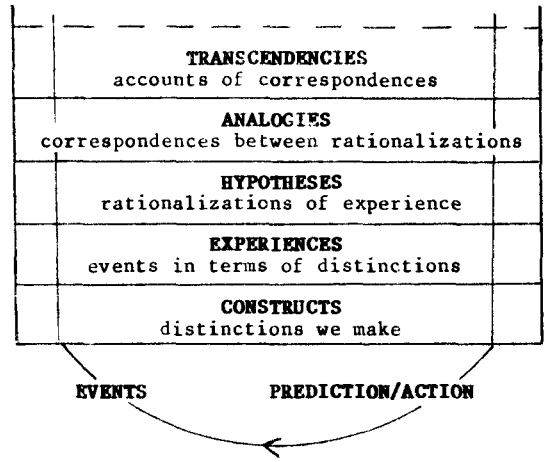

Fig. 9 Knowledge hierarchy of a personal scientist

Kelly [26] places the major emphasis of his work on the notion of **constructive alternativism**, that we have a choice in our construct systems at every level in the hierarchy and that real-world problems may often be solved by exercising this choice. Note that this should not be interpreted as an idealist position that ascribes all phenomena to our interpretation of them. Since the construct hierarchy also leads to decision and action, changes in it may equally affect the real world. Kelly and Brown are both neutral to any philosophical stance such as idealism versus realism; it is the distinctions which a philosopher makes that determines his stance and these can be analysed in terms of the epistemological and knowledge hierarchies. Kelly certainly saw his theory as reflexive and the only fundamental principle, apart from that of a construct itself, being that of constructive alternativism. This is important in its applications as a foundation for expert systems since one is attempting to capture the actual processes of the expert, not model them through some pre-conceived system that may not encompass them or distort them.

The hierarchy is strongly idiosynchratic and formalization does not imply uniformity. Constructive alternativism emphasizes the **dynamic** and **personal** nature of the construct system. It is subject to change according to feedback from failure to anticipate events. It is individualistic in the constructs used, in the vocabulary used to name the constructs, in the relations between constructs in the hierarchy, and in those constructs most likely to change when necessary. Knowledge engineering for expert systems may be seen as a process of making overt the construct system of the expert [55]. Problems arise because of the individualistic nature of construct systems. A person may be able to use his construction system whilst having no basis for communicating it to others. Two people may use exactly the same construct yet refer to it by different names. Two people may use the same names for a constructs and yet use it in different ways. Two people may use similar constructs at the lower level of the hierarchy and yet have them organized in different systems such that their reactions to the same event are quite different. Two people may have similar constructs at nearly all levels of the hierarchy and yet construe a novel event completely differently.

Logics of Knowledge Systems

A logical system is a means for making inferences that derives true statements (conclusions) from true statements (premises). Thus to define a logic we need to establish what are its statements, its truth criteria and its inference procedures. An axiomatically defined logic has a syntax stated to generate well-formed statements, axioms stated to be primitive truths, and inference rules stated to derive true conclusions from true premises.

Logics may be established within the hierarchy of distinctions/constructs by taking a statement to be a distinction made, truth to be a distinction about statements and inference rules to be a distinction about what may be inferred from a statement. The hierarchy is naturally recursive in that it results from making distinctions about the system whereby we acquire and process knowledge. Different forms of logic and different notions of truth and inference arise as we make different forms of basic distinction.

Distinctions About The Origins of Distinctions

Some very basic distinctions about the origins of distinctions at the highest level of the hierarchy lead to the main classifications of philosophical logic. If we make no distinctions about the origins of distinctions then truth is what is asserted to be true and inferences are legitimated by making them. This leads either to a anarchistic theory in which anything goes [7] or to a pragmatic theory of reasoning and a utilitarian theory of truth based on an assumption of rationality, that we distinguish something as true because it is useful to do so and reason in such a way that it works for our purposes.

Three distinctions about the origins of distinctions underly the deeper theories of reasoning and truth [47].

(1) If we distinguish those distinctions that are **necessary** in that they seem to impose themselves upon us then we generate what Popper [43] terms World 1, that of physical reality with a **correspondence** theory of truth and **causal** theory of inference.
(2) If we distinguish those distinctions that are made by **agents** in that they seem to be freely chosen then we generate what Popper terms World 2, that of subjective experience with a **performative** theory of truth and **conventionalist** theory of inference.
(3) If we distinguish those distinctions that are **independent** in that they seem to exist in their own right then we generate what Popper terms World 3, that of statements in themselves with a **coherence** theory of truth and **structuralist** theory of inference.

Conversation Theory

Figure 10 shows the way in which these distinctions about the origins of distinctions lead to notions of worlds, truth and inference. Considering entities as coherent fragments of the worlds and individuals as autonomous entities leads to the notions of Pask's [40] **conversation theory** which has been taken as a foundation for expert system development [3]. Pask's concepts of conversations and individuals are sufficiently widened for the theory to encompass all aspects of knowledge acquisition and processing. Conversations with nature model experience of the physical world and conversations with oneself model the restructuring of information at one level into knowledge at a higher level.

The definitions of entities and individuals can be formalized by noting that: an entity is a distinguished part of a world; a coherent entity is one that is relatively stable in that the relations of its parts one with another are more stable than with those outside it; an autonomous entity is one which is relatively uncontrollable in that the relations of its parts to those outside it are less stable than the relations of parts outside it with one another. Stability can be formalized in relation to pairs of distinctions in opposition except for unstable entities, and opposition can be formalized as pairs of distinctions distinguished in that they never both mark the same entity.

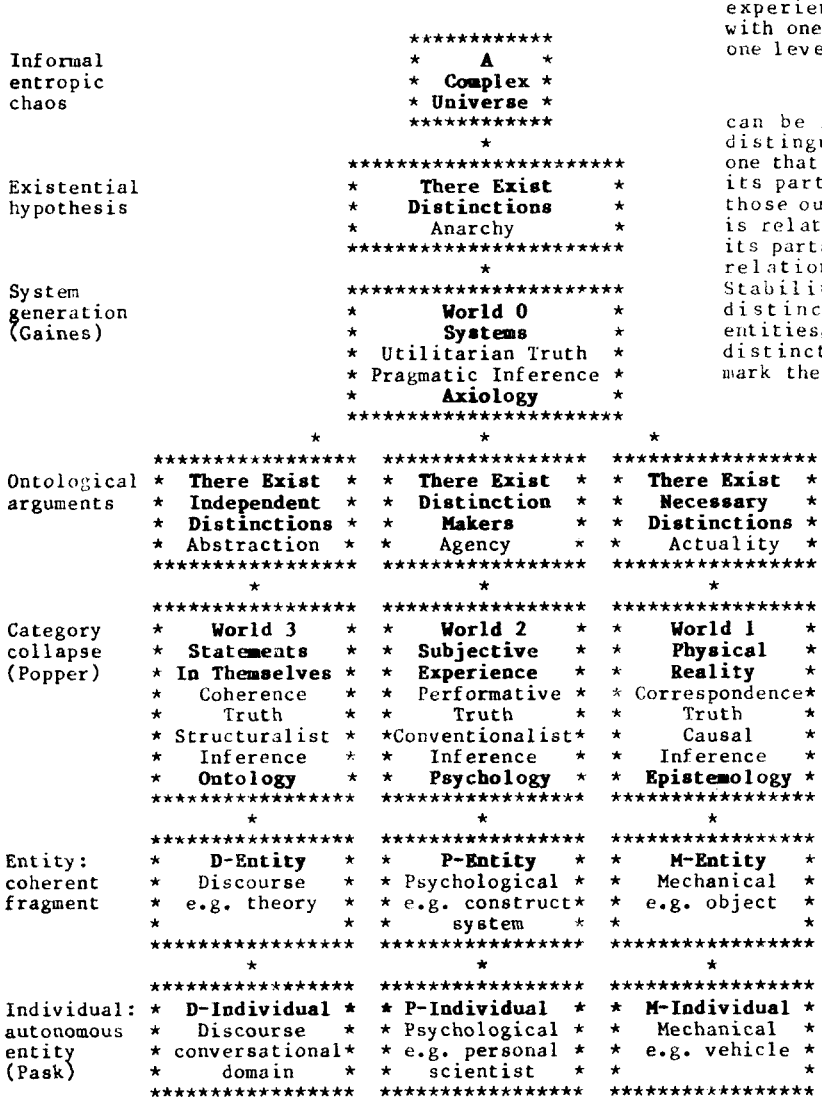

Fig. 10 Distinctions About The Origins of Distinctions Generating Worlds, Notions of Truth and Inference, Conversation Theory and Notions of Entities and Individuals

Conceptual Dependencies

These additional distinctions generate notions of time and the various relationships between objects, people and ideas that Schank has taken as a basis for conceptual dependency theory [50]. Each distinction defines a sub-world of space, time, social relations, and so on, that inherits systems of truth and inference from its parent world and adds further constraints to them. In this way various modal logics arise of spatiality, temporality, morality, and so on.

The basic axioms of inference will usually start with requirements that it is transitive, so that if A implies B and B implies C then A implies C. Other axioms such as A implies A, A implies that B implies A, and so on, will be added as the semantics of the situation requires. The Standard Uncertainty Logic described previously that results from a non-truth-functional reading of the axioms for Lukasiewicz infinitely-valued logic seems a suitable starting point for most situations since its semantics are extremely weak yet its inference power is adequate for many of the procedures used in expert systems [18].

Summary and Conclusions

This paper has argued that we are in a phase of discontinuity in the development of computing systems in which it is necessary to go back to fundamentals. We have developed information processing systems with the storage capacity to match the complexity of a substantial parts of the world and this technology is well-founded. We are beginning to develop knowledge-processing systems that can mimic people in coping with the complexity of the world but they can operate only in small information domains and the underlying knowledge technology is not well-founded.

The dependence of FGCS on knowledge processing and, coupled with the assumption that logic programming will be a key technology, suggests that we should examine the rationale behind standard logic. Modern formal logic was developed as a foundation for arithmetic and psychological considerations were ruled out as irrelevant. Many aspects of the debate about this decision are relevant to developments in artificial intelligence.

Non-standard logics and models of human knowledge processing are recent developments that lack the extensive, coherent literature of standard logic. However, it is important to consider, particularly in hardware design, that there may be systems of logic more appropriate to knowledge processing. An inappropriate system will not just be slower but may run up against computational complexity barriers that make useful inference impossible.

Logical and psychological system theories based on an axiom of unrestricted predication are plausible and attractive alternatives to standard logic. In conclusion, it is suggested that the architecture of logical inference processors should take such systems into account, and that further investigation of the logical, psychological and systemic foundations of knowledge should be a major component of the development of expert systems and future generation computing systems.

Acknowledgements

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