on the imaginary axis. This requires that

$$a = c\left(1+\frac{R_d}{R_e}\right)+dC_3R_d \quad \ldots\ldots\ldots \quad (16)$$

If eqn. 16 is satisfied, the circuit oscillates at a frequency given by

$$f_0 = \frac{1}{2\pi}\sqrt{\left\{\frac{d(1+R_d/R_e)-b}{cC_3R_d}\right\}} \quad \ldots\ldots \quad (17)$$

provided that

$$b < d\left(1+\frac{R_d}{R_e}\right) \quad \ldots\ldots\ldots \quad (18)$$

The circuit becomes an oscillator when $R_a = 9{\cdot}05\,\text{k}\Omega$, $R_b = 9{\cdot}10\,\text{k}\Omega$, $R_c = 9{\cdot}10\,\text{k}\Omega$, $R_d = 9{\cdot}24\,\text{k}\Omega$, $R_e = 22{\cdot}0\,\text{k}\Omega$, $R_1 = \infty$, $R_2 = 8{\cdot}95\,\text{k}\Omega$, $C_1 = 0{\cdot}0865\,\mu\text{F}$, $C_2 = 0{\cdot}0192\,\mu\text{F}$ and $C_3 = 0{\cdot}02\,\mu\text{F}$.

For these values of the components, b is negative, while d is always positive so that eqn. 18 is satisfied. Eqn. 16 is nearly satisfied. Eqn. 17 gives $f_0 = 1{\cdot}50\,\text{kHz}$, while the observed frequency of oscillation is 1·50 kHz.

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MEMORY MINIMISATION IN CONTROL WITH STOCHASTIC AUTOMATA

Indexing terms: Stochastic systems, Automata theory Minimisation of switching nets

Stochastic automata have been shown to require less states than deterministic automata in the solution of certain recognition and hypothesis-testing problems. This letter extends the result to a class of control problems involving the regulation of a discrete dynamical system.

One of the major properties of stochastic machines is that they offer the possibility of memory savings over equivalent deterministic machines. Rabin[7] (theorem 4) has shown that there are regular events accepted by a 2-state stochastic automaton which are only accepted by a deterministic automaton with an arbitrarily large number of states. Gaines[3] has shown that there are discriminations which cannot be learnt by an adaptive-threshold-logic element with discrete bounded weights, unless the weight changes are non-deterministic. Hellman and Cover[4] have shown that, for deterministic automata of arbitrary size, there exist hypothesis-testing problems which they cannot solve, but for which a 2-state stochastic automaton has an arbitrarily small error.

The present letter extends the class of problems for which stochastic behaviour is known to be advantageous by obtaining a similar result for certain abstract regulatory-control problems.

We shall consider an abstract formulation of the problem of regulating a discrete dynamical system to maintain its state within a prescribed region. Consider the finite automaton (I, P, S, σ, π), where I is a finite set of inputs, $P \equiv (0, 1)$ is a binary set of possible outputs, S is a finite set of states, $\sigma : S \times I \to S$ is the next-state function, and $\pi : S \to P$ is the output function. π is to be regarded as a performance function, since we shall consider the problem of regulating the inputs to the automaton to cause its output to become, and remain, 1.

Suppose that there is some distinguished element $\lambda \in I$ (the 'zero' input for the autonomous system), and consider the sets of states

$$W \equiv \{s: \pi(s) = 1\}$$

$$A \equiv \{s: \forall\, n \geqslant 0, \sigma(s, \lambda^n) \in W\}$$

$$B \equiv \{s: \exists\, n: \sigma(s, \lambda^n) \in A\}$$

W is the subset of S in which it is desired that the state should reside, A is a weak attractor within W, and B is its region of attraction.[2] We shall assume that both B and $S-B$ are nonempty, so that the autonomous system has a region of local asymptotic stability but is not asymptotically stable in the large, and consider the family of control automata, whose inputs are from P and whose outputs are in I, which induce global stability. For this family to exist, it is necessary that B be *reachable*[1] from S, i.e.

$$\forall\, s \in S, \exists\, u \in F_I : \sigma(s, u) \in B$$

where F_I is the free semigroup generated by I.

Consider, as a regulator for the dynamical system (plant), the 2-state stochastic automaton whose states are a_0 and a_1, whose inputs are from P, whose outputs are in I, whose state transitions are determined by the following table of probabilities that the next state will be a_1:

	Previous state		
Input		0	1
	a_0	0·5	0·5
	a_1	0·0	1·0

and whose output is λ in state a_1 and randomly chosen uniformly from I when in state a_0. If the automata are coupled, it is clear that the condition in which the state of the plant is within A and that of the stochastic regulator is a_1 is a stable one. However, there is a finite probability of attaining this condition from any other of the coupled automata, and hence the probability of the automata being in this condition tends to unity; we may say that the coupled system is stochastically asymptotically stable in the large.[5]

Consider now the regulator to be an arbitrary deterministic automaton with n states $(P, I, S_c, \sigma_c, \pi_c)$, with inputs from P, outputs in I, a state set S_c, a next-state function $\sigma_c: S_c \times P \to S_c$, and an output function $\pi_c: S_c \to I$. Suppose now that the plant is such that there is a 1 : 1 mapping $\eta: S_c \to S$, which is not onto in that S has $n+1$ elements so that there is one state of the plant, α say, which is not the image of any element of S_c. Suppose also that the next-state function of the plant is selected to be such that

$$\forall\, s_c \in S_c, \forall\, i \in I, \sigma\{\eta(s_c), i\} = \begin{cases} \eta\{\sigma_c(s_c, 0)\} & i = \pi_c(s_c) \\ \alpha & \text{otherwise} \end{cases}$$

$$\forall\, i \in I, \sigma(\alpha, i) = \alpha$$

The output function is selected to be such that

$$\forall\, s \in S \quad \pi(s) = \begin{cases} 1 & s = \alpha \\ 0 & \text{otherwise} \end{cases}$$

and that, if the initial state of the regulator is s_c, the initial state of the plant is chosen to be $\eta(s_c)$.

This type of plant will be termed a 'frustration automaton' for the deterministic regulator, in that it ensures that the plant output remains zero when coupled to the regulator, while also satisfying the reachability criterion for global stability to be possible with the stochastic regulator. Indeed the plant is such that *any* control policy other than that of the regulator in use ensures global stability!

Having established the existence of a frustration automaton for a regulator of known initial state and structure, it is immediately possible to generalise the result to show that an automaton exists which is a universal frustration automaton for all deterministic regulators with n states or less. The plant will have an initial transient in which its output is zero and in which it performs state and structure identification experiments.[6] These are trivial in that the input of the regulator (output of plant) is to be maintained zero for all time, and hence the behaviour of the regulator will be cyclic with period less than or equal to n. The plant will then assume the form of a frustration automaton for the particular regulator which it has identified.

An explicit construction of such a universal frustration automaton for regulators with n or less states is as follows. Let the plant be connected to the regulator at time $t = 0$, and let the output of the regulator at time $t = i$ be $R(i)$, while the output of the plant is $P(i)$ $(i = 1, 2, 3, \ldots)$. Let the output of the plant be defined to be

$$P(i) = \begin{cases} 0 & 1 \leqslant i \leqslant 2n \\ P(i-1) & i > 2n \\ 1 & \text{otherwise} \end{cases} \quad \text{and} \quad R(i) = R(m-n+i)$$

where $m \geqslant 0$ is defined to be the minimum integer such that

$$R(n+i) = R(m+i) \qquad (1 \leqslant i \leqslant n)$$

Such an m must exist in the range $0 \leqslant m < n$, since the regulator is driven by a constant input of 0 and hence its output must become periodic after a time of at most n.

This plant can be realised as a finite-state deterministic machine (it need only remember the last $2n$ outputs of the regulator), and its overall behaviour is to identify the regulator over a period of $2n$ and then switch its output to 1 if and only if the output of the regulator once deviates from its identified cycle. In these terms, the success of the stochastic regulator in dealing with the same plant may be seen to be due to the inherent acyclicity, or unpredictability, of its output.

Thus we have the result that there exists, for any n, a discrete dynamical system which cannot be regulated by any deterministic automaton with n states or less, whereas it can be (stochastically) regulated by a stochastic automaton with two states.

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CONTACT EFFECTS IN GUNN DIODES

Indexing terms: Gunn diodes, Contacts

This letter shows how the splitting of the forward and reverse current/voltage curves of a Gunn diode may be interpreted in terms of different models of contact damage, and proposes a simple test technique to obtain a quantitative measure of contact quality. Experimental verification of this technique is presented, showing a correlation between device efficiency and a contact parameter β.

This letter shows how some of the pathological features commonly observed in the subthreshold current/voltage (I/V) characteristics of a Gunn diode may be interpreted.

Fig. 1 shows the experimental I/V characteristics of two representative X band Gunn diodes with alloyed-metal cathode contacts. These curves were measured using short ($\sim$200 ns) pulses to avoid thermal effects. It is immediately evident from this Figure that, although the Gunn diode is nominally a symmetrical device, the forward (substrate positive) and reverse (substrate negative) directions are far from equivalent in practice. This well known anomaly takes two forms. Type-A curves show positive splitting ($I_F < I_R$) at all voltages, while the much less common type-B curves show positive splitting only above some crossover voltage, the curves being split negatively below this point.

It has been suggested by Hasty and coworkers[1] that the behaviour designated type A is due to a region of reduced donor density associated with an alloyed-metal contact. However, it will shortly be shown that this 'donor notch' model of an imperfect contact in fact gives rise to curves which are clearly of type B. One must therefore seek some other explanation for type-A behaviour.

The alloying of a metal contact is likely to produce considerable crystallographic damage in that region of the device close to the metallised layer, and the most important effect of this damage will be a considerable increase in the number of scattering centres in the affected region. This will certainly produce a local decrease in the carrier low-field mobility, and the work of Ruch and Fawcett[2] suggests that it is reasonable to argue that the overall velocity/field curve will also be locally degraded.

This situation can readily be treated by a 1-dimensional model closely related to the Hasty 'donor notch' model. The diode is regarded as having a completely flat donor-concentration profile with two ohmic contacts. Associated with one of these contacts, but lying at least partly in the active layer, is a region about 1 μm wide in which the carrier low-field mobility, and hence the local effective velocity/field curve, are degraded by a specified amount. In this model, these two effects are related in what seems a qualitatively reasonable way by using the Thim[3] approximation to the static velocity/field curve:

$$V(E) = \{\mu_0 E + V_s(E/E_0)^4\}/\{1 + (E/E_0)^4\}$$

It has been assumed in what follows that the saturation velocity V_s is independent of μ_0. However, this is not a critical feature of the model, since any reduction of saturation velocity accompanying a reduction in μ_0 would tend to accentuate the effects to be described.

Fig. 2 shows a typical pair of I/V curves calculated from the model just described, compared with a typical pair of curves calculated using the Hasty 'donor notch' model. Both these pairs of curves were calculated numerically using a modified version of a program used to simulate the behaviour of Gunn devices in microwave circuits, although in this case the calculation was terminated when a time-independent field distribution could no longer be found. It is immediately clear that the two models predict pairs of curves which are qualitatively quite different. The *donor-notch model* predicts curves which split negatively at low voltages and show the crossover which is the trademark of type-B behaviour, while the *mobility-notch model* predicts the much more common type-A curves, which show only positive splitting and have no crossover.

It is easy to see qualitatively that the difference between the two models is systematic, and arises because positive splitting occurs when a depletion layer forms at the downstream edge of the notch when the device is forward biased. This happens only when the carriers in the notch travel slower than those in the bulk of the sample, a condition which is fulfilled at all fields in the mobility-notch model, but which only holds for notch fields above threshold in the donor-notch model.

The success of the mobility-notch model in explaining the most commonly observed form of I/V-curve anomaly suggests