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EXPLORING PERSONAL SEMANTIC SPACE

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This paper presents an overview and summary of an ongoing research programme concerned with the development of man-computer symbiotic systems for the amplification of human intelligence, creativity and similar capabilities necessary to cope with complex worlds of knowledge, decision and action. Our overall objective is to provide both formal and technological foundations for the role of man as a personal scientist, and for a group of people as a commune of science, exploring a world, forming hypotheses, using them as a basis for actions and changing them as a result of feedback from the consequences of those actions. To achieve this objective involves both practical work on actual man-computer systems applied to real problems and foundational work on the logics and systems theories underlying the acquisition of knowledge. In this paper we present salient aspects of both the theory and practice and discuss their inter-relationships with particular emphasis on the foundational and empirical semantics of the search for knowledge.

1. THE APPROACH

Three main themes are developed in this paper. Firstly, that man-computer interaction in a conversational, dialectical mode offers new opportunities for the empirical exploration of semantic domains. Secondly, that the semantic structures underlying both personal constructs (KELLY 1955) and semantic differentials (OS-

GOOD, SUCI & TANNENBAUM 1957) may be treated as logical spaces of predicates applying to entities and analysed in terms of the entailment relations between these predicates. Thirdly, that the process of communication between individuals given the personal, conventional nature of these predicates depends on the normative influence of world 3 domains (POPPER 1968) in forcing the formation of P-individuals (PASK 1975) with related construct systems.

1.1. PRESUPPOSITIONS ABOUT THE ROLE OF LANGUAGE

To avoid misconceptions about terminology it is worth emphasizing at this stage that we are using the term semantics in a rather broader sense than 'linguistic semantics'. There is often a preconception that the search for meaning is essentially linguistic and that the nature of meaning can be comprehended fully through the study of language and linguistic acts. The relation between language and human thought processes is itself a topic of major interest to us and we would not want to presuppose an essential link. Our bias is to presuppose otherwise and assume that action is primitive and that a person rationalizes the basis for action as being knowledge and the process of determining action as decision. The linguistic expression of all these phenomena is an explanatory rationalization of them that is itself significant as an act that can modify future related phenomena and their interpretation. The linguistic rationalization of an action is not the full basis for it and neither is the linguistic expression of knowledge a full representation of it.

These are intended to be commonsense assertions rather than deep theoretical distinctions. We make them at this stage because both our theory and our technology involves the use of linguistic labels and it would be easy to regard them as central to the topic of discussion. However, this would obscure a number of key questions that are of interest to us:

1.1.1. What are the underlying (pre-linguistic) processes of human action?

1.1.2. How can we access these processes within the individual?

1.1.3. How do different individuals come to have a common basis for interaction (competetive, collaborative, or communicative)?

1.1.4. What are the natures of the logics underlying human action?

1.1.5. What are the bases of rationality in human action?

1.1.6. What are the roles of language in human action?

These are only some of the questions of interest but they are sufficient to show the level of presupposition at which we wish to operate and also the extreme pluralism of the points of view that we are taking. Presuppositions of existence are often apparently weak whilst being extraordinarily strong; if we presuppose that something exists then we may well find it. The presupposition that there exists an underlying rationality for the behaviour of even one person in one domain of activity over a limited period is strong indeed. To presuppose also that this is unique and applies in other circumstances and to other people is an extraordinarily strong postulate. It seems to us that it is reasonable if we find a common rationality to ask how it can possibly have arisen - what is the common experience, the normative force, that has given rise to such a phenomenon?

1.2. PERSONAL SEMANTIC SPACE

Thus our prime initial objective in empirical studies has been to explore the personal semantic space of the individual and to do so with the minimum injection of structure from the process of exploration. In this work we have been very much guided

by the epistemological theories of the clinical psychologist George Kelly. These are expressed in his seminal work The Psychology of Personal Constructs (KELLY 1955) which provides a remarkably far-reaching and well-structured foundation for the study of human action. His work is anchored very firmly both in its close correspondence to the actual behaviour of people and in its coherent and consistent philosophy. This is not to say that Kelly fully worked out a logically, philosophically and psychologically complete model of knowledge acquisition and action. His attempts to link his work to other philosophical studies of epistemology, his attempt to present it axiomatically, and his embodiment of it as an empirical tool through the repertory grid, are all sketches which neither the mathematics nor the computer technology of his time were able to sustain.

In the quarter century since the publication of Kelly's book there have been many developments that relate to it and form a basis for a fresh impetus during the next twenty five years. In philosophy the balance has been struck between the extremely personal epistemologies of the existentialists and the extremely impersonal epistemologies of the logical positivists. Attempts, such as those of AYER (1936), to define precisely the one acceptable method of legitimating belief have died down, and Kelly's constructive alternativism has become fashionable in conventionalist and pluralist positions such as those of GELLNER (1974) and FEYERABEND (1975). In Kelly's work we can find incorporated both KUHN's (1962) emphasis on the importance of the paradigm and hence the possibility of revolutionary changes in viewpoint, and POPPER's (1972) emphasis on falsifiability as the prime test of meaningful belief. Modern philosophy has swung the focus of attention from science to the scientist, a viewpoint which makes Kelly's work now appear central to the key issues.

Positivist science advanced as rapidly as it did because of its very close links with formal logic. Kelly himself was probably influenced by this in his attempt to present his own position axiomatically as a "fundamental postulate" and a set of eleven "corollaries". However, the possibility of forming logical foundations for his theories, let alone axiomatizing them, was not

within the realms of the mathematics open to him at the time. His founding of his epistemology on the process of making a bipolar distinction is a precursor to BROWN's (1969) fundamental reformulation of propositional logic in his "calculus of distinctions". His concept of a construct applied to elements and having a range of convenience requires a modal logic incorporating notions of relevance, and the theory underlying these was only formalized during the mid 1960's (SNYDER 1971, ANDERSON & BELNAP 1975). The formalization of modal logic has been very fruitful in establishing semantic foundations for natural language (CRESSWELL 1973), and its basis in the concept of possible worlds (LEWIS 1973, BRADLEY & SWARTZ 1979) seems very close to the model that Kelly needed for the dynamics of construct formation and modification. A related development in recent years has been that of multivalued logical foundations for set theory such as ZADEH's (1976) fuzzy logic, and the application of this also to modelling human semantic processes has much in common with Kelly's approach.

The development of Kelly's work to date has tended to neglect the epistemological foundations and concentrate upon the clinical applications (SLATER 1976) and its integration into the mainstream of work on personality (BANNISTER & FRANSELLA 1971, HOGAN 1976). Because of the experimental nature of much of this work the analysis of Kelly's repertory grids through computer programs has itself become a significant line of development (SLATER 1977, SHAW 1980). The on-line application of computers to operationalize Kelly's construct theory and to reflect to an individual his role as a personal scientist adds a new dimension to the work. We can see the beginnings of the man-machine symbiosis (LICKLIDER 1960) promised in the early days of computing, in which the logical processing power of the computer is used to complement the creative imagination of the person.

1.3. THE STRUCTURE OF THIS PAPER

Section 2 of this paper outlines Kelly's personal construct theory and gives his system of postulate and corollaries together with brief explanations. It continues with a description of his repertory grid technique for exploring a person's construct system, and then goes on to describe the operation of a computer program, PEGASUS, for the interactive elicitation and analysis of repertory grids and construct systems. There follows an outline of further programs, PEGASUS-BANK and SOCIOGRIDS, which can deal with the construct systems of more than one person. The section finishes with the discussion of the importance of dialectical interaction with computers in the elicitation of construct systems.

Section 3 is concerned primarily with the logical foundations of the repertory grid and personal construct theory. It develops a logical model of constructs as predicates applying to elements and shows how a repertory grid may be analysed in terms of entailment relations. This is compared and contrasted with conventional, distance-based methods of grid analysis through examples of the different techniques applied to a particular grid. The programs ENTAIL (logical entailment analysis), INGRID (principal components analysis), QARMS (Q-analysis), and FOCUS (clustering), are all compared for their differing analyses of semantic space. Extensions of the logical approach to fuzzy semantics, the construct hierarchy, opposites and relevance are also developed.

Section 4 is concerned with the interplay between the conventionalist epistemology of personal construct theory and the notion of socially available real worlds. It puts forward a model in which a P-individual within a particular person uses a particular construct system by which to surrender himself to a particular third world in order to catch its essence. It also shows how essential conflicts are resolved through the formation of multiple P-individuals in the same person.

2. PERSONAL CONSTRUCT THEORY AND PRACTICE

Kelly put forward the idea of an individual using "personal constructs" as filters through which he perceives events:

> "Man looks at his world through transparent templets which he creates and then attempts to fit over the realities of which the world is composed." (Kelly 1955, pp. 8-9)

He continually emphasizes the epistemological status of these constructs in predicting and controlling the world and their ontological status as personal conjectures rather than reality-derived absolutes:

> "Constructs are used for predictions of things to come, and the world keeps on rolling on and revealing these predictions to be either correct or misleading. This fact provides the basis for the revision of constructs and, eventually, of whole construct systems." (Kelly 1955, p. 14)

When it came to the formal and practical representation of constructs Kelly took them to be bipolar in nature such that each event construed was classified relative to the two poles of each construct. In essence Kelly placed the same fundamental emphasis as did Spencer Brown in his seminal work, Laws of Form, on the human, creative operation of "making a distinction":

> "The theme of this book is that a universe comes into being when a space is severed or taken apart... By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance." (Brown 1969, p.V)

2.1. KELLY'S POSTULATE AND COROLLARIES

Kelly (1955 Chapter 2) develops his theory through a set of specific assertions which are worth considering in some detail.

2.1.0. *"Fundamental Postulate:* A person's processes are psychologically channelized by the way in which he anticipates events."
This fundamental emphasis on the role in behaviour of a view to the future is what distinguishes Kelly's particular approach to psychology. He saw man as driven by the need to cope with coming events in the world and all other aspects of behaviour as deriving from this. He notes in expanding upon this point that "a person's processes, psychologically speaking, slip into the grooves which are cut out by the mechanisms he adopts for realizing his objectives." (p. 49)

2.1.1. *"Construction Corollary:* A person anticipates events by construing their replications."
He notes that "By construing we mean placing an interpretation: a person places an interpretation on what is construed. He erects a structure, within the framework of which the substance takes shape or assumes meaning." (p. 50) He emphasizes that "Construing is not to be confounded with verbal formulation." (p. 51)

2.1.2. *"Individuality Corollary:* Persons differ from each other in their constructions of events."
This is Kelly's stress upon the personal nature of construing but he emphasizes also "persons can find common ground through construing the experiences of their neighbours along with their own", although "individuals can be found living out their existence next door to each other but in altogether different subjective worlds." (p. 56)

2.1.3. *"Organization Corollary:* Each person characteristically evolves, for his convenience of anticipating events, a construction system embracing ordinal relationships between constructs."
Kelly recognized that "Different constructs sometimes lead to incompatible predictions" (p. 56) and saw a hierarchical relationship that ordered constructs relative to one another

as a basis for resolving such contradictions. He saw the hierarchy as one in which the stability of the upper levels allowed for change in the lower ones and that the arrangement of constructs in a particular hierarchy was more a determinant of personality than the constructs themselves.

2.1.4. *"Dichotomy Corollary:* A person's construction system is composed of a finite number of dichotomous constructs." Kelly saw concepts of similarity and contrast between elements forming the two "poles" of a construct. This does not necessarily involve a binary evaluation of elements since they may lie somewhere between the poles.

2.1.5. *"Choice Corollary:* A person chooses for himself that alternative in a dichotomized construct through which he anticipates the greater possibility for extension and definition of his system."
This is Kelly's basis for human action which he saw as an alternative to concepts of motivation - "there is a continuing movement toward the anticipation of events, rather than a series of barters for temporal satisfactions."(p. 68)

2.1.6. *"Range Corollary:* A construct is convenient for the anticipation of a finite range of events only."
This may be thought of as introducing a concept of relevance into the definition of construct systems.

2.1.7. *"Experience Corollary:* A person's construction system varies as he successively construes the replications of events." This expresses the essential fallibility and conjectural nature of constructs. They are hypotheses to be judged by their utility.

2.1.8. *"Modulation Corollary:* The variation in a person's construction system is limited by the permeability of the constructs within whose ranges of convenience the variants lie."

Kelly sees a "permeable" construct as one "so constituted that new experience and new events can be discriminatively added to those which it already embraces." (p. 81)

2.1.9. "Fragmentation Corollary: A person may successively employ a variety of construction subsystems which are inferentially incompatible with each other."
Through this corollary Kelly allows for the operations of different psychological systems and hence different methods of inference and forms of rationality within a single individual.

2.1.10. "Commonality Corollary: To the extent that one person embraces a construction of experience which is similar to that employed by another, his psychological processes are similar to those of the other person."
This is complementary to the individuality corollary and allows for the "fact that certain groups of people behave similarly in certain respects." (p. 93)

2.1.11. "Sociality Corollary: To the extent that one person construes the construction processes of another, he may play a role in a social process involving the other person."
Kelly is here allowing the construal processes of one person to become elements for construal by another and hence to model the process by which we anticipate the actions of others.

2.2. THE REPERTORY GRID

We have already noted that Kelly saw construing as not necessarily involving linguistic labels:

> "A person may construe his experience with little recourse to words, as, for example, in certain conditioned reflexes. Even those constructions which are symbolized by words are not necessarily similar just because the words are similar. Conversely, two persons may be using essentially the same construction of their experience, although they express themselves in quite different terms." (Kelly 1955, p. 92)

In this repertory grid technique Kelly attempted to get people to make their construct usage explicit by defining distinctions between elements, i.e. by assigning each element under consideration to one pole or the other of a construct. This usually involves the poles in being labelled in some way but these are treated as memory aids to the person whose 'meaning' is defined by the way in which they are used rather than by the actual labels themselves.

Figure 1 is a repertory grid from SHAW (1980, p. 79) showing Jane's allocation of twelve acquaintances to the poles of eight constructs. It is a particularly good illustrative example because Jane has labelled the poles with an entire constellation of explanation rather than a single word. It can be seen that the single word she picks out (underlined) is not necessarily one that one would agree with in paraphrasing the whole. The fact that she has given far more background explanation to the poles of her constructs than is usually available also makes it easier to assess the prima facie meaningfulness of any analysis of her grid. The only difference between Figure 1 and Figure 6.4 in the book is that Shaw uses the letter "X" for the assignment to the left-hand pole and the letter "O" for the assignment to the right-hand pole, whereas we have used the numbers "1" and "0" respectively. This change to numerals is deliberate because we wish to examine how the values in the grid may be viewed in two ways: as numerical values; and as logical values.

KELLY (1955) presented constructs as binary categories and based his own methodology for eliciting constructs on this. However, other workers found the need for 'shades of grey' between the two poles of a construct and in a later work Kelly notes that this is consistent with his notion of a construct:

> "The construct, of itself, is the kind of contrast one perceives... while constructs do not represent or symbolize events, they do enable us to cope with events, which is a statement of a quite different order... They also enable us to put events into arrays or scales, if we wish." (Kelly 1970, pp. 13-14)

It is common in many practical applications of repertory grids to use an N-point scale with 1 being an assignment to the left-hand pole and N being an assignment to the right-hand pole, and

			ELEMENTS											
	I	O	1	2	3	4	5	6	7	8	9	10	11	12
C1	Intensity. They both are interested in other people. Concerned with world problems. Ambitious. Slightly detached.	Humorous. Creative. Unconventional approach to work & relationships. Exciteable.	1	1	0	0	1	0	0	1	0	1	0	0
C2	Individualistic. Musical. Calm(exteriorally). Unconventional. Non-aggressive. Loyal. Interested in myth & fantasy. Homely. Land-loving. Tending toward introversion. Unusual humour.	Self aware. Controlled. Sporting. Experienced in relationships. Attracted to sophistication & the exotic. Extroverted. Light hearted.	0	0	0	0	1	1	0	0	0	0	1	0
C3	Generous. Interested in history. Slow living. Perfectionist in work. Unusual relationships.	Direct. Political. Super active. Strong integrity. Committed.	0	0	0	0	1	1	1	0	1	0	1	0
C4	Ambitious. Questioning. Quick minds. Confident. Interested in "societies ills."	Artistic. Capable. Gentle. Romantic. Exploratory.	1	1	1	1	0	0	0	1	0	1	0	1
C5	Outdoor enthusiasts. Anxious to succeed. Anxious about success with other sex. Active. Enigmatic. Need mental stimulation.	Creative. Enjoys comfort. Relaxed.	1	1	1	1	0	0	0	1	0	0	0	1
C6	Enjoy intellectual discussion. Difficult to understand initially. City livers. Seek challenges. Insecure backgrounds.	Affectionate. Humble. Sensitive. Musical. Involved with those immediately around. Compassionate. Philosophical.	0	1	1	1	0	0	1	1	1	1	0	1
C7	Energetic. Sociable. Politically concerned interests. Dynamic. Restless. Factual approach as opposed to interest in fantasy world.	Thorough. Care for detail. Extremely creative. Not concerned with social success. Gentle. Perceptive.	1	1	1	1	0	0	0	1	1	0	0	1
C8	Both need company. Gregarious. Prepared to compromise. Factual approach. Enjoy discussion.	Musical. Scientific but also keen on the "unreal" world. Fantastical.	1	1	1	1	0	0	0	1	1	1	0	1

Fig. 1. Jane's Repertory Grid

intermediate numbers representing some form of 'intermediate' assignment. N is usually odd, 5 or 7, to allow a 'neutral' mid-point to the scale. In section 2.3 we give an example of computer elicitation and analysis of a grid using a 5-point scale.

There is an obvious resemblance between Kelly's repertory grid and the semantic differential methodology of OSGOOD, SUCI and TANNENBAUM (1957). Both techniques eventually produce a matrix of assignments of elements assigned to bipolar rating scales, and both techniques have been applied to similar areas of study (SLATER 1976, SHAW 1980, SNIDER & OSGOOD 1969). The motivation of the work on semantic differentials has been to provide a psychological theory of meaning in which individual words can be seen to be compounded from relatively few components in a basic semantic space. In itself this is not inconsistent with Kelly's theory but it tends to neglect two features of semantic space that he regarded as epistemologically important: firstly, the individual nature of semantic space; and, secondly, the hierarchical structure of semantic space. In terms of the practical methodologies this difference in emphasis shows up in the use of elicitation techniques for both constructs and construct hierarchies in work based on Kelly's viewpoint, whereas that deriving from the semantic differential uses consensual constructs over a population. Much of the technical content of this paper is applicable to both approaches and we shall also be concerned with how the gap between the individual and the community is bridged.

2.3. PEGASUS: A PROGRAM FOR THE EXPLORATION OF PERSONAL SEMANTIC SPACE

PEGASUS (Program Elicits Grids And Sorts Using Similarities) is an interactive computer program which elicits a grid from an individual, simultaneously acting as a psychological reflector by heightening his awareness and deepening his understanding of himself and his processes. This is done by continual real-time

feedback commentary on highly related elements or constructs, together with the encouragement to differentiate between them.

Before choosing his elements the user is asked to think about his purpose for eliciting the grid. This is of great importance for the interaction which is to follow, since the elements are dependent on the purpose and the constructs are dependent on the elements. By using combinations of reviewing the purpose and adding and deleting constructs and elements, a depth of interaction can be achieved which could not at first be envisaged. Thus the user is given the opportunity to reflect on his understanding of the topic area to examine and explore his thoughts and feelings in this atmosphere of heightened awareness of personal knowing. His perception may be changed in a way which by other means can take years to accomplish.

In PEGASUS the computer is acting as a cognitive mirror, reflecting back to the user his models of construing. The essence of learning is constructive and creative change. For the learner himself, learning is the revision of his cognitive model in order to make his anticipation of events more effective, that is in the way he perceives and construes events and behaves in the situation. PEGASUS actively encourages the consideration and revision of tentative hypotheses of the personal scientist approach, hence supporting the reconstruction of cognitive models and the change which is the 'seeing' and learning of constructive alternativism.

The program is divided into six main sections and these are illustrated here through an example taken from SHAW (1980). The first section is the Basic Grid in which explanation of grids and the use of the terminal are given, and the first four constructs elicited. The conventional method is used to elicit the first few constructs, that is to present a triad of elements to be divided into a pair and a singleton indicating the poles of the construct. After the user has named the poles the computer assigns a 1 to the pair and a 5 to the singleton, and he has then to assign ratings from 1 to 5 to each of the other elements. These are then retyped in groups to highlight the relative position of each element with respect to the others, and an option to change the ratings is given.

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THIS IS A PROGRAM TO ELICIT A KELLY REPERTORY GRID.
PLEASE READ CAREFULLY EVERYTHING THAT IS PRINTED AND
MAKE SURE YOU UNDERSTAND WHAT YOU HAVE TO DO.
A REPERTORY GRID IS A TECHNIQUE DEVISED BY KELLY TO
HELP YOU EXPLORE THE DIMENSIONS OF YOUR THINKING.


YOU MUST DECIDE ON A PURPOSE FOR DOING THE GRID AND
KEEP THIS IN MIND WHEN YOU CHOOSE THE ELEMENTS--THE
THINGS YOU ARE GOING TO THINK ABOUT DURING THE PROGRAM.
THESE ELEMENTS WILL THEN BE USED TO ELICIT CONSTRUCTS.


YOU ARE LIMITED TO 25 LETTERS AND SPACES FOR YOUR ELEMENT
AND CONSTRUCT NAMES.
IF YOU MAKE A TYPING ERROR PRESS THE DELETE (OR RUBOUT) KEY AS MANY
TIMES AS YOU WANT TO ERASE A CHARACTER THEN CARRY ON.
THROUGHOUT THIS PROGRAM THE QUESTION WILL BE ASKED --
DO YOU NEED HELP. EACH TIME JUST TYPE  YES IF YOU DO AND
PRESS THE RETURN KEY
BEFORE YOU START THIS GRID WHAT IS YOUR NAME OR IDENTIFICATION

 ?ARTHUR

THIS PROGRAM INCORPORATES FOUR VERSIONS OF PEGASUS.
 1. A PEGASUS GRID ELICITATION STARTING A NEW GRID;
 2. A STRAIGHT KELLY REPERTORY GRID ELICITATION
    WITHOUT COMMENTARY (MIN-PEG);
 3. A PEGASUS GRID ELICITATION USING A STORED BANK
    OF CONSTRUCTS (PEG-BANK);
 4. A PEGASUS GRID ELICITATION WITH PART ALREADY
    ELICITED BY YOU RECENTLY (PRE-PEG).
WHAT IS THE NUMBER OF THE VERSION YOU WISH TO USE ?1


TYPE IN ON ONE LINE YOUR PURPOSE FOR DOING THIS GRID


 ?EXPLORING LEARNING SITUATIONS


NAME SIX ELEMENTS.
YOU MUST CHOOSE A SET OF SIX ELEMENTS KEEPING IN MIND
WHY YOU WANT TO DO THIS GRID. THEY COULD BE PEOPLE, EVENTS,
PIECES OF MUSIC, PICTURES, BOOKS, OR WHAT YOU WANT BUT
WHATEVER YOU CHOOSE THEY MUST BE OF THE SAME TYPE AND
EACH MUST BE WELL KNOWN TO YOU. TRY TO CHOOSE SPECIFIC
THINGS. NOW TYPE EACH ONE AFTER EACH QUESTION MARK.
DO NOT FORGET TO PRESS THE RETURN KEY AFTER EACH.


ELEMENT  1  ?LECTURE
ELEMENT  2  ?TUTORIAL
ELEMENT  3  ?SEMINAR
ELEMENT  4  ?PRACTICAL
ELEMENT  5  ?FILM
ELEMENT  6  ?LIBRARY
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Fig. 2(a). PEGASUS Interaction: Introductory Section

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TRIAD FOR ELICITATION OF CONSTRUCT 2
 4 PRACTICAL
 5 FILM
 6 LIBRARY


NAME THE PAIR


DO YOU NEED HELP ?NO


 ?4

 ?6


NAME THE POLES OF YOUR CONSTRUCT


DO YOU NEED HELP ?YES


NOW I WANT YOU TO THINK ABOUT WHAT YOU HAVE IN MIND WHEN YOU
SEPARATE THE PAIR FROM THE OTHER ONE.HOW CAN YOU DESCRIBE
THE TWO ENDS OR POLES OF THE SCALE WHICH DISCRIMINATE


PRACTICAL AND LIBRARY FROM FILM
JUST TYPE ONE OR TWO WORDS FOR EACH POLE TO REMIND YOU WHAT
YOU ARE THINKING OR FEELING WHEN YOU USE THIS CONSTRUCT.



LEFT POLE RATED 1 -- ?FLEXIBLE
RIGHT POLE RATED 5 -- ?RIGID


TYPE IN THE RATINGS


DO YOU NEED HELP ?NO


 4 PRACTICAL    1
 6 LIBRARY      1
 5 FILM         5
 1 LECTURE      ?4
 2 TUTORIAL     ?4
 3 SEMINAR      ?3


POLE 1 --FLEXIBLE

 4 PRACTICAL    1
 6 LIBRARY      1


 3 SEMINAR      3

 1 LECTURE      4
 2 TUTORIAL     4

 5 FILM         5

POLE 5 --RIGID


DO YOU WANT TO CHANGE ANY OF THESE VALUES ?YES
DO YOU NEED HELP ?NO


HOW MANY ?1
ELEMENT NUMBER ?4
NEW RATING FOR PRACTICAL ?2


POLE 1 --FLEXIBLE

 6 LIBRARY      1

 4 PRACTICAL    2

 3 SEMINAR      3

 1 LECTURE      4
 2 TUTORIAL     4

 5 FILM         5

POLE 5 --RIGID


DO YOU WANT TO CHANGE ANY OF THESE VALUES ?NO

DO YOU WANT TO CHANGE THE POLE NAMES ?NO
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Fig. 2(b). PEGASUS Interaction: Triadic elicitation of a Construct

The second section, Construct Match, provides feedback when two constructs are highly related.

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THE TWO CONSTRUCTS YOU CALLED
 1 INVOLVEMENT--REMOTENESS
 2 FLEXIBLE--RIGID
ARE MATCHED AT THE  75 PERCENT LEVEL.
THIS MEANS THAT MOST OF THE TIME YOU ARE SAYING
INVOLVEMENT YOU ARE ALSO SAYING
FLEXIBLE
AND MOST OF THE TIME YOU ARE SAYING
REMOTENESS YOU ARE ALSO SAYING
RIGID


THINK OF ANOTHER ELEMENT WHICH IS EITHER INVOLVEMENT AND RIGID
OR FLEXIBLE AND REMOTENESS
IF YOU REALLY CANNOT DO THIS THEN JUST PRESS RETURN AFTER THE
FIRST QUESTION MARK BUT PLEASE TRY. THEN YOU MUST GIVE
THIS ELEMENT A RATING VALUE ON EACH CONSTRUCT IN TURN.
AFTER EACH QUESTION MARK TYPE A VALUE FROM 1 TO 5
WHAT IS YOUR ELEMENT ?PROGRAMMED TEXT
RATINGS :


INVOLVEMENT--REMOTENESS ?2
FLEXIBLE--RIGID ?5


DO YOU WANT TO CHANGE ANY OF THESE VALUES ?NO

ELEMENT 7 --PROGRAMMED TEXT
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Fig. 2(c) PEGASUS Interaction: Elicitation of an Element through Feedback about a Construct Match

The user is first asked to add an element which is either at pole 1 on the first construct and pole 5 on the second or vice versa. If he can add a new element it must be rated on all the constructs so far elicited, but if he cannot split the two constructs this way he is asked if he would like to delete a construct, combine two constructs into one, or just carry on.

When four constructs have been entered, the program moves into the third section, Element Match, and begins to calculate matching scores between elements. Each element is matched with every other on the basis of the ratings used, and a comment made

on the highest match if it meets the set criterion. The first choice offered is to add a new construct on which the two elements are placed at opposite poles, and then all the elements must be rated in the usual way. Alternatively an element may be deleted, or no action may be taken. Section four allows the user to Finish at this stage in the cycle. Section five, Review, gives the choice of adjusting or redefining the purpose, and altering the level of match on which feedback commentary is given. There is an opportunity to see a focused version of the grid to date, and to delete any element or construct which is felt to be unsatisfactory in any way. In the sixth section, Alternative Elicitation, the user is given the freedom to add an element or to add a construct without using a triad if he so wishes.

When the grid reaches the maximum size allowed, or if the user chooses to finish before that, the results are analysed using the FOCUS procedure. This is a two-way hierarchical cluster analytic technique which systematically reorders the rows of constructs and columns of elements to produce a focused grid which shows the least variation between adjacent constructs and adjacent elements. The printout shows the trees of the elements and constructs as well as the focused grid with the element and construct labels.

As a PEGASUS elicitation proceeds this FOCUS algorithm is used to offer to the user a possible explanation and interpretation of his meaning system in the terms of the similar patterns he uses in supposedly different circumstances. Cross-references are mapped across the grid and exhibited to the user in such a way as to offer him the facility to reconsider and change anything he feels to be inappropriate, which in turn enables him to be more aware of the links he is implicitly holding in his cognitive model. Here the conversation can be seen as a feedback loop with the computer acting as the error regulator by representing the content of the conversation and the implicit links within the conversation. In this way the participative analysis extracts and displays the essence of the subjectivity and personally meaningful relationships in the grid.

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THE TWO ELEMENTS  2 TUTORIAL AND  3 SEMINAR
ARE MATCHED AT THE  87 PERCENT LEVEL.
THIS MEANS THAT SO FAR YOU HAVE NOT DISTINGUISHED
BETWEEN TUTORIAL AND SEMINAR
DO YOU WANT TO SPLIT THESE ?YES
HELP ?YES


THINK OF A CONSTRUCT WHICH SEPARATES THESE
TWO ELEMENTS AND THEN KEEPING THIS IN MIND


ACCORDING TO HOW YOU FEEL ABOUT THEM PLEASE ASSIGN TO EACH
OF THE OTHER ELEMENTS IN TURN A PROVISIONAL VALUE FROM 1 TO 5



NAME THE POLES OF YOUR CONSTRUCT


LEFT POLE RATED 1 -- ?SMALL GROUP
RIGHT POLE RATED 5 -- ?LARGE GROUP


TYPE IN THE RATINGS


 2 TUTORIAL     1
 3 SEMINAR      5
 1 LECTURE      ?5
 4 PRACTICAL    ?4
 5 FILM         ?5
 6 LIBRARY      ?1
 7 PROGRAMMED TEXT           ?1


POLE 1 --SMALL GROUP

 2 TUTORIAL     1
 6 LIBRARY      1
 7 PROGRAMMED TEXT           1



 4 PRACTICAL    4

 1 LECTURE      5
 3 SEMINAR      5
 5 FILM         5

POLE 5 --LARGE GROUP
```

Fig. 2(d). PEGASUS Interaction: Elicitation of a Construct through Feedback about an Element Match

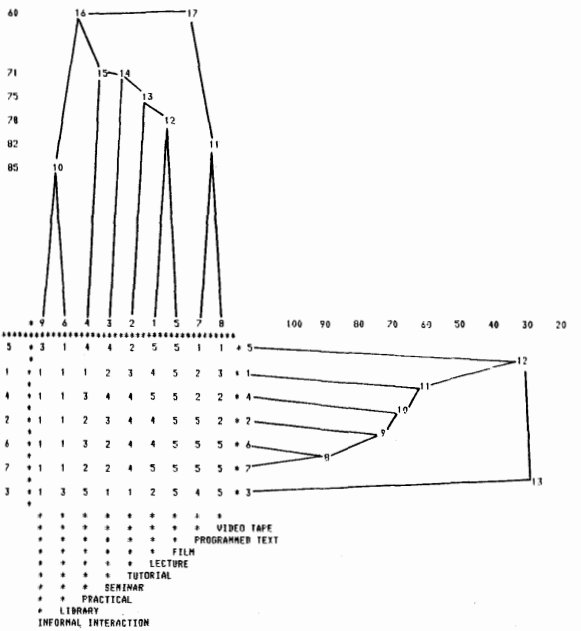
```
DO YOU WANT TO FINISH NOW ?YES
DO YOU WANT:
    1) A COMPLETE PRINTOUT OF THE ANALYSIS OF YOUR GRID
    2) ONLY THE RESULTS OF THE ANALYSIS
WHAT IS THE NUMBER OF YOUR CHOICE ?2


CONSTRUCT  3  REVERSED

TO DRAW THE CONSTRUCT TREE:              MATCH

JOIN 6  AND 7  INTO CLUSTER 8            88.89 %
JOIN 2  AND 8  INTO CLUSTER 9            72.22 %
JOIN 4  AND 9  INTO CLUSTER 10           66.67 %
JOIN 1  AND 10 INTO CLUSTER 11           61.11 %
JOIN 5  AND 11 INTO CLUSTER 12           33.33 %
JOIN 12 AND 3  INTO CLUSTER 13           27.78 %


FOCUSED GRID
```




```
                SMALL GROUP   5 ---  LARGE GROUP
                INVOLVEMENT   1 ---  REMOTENESS
             SELF-ORGANISED   4 ---  STAFF-ORGANISED
                   FLEXIBLE   2 ---  RIGID
           VARIABLE CONTENT   6 ---  SPECIFIC CONTENT
                       LIKE   7 ---  DISLIKE
              NO EQUIPMENT    3 ---  EQUIPMENT

TO DRAW THE ELEMENT TREE:                MATCH

JOIN 9  AND 6  INTO CLUSTER 10           85.71 %
JOIN 7  AND 8  INTO CLUSTER 11           82.14 %
JOIN 1  AND 5  INTO CLUSTER 12           78.57 %
JOIN 2  AND 12 INTO CLUSTER 13           75 %
JOIN 3  AND 13 INTO CLUSTER 14           71.43 %
JOIN 4  AND 14 INTO CLUSTER 15           71.43 %
JOIN 10 AND 15 INTO CLUSTER 16           60.71 %
JOIN 16 AND 11 INTO CLUSTER 17           60.71 %

THIS IS ARTHUR'S GRID
PURPOSE:  EXPLORING LEARNING SITUATIONS
```

Fig. 2(e): PEGASUS Interaction: Part of Review Section

2.4. EXTENDING THE ANALYSIS TO A COMMUNE OF SCIENCE: PEGASUS-BANK AND SOCIOGRIDS

From the example given it can be seen that the PEGASUS computer program acts as a dialectical partner to the person in aiding him to understand his personal semantic structure for a world. The techniques used in this operational form of Kelly's personal construct theory may be extended from the individual personal scientist to the group who through their construct structures form a commune of science. PEGASUS-BANK is a development of PEGASUS which allows the user to complete a grid on a topic area and get ongoing feedback on the relationships between his constructs and those of an "expert" or the consensually validated definitions which represent public knowledge in the area. This is done by storing the "expert" grid in the computer, and as the user elicits a grid in the usual PEGASUS way, the feedback is given not only in terms of how the user's constructs map on to each other, but also how they map on to the bank of stored constructs (SHAW 1981).

This technique can also be used to help two people to come to an understanding of each other. One may elicit a grid which is stored in the computer for the other to use as he elicits his own grid using the same elements. At each stage the bank may be increased or modified hence encouraging each of the participants to take on the construct system of the other by mapping out the similarities between the patterning, and hence meanings can be exchanged between the pair.

The SOCIOGRIDS program was devised in order to explore the similarities and differences in construing between members of a group. This technique is based on the assumption in Kelly's commonality corollary that there may be shared meaning among a group of individuals. Starting with the negotiation of a common set of elements by the group concerned, this program analyses the set of repertory grids elicited from the individuals in the group. The constructs elicited are personal to the individual, and the similarity between constructs is measured using an operational

definition of how the construct orders the elements rather than on any literal or conceptual similarity.

Using the SOCIOGRIDS technique each individual in the group can have feedback on his own construing of the topic from a focused grid such as the one in figure 2(e). In addition a "mode" grid of the most commonly used constructs from the group is extracted and focused. This then shows the content of the topic which the members of the group have in common. Each construct in the mode grid has been obtained from an individual in the group and is in no way changed when used in the mode. This grid is not a consensus grid which averages out the individualities to produce a pale imitation of the group, but is strongly weighted towards the commonality of construing within the group. These constructs are selected on the basis of how they order the elements not on linguistic labels, and hence tend to be tightly clustered into subsets of comparable meanings.

Figure 3 illustrates a mode grid from a group of art students who were concerned to examine the differences in the construing of the students and the tutor in the field of graphic art. The elements are pieces of graphic art contributed by individual members of the group and discussed in the group before being included. The first two main clusters which appear can be seen to express similar ideas in different terminology.

The mode grid can then be used as a common referent with which to compare each member of the group. Figure 4 shows the combined grids of the mode and grid 7 focused together, indicating the extent of the interaction between the two perspectives. In fact, this combination shows the most interaction, the least being shown by the combined grids of the mode with grid 6.

The program also produces a set of "socionets" from the measure of similarity between pairs of individual grids. The socionets show the rank ordering of the most highly related pairs of individuals in the group and hence expose those members of the group who have the most in common and those with strongly individualistic viewpoints. Figure 5 shows the set of socionets from

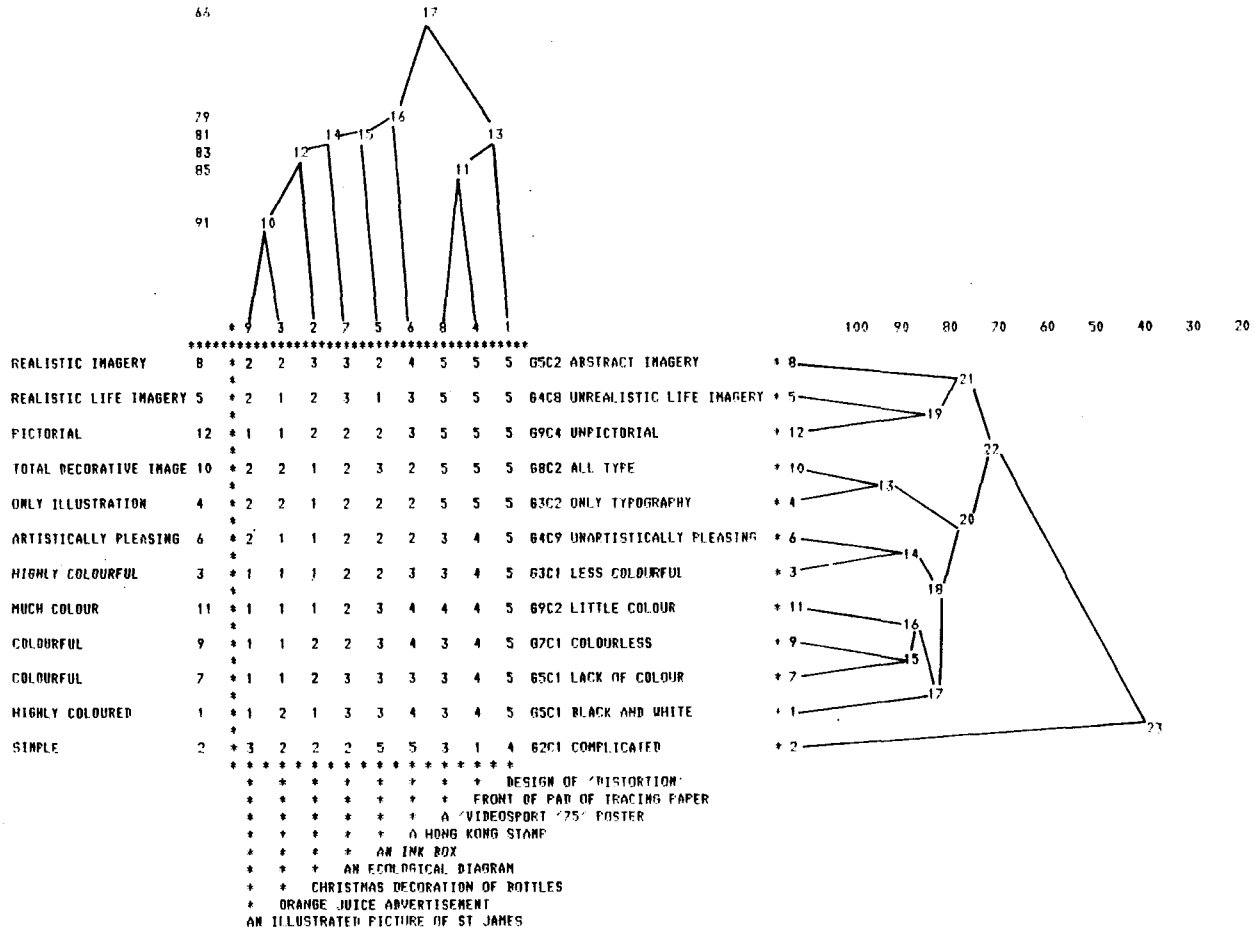


Fig. 3. The Mode Grid on Graphic Art Objects

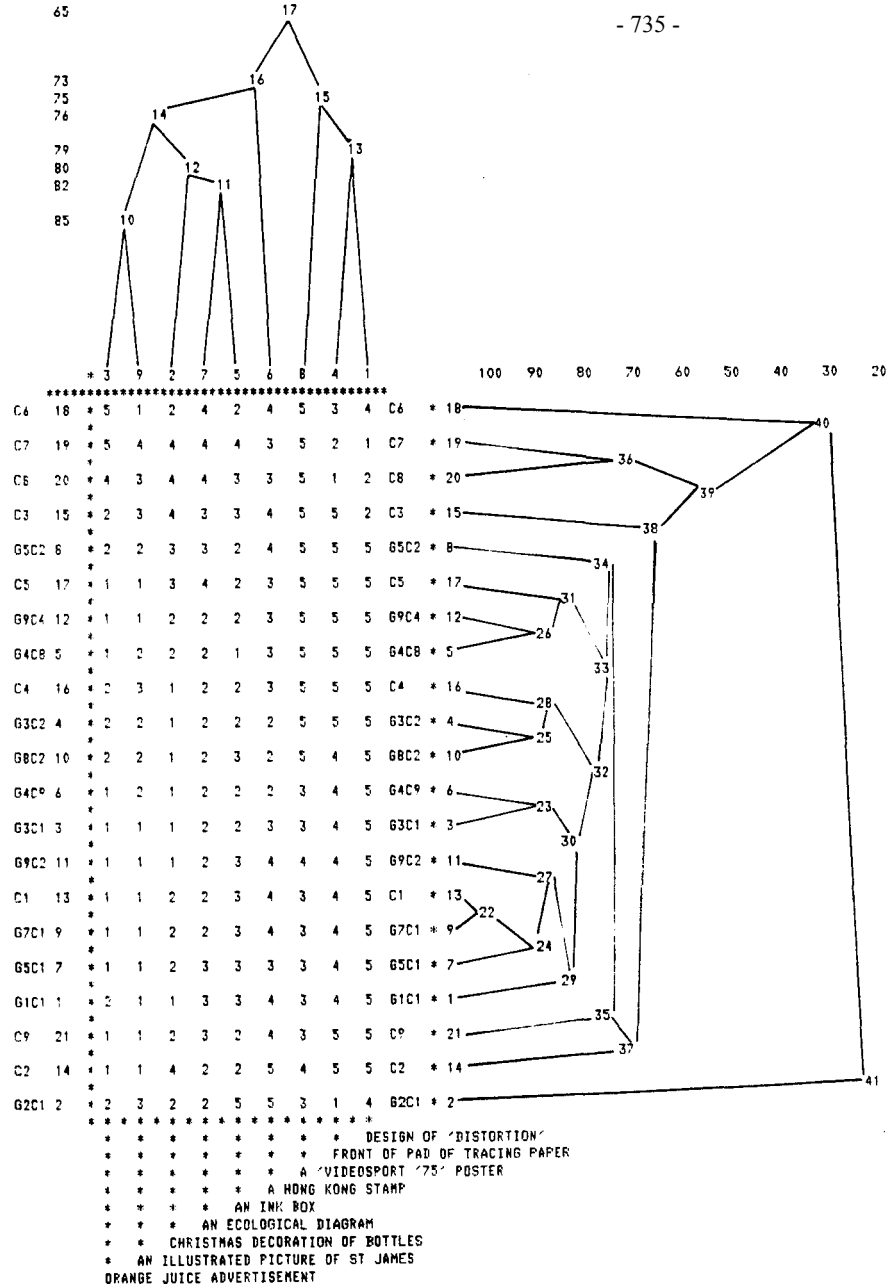


Fig. 4. The Combined Grids of the Mode and Grid 7

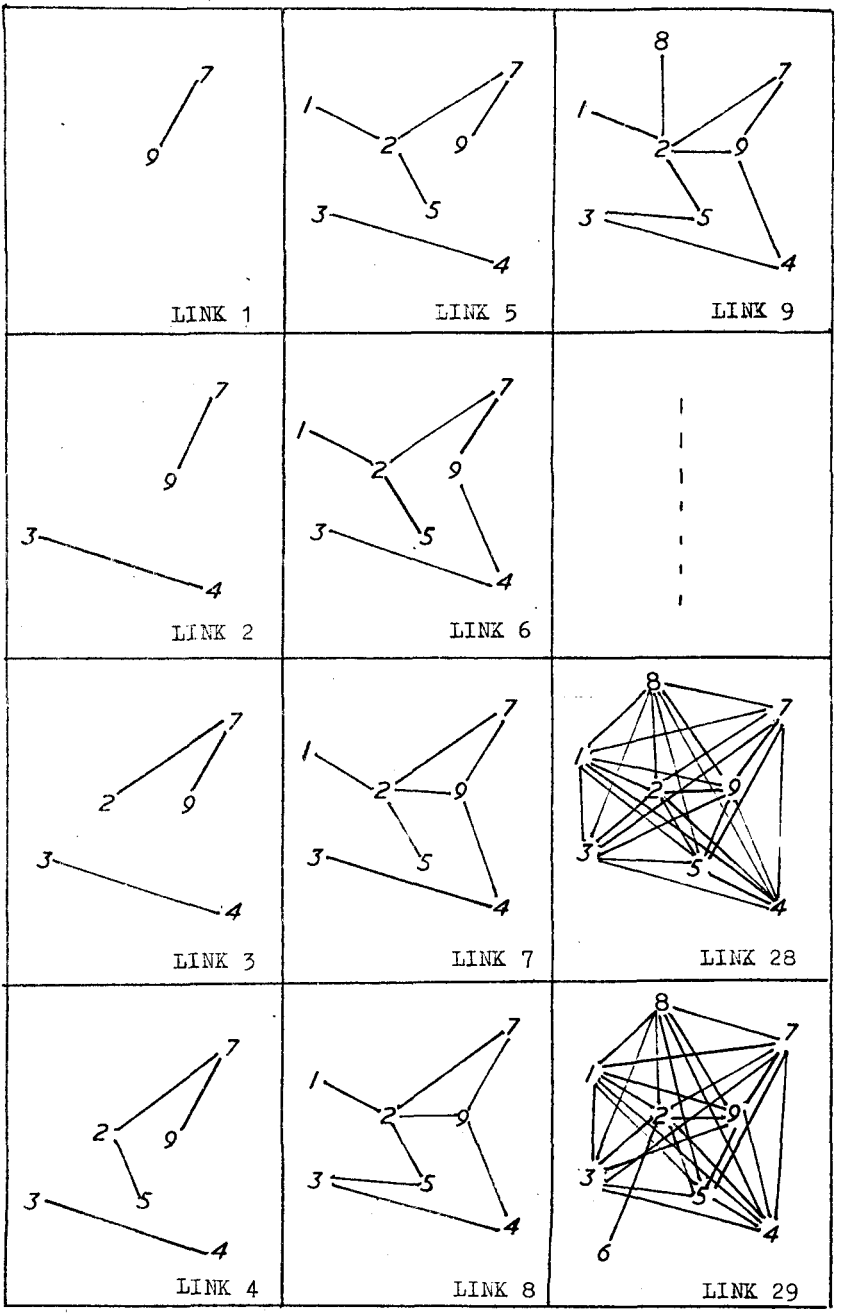


Fig. 5. SOCIOGRIDS Analysis of Relations Between Participants

the group of art students. Firstly two highly related pairs emerge, 7 with 9 and 3 with 4. Gradually 2 is revealed as the person who has most in common with all the others, and the tutor joined the group at link 9. Not until all the other links have been made at stage 28 does person 6 join the group. He clearly shows a strongly individualistic point of view, and in fact chose to leave the group shortly after this time, not having seen these results.

These techniques have been applied to a wide variety of situations (SHAW 1980) including industrial inspection of knitwear, appraisal of subordinates and interactions in groups of students. They can be used to chart changes in the attitudes in a group over a period of time, and to investigate alternative points of view which may lead to a conflict of interests. It has been our experience that participants in such an exercise become very involved in the process of reviewing similarities and negotiating differences within the group, and find it a relevant learning experience.

2.5. THE SIGNIFICANCE OF COMPUTER-BASED INTERACTION

It would be easy to assume that interactive programs such as PEGASUS are merely more convenient ways of eliciting construct systems through extensions of Kelly's repertory grid and do not themselves add anything qualitatively new to the process. However, such an assumption would be missing certain crucial psychological factors in the man-computer situation and its differences from the man-man situation. We have observed informally in making PEGASUS available to a wide range of people in a variety of situations that those coming to it for the first time often seem to find it a very dramatic experience. They react to it intensely and become gripped by the interactive process of construct elicitation. They also feel that they are learning something new from the process and are prepared to use this in determining their behaviour.

Probably such involvement is also significant in the elicitation of construct systems by a person rather than computer inter-

action. However, we believe there are certain quite fundamental differences when the elicitation is done in such a way that interpersonal interaction is clearly absent. In particular, when a person is feeding back comments and guidance it is a natural and ready assumption that the constructs are being injected rather than elicited. It is easy for the subject to believe that the elicited constructs do not come from himself but that a tutorial or debating siutation with another person is taking place. It is necessary to persuade him that this is not so and the persuasion has to be stronger the more striking and significant the constructs elicited. However, when a computer is the tool by which his construct structure is being reflected or laid bare then such an assumption of outside injection and interference is far less tenable.

When constructs are being elicited by a computer program then it is more likely to be accepted that it is precisely and only oneself that is being portrayed. We 'trust' a computer program to be doing just what it appears to be doing without deeper motivations and without attempting to persuade us to its point of view. No-one is telling the user anything. He is seeing in interacting with PEGASUS, possibly for the first time, the basis for his own thought processes. Very often extreme surprise is the first reaction. If another person were eliciting the construct structure then the surprise would be taken as an indication that he was incorrect and one would ignore him or argue with him. With computer elicitation it is more likely that one will accept the reflected structures as being self-generated and the surprise acts as motivation to know more.

That this knowledge can be totally private to oneself is another important feature of interaction with the computer. We do not like, as Kelly put it, to be "caught with our constructs down". When another person is involved we are more reluctant to expose and explore our constructs the more surprising they are; perhaps because the surprise is often the result of a conflict between our ostentative value judgements and the basis of our behaviour. Or it may just be sloppy verbal behaviour: that we are naming two distinct constructs with the same label. For example,

in using PEGASUS a scientist found that he was using the word "time" to label several different elements and generating confusion in his arguments because of this.

Another reason that we are reluctant to explore construct structures freely in interacting with another person, particularly a professional person, is that we are acutely aware of the possible 'waste' of their time. This phenomenon has been noted (CARD et al 1974) as accounting for a major part of the preferences expressed by patients to be interviewed through an interactive computer program rather than their doctor. There are many pressures and artefacts of interpersonal relationships that can totally obscure and undermine such reflective processes as we require in the elicitation of personal constructs.

It is interesting to note that this argument has been put in reverse: ADAMS (1979) notes that children learn quickly to play games on a personal computer and conjectures that this is because of the lack of interpersonal complications. She suggests, however, that "one of the benefits of game-playing is that a child learns how to behave with and towards others, how to cope with success and failure, and what effect it has on others. In the human-computer relationship the child does not learn these valuable social skills." We are arguing conversely that the need to be deploying such "social skills" is a load that can seriously detract from the exploration of the self.

A notable technical feature of PEGASUS that profoundly affects human reactions to it is that relationships between constructs may be inferred instantly and queried with the user. This immediate analysis and feedback is a key factor in most applications of interactive computers and can go way beyond what any manual analysis can accomplish. Instant feedback whilst one remembers one's line of reasoning is very different from delayed analyses that are delivered at a later time when the entire context of the replies one has been giving may have been forgotten. Construct structures in particular have a high degree of context-dependence. It is often the relationship between the structures elicited and the <u>role</u> we are adopting in answering the questions

that elicit them which is of prime interest to us. Using WOLFF's (1976) terminology, we surrender ourselves to a particular role and become a "physicist", a "mathematician", a "manager", a "father", etc., and it is the analysis of our construal of the world in the specific role which we are attempting to catch. These concepts will be developed further in Section 4.

2.6. SUMMARY

This section has provided some background on Kelly's approach to epistemology and personal semantics. It has demonstrated that this approach can be operationalized to provide an empirical basis for exploring personal semantic space. It has also demonstrated that the approach can be extended to investigate the interplay between the semantic structures in a group of people having some communality. Finally, it has been argued that the computer provides a new dimension in the dialectical exploration of semantic space. In the next section we develop more rigorous, logical foundations for the analysis of personal semantic space.

3. LOGICAL ANALYSIS OF SEMANTIC SPACE

In section 2 we have shown how the semantic space of a personal scientist and a commune of science may be explored using computer systems for construct elicitation and analysis. In both our work on the technology of dialectical computer systems and on the fundamentals of human reasoning we have felt the need for an analysis of semantic space that goes beyond the distance-based measures commonly used (and used in FOCUS and SOCIOGRIDS). At the beginning of section 2 we noted the common foundation underlying the work of Kelly and that of BROWN (1969) in his Laws of Form. It casts an interesting light on the further development of Kelly's work that Brown goes on to use the notion of a

distinction to develop a logical "calculus of distinctions" with fewer primitives than the classical propositional calculus which he claims avoids the paradoxes of previous approaches. In his own practical development of a personal construct technology through the "repertory grid" and the extraction of "factors" from it Kelly treats constructs as if they gave a vector of measurements of the event rather than a logical representation of it. This approach seems to have been followed also by all later workers on the analysis of the repertory grid through a variety of methods such as principal components analysis. In the following sections we show that the analysis of construct systems as logical structures both encompasses many of the advantages of existing methods and also leads to interesting new directions of analysis.

This section deals with the analysis of the grid rather than its elicitation but it is worth emphasizing at this stage that our prime motivation for the form of logical analysis developed here was to extend techniques for the interactive elicitation of grids through feedback of the analysis. We have been aiming to develop a conversational, dialectical, system of computer programs for the self-reflective study of one's role as a personal scientist. With this in mind it has been important to develop forms of analysis that can support a conversation by commenting upon its contents without introducing new constructs beyond those the user already employs. It is this which has led us to a logical analysis in which the meta-language used for comments on construct structures is essentially the same as the object-language in which the information defining these structures is given.

We will first review various distance-based methods of repertory grid analysis and then compare them with a logical approach.

3.1. DISTANCE-BASED REPERTORY GRID ANALYSIS

We will concentrate initially on the relations between the constructs in a grid such as that shown in Figure 1. For any given construct we may regard the numbers in the grid as a vector of values giving the assignment of each element in turn to

one or other of the poles of the construct. From this point of view each construct becomes represented as a point in a multi-dimensional space whose dimension is the number of elements involved. The natural relation to examine between constructs is then the distance between them in this space. Two constructs which are zero distance apart are such that all elements are construed in the same way in relation to them and hence we might infer that they are being used in the same way - in some way they are equivalent constructs. For constructs which are not equivalent we may analyse the entire constellation in space to determine a set of axes such that the projection of each construct onto the first axis accounts for most of the distance between them, the projection on the second axis accounts for most of the remaining distance, and so on. This is a principal components analysis of the construct space (SLATER 1977), and it is related to the factor analysis of semantic space used by OSGOOD, SUCI and TANNENBAUM (1957). We may also group constructs together that are close together in space using some form of cluster analysis (DURAN & ODELL 1974).

All of the techniques based on such a numerical spatial view of construct structures depend on the notion of constructs being equivalent if they are represented by the same point in space and somehow nearly equivalent if they are represented by points close to one another. Principal components analysis goes even further and assumes that the distances between points are themselves meaningful and that the distribution of points in space gives an indication of meaningful directions in that space. However, it is most often used just as a basis for clustering constructs according to their distance apart on the two principal dimensions so that the notion of the 'meaning' of these dimensions does not necessarily arise.

The grid of Figure 1 was analysed using SLATER's (1977) INGRID program for determining the principal components. Figure 6 shows the twelve elements and the two poles of each of the eight constructs plotted against the first two principal components. In this section we will concentrate on the construct analysis

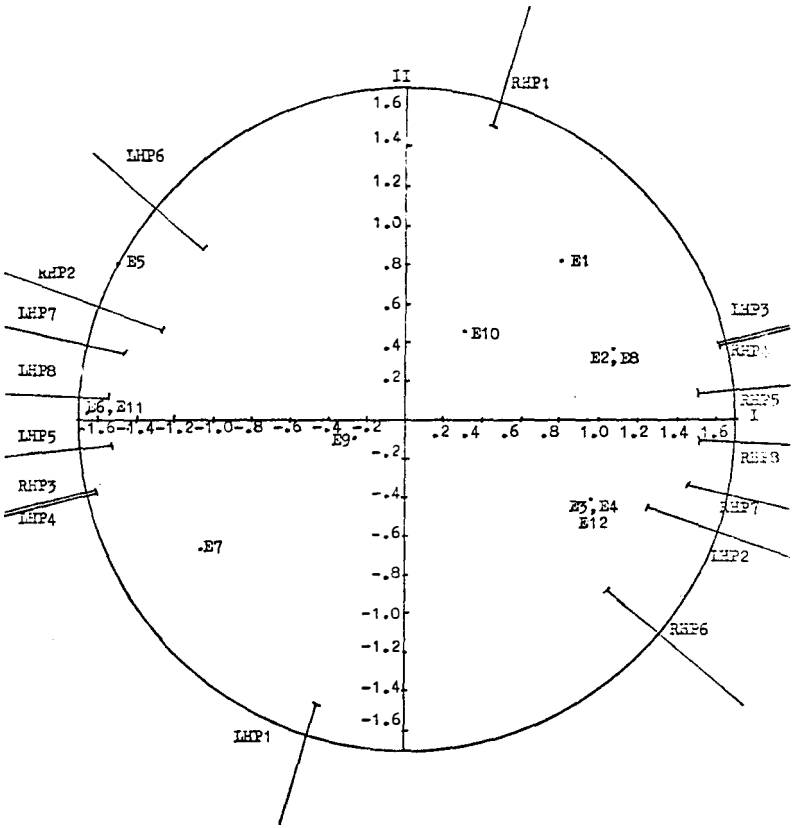


Fig. 6: Principal Components Analysis of Jane's Grid by INGRID

and treat the element analysis later in section 3.5. It can be seen that the left-hand poles of constructs 4, 5, 7, and 8, form a fairly tight cluster together with the right-hand poles of constructs 2 and 3. The left-hand pole of construct 6 is associated more loosely with this but both poles of construct 1 are isolated well away from the cluster. There is a mirror image cluster of the right-hand poles of constructs 4, 5, 6, 7 and 8, together with the left-hand poles of constructs 2 and 3. Because the assignment of elements to poles is such that the vector of assignments to the left-hand pole is the reverse of that to the right-hand pole such a mirror image is bound to occur with conventionally elicited grids.

ATKIN's (1974) Q-Analysis provides an alternative means of analysing the structure behind a matrix of data such as that of Figure 1. In terms of the present discussion it is convenient to regard it as a form of hierarchical cluster analysis based on a distance-measure, although it is conventionally presented in combinatorial topology terms. The data of Figure 1 was analysed using a program QARMS (Q-Analysis of Relations in Multilevel Structures) that can also deal with grids using rating scales as well as the binary data shown. The results are shown in Figure 7 with the connectivities also drawn out as a hierarchical cluster.

SHAW's (1980) FOCUS algorithm has already been shown in action in Figure 2(e). It is another distance-based grid analysis technique that sorts the constructs into a linear order such that constructs closest together in the space are also closest together in the order. It has the advantage in presentation that the sorting is used only to re-present the original grid reorganized by the 'neighbourness' of constructs and elements. It is left to the user to construe meaning into the result and confirm this directly in terms of the original data. Figure 8 shows the grid of Figure 1 as processed by FOCUS. Note that the letters "X" and "O" have been replaced by the numerals "1" and "0" used in section 2.2. In re-organizing the grid FOCUS has also reversed constructs 2 and 3. Concentrating on the construct analysis again, it can be seen that constructs 3 and 4 are equivalent and close

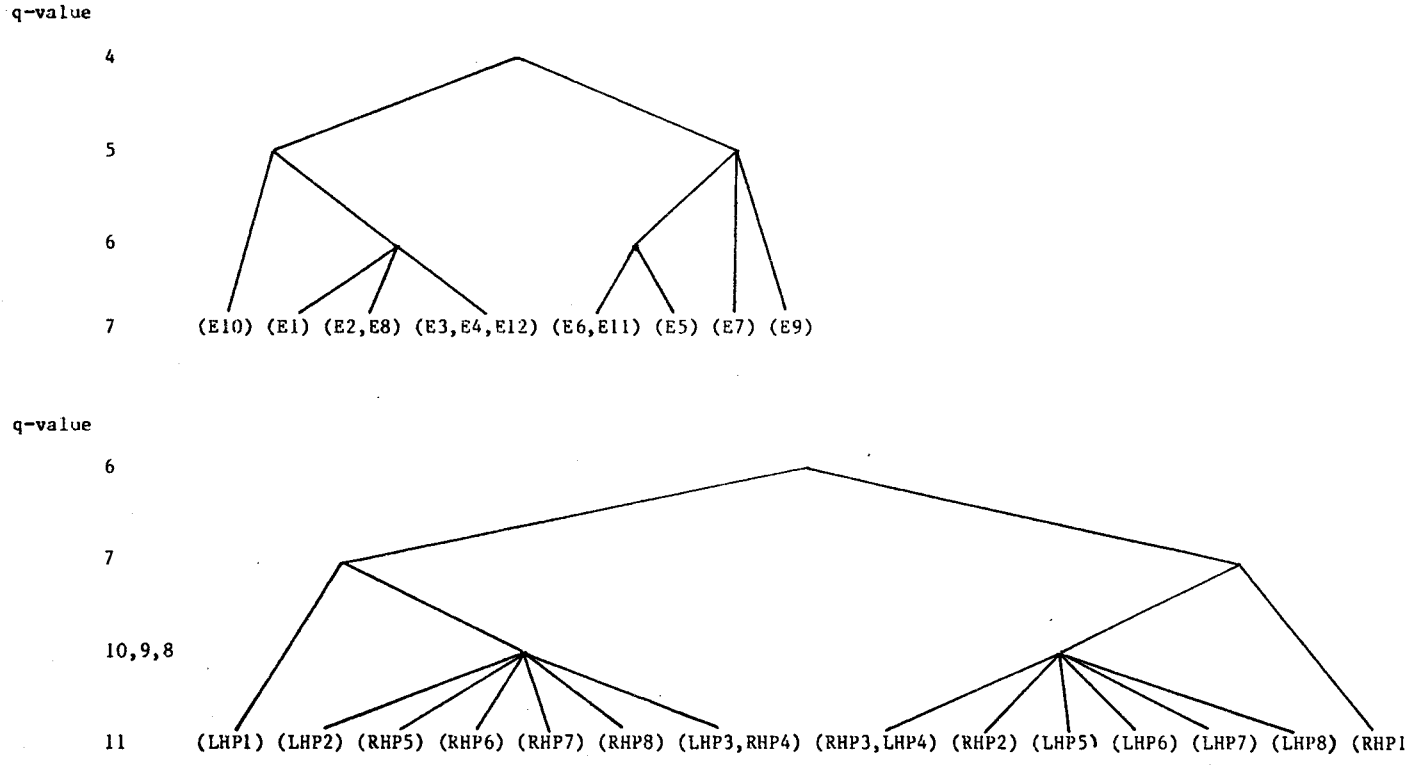


Fig. 7. Q-Analysis of Jane's Grid by QARMS

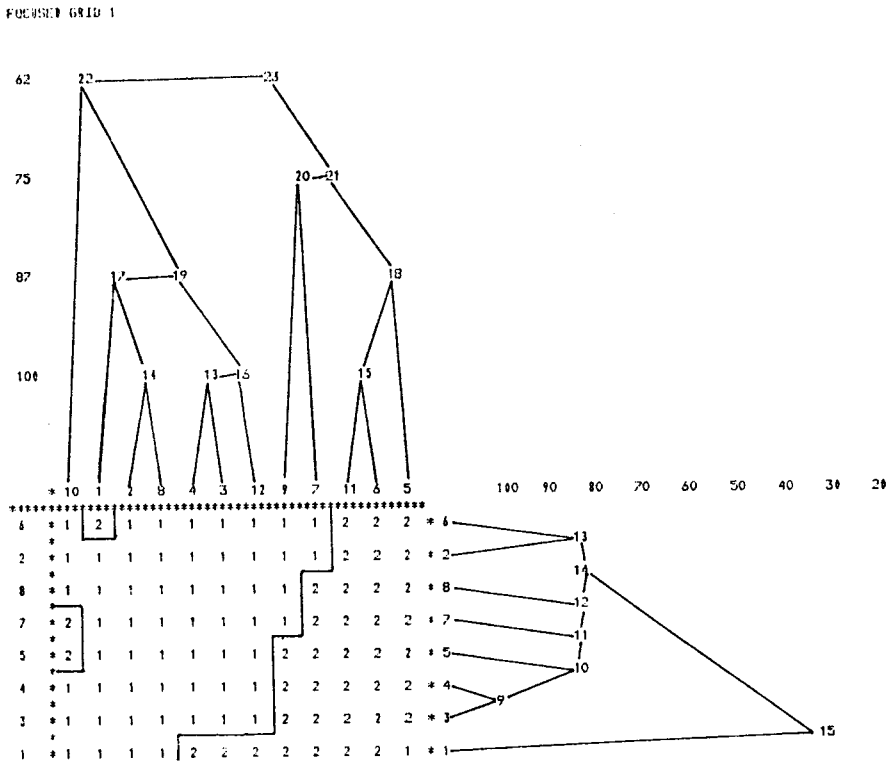


Fig. 8. Cluster Analysis of Jane's Grid by FOCUS

neighbours of 5, 7 and 8, and that this cluster is itself a close neighbour of the cluster formed by 2 and 6. Construct 1 is not linked into the other constructs at a meaningful level.

Thus, for this example at least, the actual clusters produced by FOCUS, QARMS and INGRID do not differ in any meaningful way. In general, since all these techniques use distance measures to produce clustered data, one would expect the results to be similar. The objective of FOCUS is not to produce a different analysis in terms of clusters but rather to present the analysis in terms of the data that produced it. In this way those who produced the grid are able to see how the decisions they made in doing this affect the actual analysis.

The reversal performed by FOCUS is an important operation in analysing a repertory grid. KELLY (1955, p. 283) uses such a process of reversal (which he calls "reflection") in his analysis of repertory grids, and the need for it clearly arises from the artificiality of the assignment of "left-hand" and "right-hand" poles to a construct. Unless some special additional rationale is in operation, what are called the left-hand and right-hand poles of a construct may be reversed without distorting the grid providing the assignment of elements to those poles is also reversed. In principal components analysis and Q-Analysis such reversals show up in the clustering of left-hand poles together with right-hand poles, for example as in Figures 6 and 7.

It is convenient to make a point here that applies to all techniques for grid analysis. Any relation we infer between constructs from a given grid are derived from the set of elements used in eliciting that grid. Hence they should be qualified by a reference to that set: not "constructs 3 and 4 are equivalent", but rather "in relation to elements 1 through 12, constructs 3 and 4 are equivalent". To the extent that we drop this qualification we are proceeding inductively rather than deductively and our conclusions may be incorrect. This applies to any conclusion that extends the relation between constructs to elements that have not been tested, for example "in relation to your close acquaintances, constructs 3 and 4 are equivalent". Interactive

elicitation of the grid through such programs as PEGASUS (section 2.3) avoids some of the problems of inductive inference by checking conclusions as they are made and allowing new data to be entered to eliminate false inferences.

In conclusion the various distance-based analyses of grids each provide related methods of clustering elements and constructs in such a way that one can provide feedback on possible structures underlying the construing. They have two factors in common that restrict their application in some contexts. Firstly, the structure exhibited is limited in its semantics to a symmetric relation of 'neighbourness' between the items clustered. Secondly, the analyses produce results about distances, components, connections, geometrical relationships, and so on, which represent a different way of looking at the data. This may be valuable in itself and may be expressed through basic notions of similarity. However, for some applications such as interactive discussion in conversational grid elicitation it would be preferable to have an analysis that expresses relations in the data in terms more immediately meaningful and directly related to the data itself. It was these considerations that led us to the logical data analysis described in the next section.

3.2. LOGICAL REPERTORY GRID ANALYSIS

There is an alternative way of looking at the grid of Figure 1 which views it not as a set of vectors in a space but instead as an assignment of truth-values to logical predicates. We may take the left-hand pole of each construct in Figure 1 to be a logical predicate that may be applied to a person and take the assignment of the value 1 to a particular element in the grid to mean that the predicate is true for that element. Conversely we may take the value of 0 assigned to an element for a construct to mean that the predicate represented by the left-hand pole of that construct is false for that element. It is convenient to use the abbreviation LHPm for the predicate that corresponds to the left-hand pole of construct m. Thus LHP5 is the predicate for the left-

hand pole of construct 5. If we then also adopt the convention that En stands for the n'th element then the notation LHPm En may be used to denote the truth value of the predicate corresponding to the left-hand pole of construct m when applied to the logical constant corresponding to the n'th element. A repertory grid, such as that of Figure 1, is then the matrix of such truth values for the m constructs and n elements involved.

Because of the inverse relation already noted between assignments to the opposite poles of a construct in a conventional repertory grid, the predicate corresponding to the right-hand pole is logically related to that corresponding to the left-hand pole. We normally require that an element be assigned to one, and only one pole, so that if LHP E is true then RHP E must be false, and vice versa. Hence, LHP E is essentially the logical negation of RHP E. For the current discussion we shall accept that this relation exists as a constraint between the two predicates corresponding to the two poles. However, it is not an essential one for the theory and we discuss in section 3.7 the possibility of relaxing it and the consequences of doing so. For this reason we shall carry out most of the discussion in terms of the left-hand poles and associated predicates primarily, noting occasionally the corresponding phenomena for right-hand poles.

First let us examine the previous relation of equivalence between constructs in logical terms. We can define two logical propositions to be equal if their truth-values are the same, and this also corresponds to their numerical truth-values being equal, e.g.

$$\text{LHPm E} = \text{LHPn E} \qquad (1)$$

We can define two logical propositions involving the same free variable as being equivalent if they are equal for all values of that free variable, e.g.

$$\forall \text{E LHPm E} = \text{LHPn E} \qquad (2)$$

and it is then convenient to drop the variable and write:

$$\text{LHPm} \equiv \text{LHPn} \qquad (3)$$

Now this equivalence between the poles of constructs clearly coincides with our previously discussed equivalence in terms of distance. If two propositions are logically equivalent in this way then the vectors of truth-values against elements are the same and hence they are zero distance apart. The converse may also be shown for any proper distance measures.

However, in terms of logical relations equality is only one of many possible relations. There are six binary logical operators between propositions that establish relations between them. Two of these relations are symmetrical and correspond to the two propositions being equal, or to one being equal to the negation of the other. This corresponds to the reversal or reflection of constructs discussed above. The other four operators are forms of implication between propositions, that one proposition being true implies that the other is also true. The four forms arise because of the possibilities of negation, that one being true implies the other is not and so on. They may all be derived from the one operator, $\supset$, where:

$$\text{LHPm E} \supset \text{LHPn E} \qquad (4)$$

means that the assignment of element E to the left-hand pole of construct m implies that it is also assigned to the left-hand pole of construct n.

In contrast to the equality relation, the implication relation is asymmetric. If we assert the implication given in (4) then we are only constraining the truth-value of LHPn E if LHPm E is true. If this is not so and element E is not assigned to the left-hand pole of construct m then we are saying nothing about its assignment to the left-hand pole of construct n. This contrasts to the equality relation asserted in (1) where the proposition LHPm E being false also leads to LHPn E being false in order to satisfy the equality.

One important property of the implication relation is its transitivity. From the way in which we have defined it we can see that if, as well as (4), we have:

$$LHPn\ E \supset LHPo\ E \qquad (5)$$

then we can derive:

$$LHPm\ E \supset LHPo\ E \qquad (6)$$

This is the normal transitivity of an implication relation in a logical calculus.

Asserting mutual implication between two propositions allows us to derive their equality. Thus adding the converse asymmetric assertion:

$$LHPn\ E \supset LHPm\ E \qquad (7)$$

to that of (4) does enable us to derive (1). From this we can see that the relation of implication is a weaker one than that of equality but closely related to it in that if we know the four implication relations between two propositions we may infer the two equivalence relations between them. These results from elementary propositional logic show that it is of interest to consider the implication relation in repertory grid analysis since the equality and equivalence relations normally analysed may be derived from it but not vice versa.

In the same way that we moved from the relation of equality between individual propositions in (1) to that of universal equivalence between them in (2), we may say that one proposition involving a free variable entails another proposition involving the same variable if it has an implication relation with it for all values of the free variable, e.g.:

$$\forall E\ LHPm\ E \supset LHPn\ E \qquad (8)$$

and it is then convenient to drop the variable and write:

$$LHPm \rightarrow LHPn \qquad (9)$$

We will read this as "the left hand pole of construct m entails the left hand pole of construct n". Clearly entailment, being derived from implication, is also asymmetric, and mutual entailment gives us equivalence in the same way as mutual implication gives us equality. Thus adding the converse entailment to (9):

$$LHPn \rightarrow LHPm \qquad (10)$$

to (9) itself allows us to derive the equivalence of (3). Note similarly that the entailment relation is transitive like the implication relation so that from (9) and:

$$LHPn \rightarrow LHPo \qquad (11)$$

we may derive:

$$LHPm \rightarrow LHPo \qquad (12)$$

We have linked the discussion of this section to personal construct theory. However we note that most of our definitions come directly from classical logic and are independent of personal construct theory. The formal mechanisms for defining entailment are rather more complex than those used here because the logic of entailment is concerned to avoid certain paradoxical results (ANDERSON & BELNAP 1975). The nature of these paradoxes does have some interest in personal construct theory because they are to do with relevance in entailment - does one proposition entail another in a relevant way or just through an artefact of the logical calculus? Similar but deeper questions arise when we consider the derivation of entailment from repertory grid data - is one construct relevant to another in the way in which it entails it or is the derived relation a fortuitous one? We consider such questions in sections 3.4 and 3.7.

It is also worth noting that our definitions of equivalence and entailment are also related to those in modal logics (SNYDER 1971). We can regard (2) and (8) as being definitions of necessary equality and necessary implication in a quantification model of a modal logic. In the context of personal constructs we can see this best by noting that two verbal interpretations of (8) are acceptable: "when you assign an element to the left-hand pole of construct m you always also assign it to the left-hand pole of construct n", or "when you assign an element to the left-hand pole of construct m you necessarily assign it to the left-hand pole of construct n". These links may be formalized through a possible worlds (BRADLEY & SWARTZ 1979) model of modal expressions by noting that each element provides a possible world for construing. Entailments according to our definition then become logical implications that are true for all possible worlds currently under consideration. This is a useful and evocative viewpoint because it links personal construct theory with the linguistic semantics of counterfactuals and presuppositions (LEWIS 1973) which is very relevant to Kelly's concept of constructs being "used for predictions of things to come". It also provides useful technical links into the formal mechanisms for treating the topological structure (LEMMON 1966) of possible worlds and its role in logic and semantics which seem equally applicable to personal construct theory.

To conclude the rather abstract discussion of this section and lead into the more concrete operational implementation of the next it is worth considering a specific example of what we mean by entailment, its asymmetry, and the derivation of equivalence from entailment but not vice versa. The poles of two constructs may be quite distinct in terms of equivalence yet closely related in terms of entailment. For example suppose that in construing people someone uses the two constructs m:runs---doesn't run and n:energetic---passive, then we might well expect to find that LHPm entails LHPn but that LHPn does not entail LHPm, that is that being a runner entails being energetic but being energetic does not entail being a runner. If we analyse such a construct

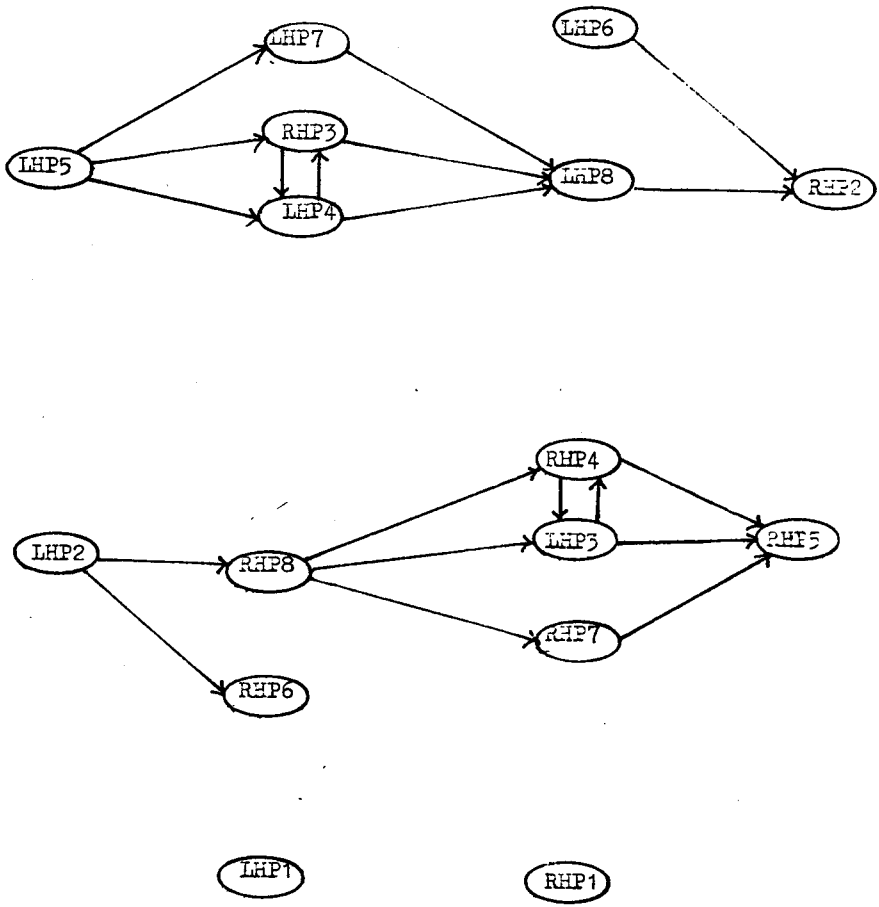


Fig. 9. Entailment Analysis of Jane's Constructs by ENTAIL

structure in terms of distance measures and hence of equivalence only then we shall not derive such asymmetrical relations between constructs even though they are meaningful and of practical interest.

3.3. ENTAIL: A PROGRAM TO DERIVE ENTAILMENTS BETWEEN CONSTRUCTS

It is simple to derive the entailment structure between the poles of constructs. We only have to check the truth of the four possible implications for all elements. Thus LHPm entails LHPn is checked by noting whether whenever an element is assigned to the left-hand pole of m it is also assigned to the left-hand pole of n. If so, then the entailment relation holds true, otherwise it is false. Clearly, as we noted above, it would also suffice to check that whenever an element is not assigned to the left-hand pole of n it is also not assigned to the left-hand pole of m. We call the program that performs this analysis ENTAIL (Entailment Nets Through Analysing Implicational Links). Note again that the inference from a particular set of elements that one pole of construct m entails one pole of construct n is an inductive one if we assume that it applies to other elements in addition to those used in its derivation.

Figure 9 shows the entailment between the poles of the constructs derived by ENTAIL from the grid of Figure 1 drawn out as a directed graph. There are effectively two main sub-graphs which are mirror images of one another plus two isolated poles. One of the graphs shows the entailments for one set of poles, and the other the entailments for the opposite poles. Note that the "reversal" of constructs 2 and 3 apparent in the INGRID, QARMS and FOCUS analyses shows up as LHP2 and LHP3 appearing in the graph of the right-hand poles of the other constructs and vice versa. Because of the essential bipolarity assumed in the elicitation of the grid the two graphs are essentially the same with the arrows and poles reversed in

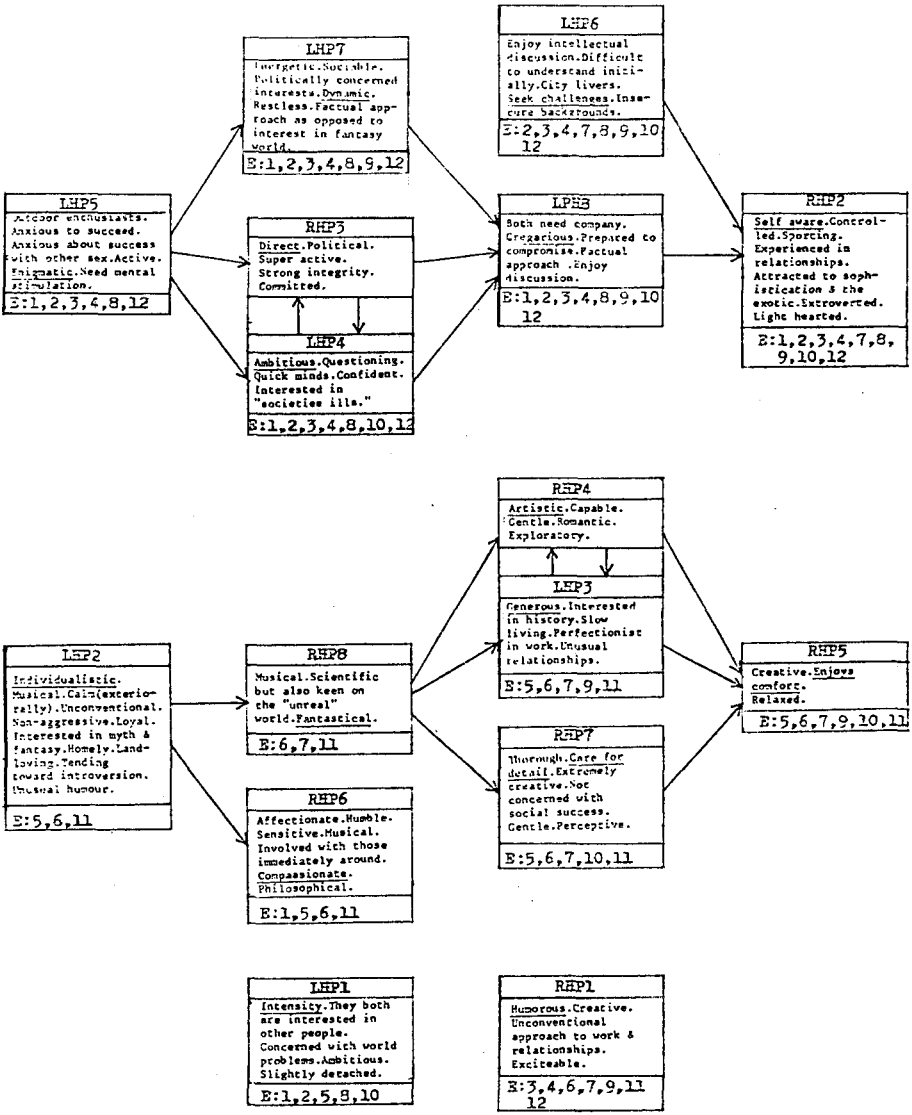


Fig. 10. Entailment Analysis of Jane's Grid by ENTAIL

one relative to the other. In section 3.7 we discuss extensions to the form of grids which would result in such pairs of graphs not necessarily having such a simple relation.

Note that we have taken advantage of the transitivity of the entailment relation not to draw in all the arrows strictly necessary. Thus we have not drawn an arrow from LHP2 to RHP7, RHP5, RHP4, and LHP3 because there is an arrow from LHP2 to RHP8 and then one from RHP8 to RHP7, RHP7 to RHP5, and so on. We can see from the figure that LHP2 entails RHP8, RHP7, RHP5, RHP4 and LHP3 by tracing through the graph. Note that the equivalence between LHP3 and RHP4 now shows up as mutual entailment.

It is very interesting to compare Figure 9 with the results of the INGRID clustering in Figure 6, the QARMS clustering in Figure 7 and the FOCUS clustering in Figure 8. We can see that the same hierarchy of clusters has turned up: (3,4); ((3,4),5,7,8); (2,6); (((3,4),5,7,8),(2,6)); with construct 1 unrelated to the others in all cases. Thus the ENTAIL analysis gives rise to the same basic clustering as did INGRID, QARMS and FOCUS. At a more fundamental technical level we would expect all such distance-based clustering techniques to show such similarity with non-pathological data. Both the fundamental and empirical similarities are important in their own right since two of these programs are widely used for grid analysis and one would hope that any new technique would continue to provide at least the same basic analysis.

However, there is additional information in Figure 9 that goes beyond that available in Figures 6, 7 and 8. This comes from the directed nature of the entailment links shown. There is equivalence only between LHP3 and RHP4 - all the other relations are one way only. To show the significance of this we have drawn out Figure 10 which gives the full text attached by Jane to each pole of the construct together with the elements assigned to that pole. The asymmetry of the entailment relation may be seen by considering that from RHP4 to RHP5 for example. We see from the descriptions of the poles that Jane is saying that any one of her acquaintances who is artistic and so on is also creative and so on. However, the converse does not hold.

From the element data in Figure 10 we can see the reason for this asymmetry. For example, from the elements assigned to RHP4 and RHP5 we can see that the entailment between them not being mutual is due to Jane's acquaintance element 10 being termed creative but not artistic. In this case only one element breaks down the equivalence. If we consider the entailment from LHP8 to LHP5, that her acquaintances who are musical are also artistic then the converse is not true of two acquaintances, elements 9 and 10. And if we consider the entailment from LHP2 to RHP5, that being individualistic entails being creative then the converse is not true for elements 7, 9 and 10. Thus the construct analysis produced by ENTAIL has reproduced the clusters shown by INGRID, QARMS and FOCUS but it has also shown up new features of the data not evident in these distance-based and essentially symmetric forms of analysis.

Note finally that the form of analysis produced by ENTAIL has some of the features of INGRID in that it is two-dimensional and some of the features of QARMS in that it provides hierarchic clusters. However, it also retains the key feature of FOCUS in that it re-presents the original data in a reorganized form. Like both INGRID and FOCUS it also shows the relation between elements and constructs, but unlike them it extends this relation to show the asymmetrical, directed entailment structure between constructs.

3.4. ENTAILMENT AND THE SUPERORDINATION/SUBORDINATION HIERARCHY

The directed graph of entailment is reminiscent of the type of structure that we get when considering Kelly's concepts of "superordination" and "subordination" between constructs. He notes that there is a natural hierarchy amongst constructs:

> "Constructs are construed by means of other constructs, and those, in turn, by still other constructs. It is thus a system is formed." (Kelly 1955, p. 479)

Entailment as defined here appears to treat constructs at the same level and yet to derive a hierarchical structure amongst them. We can see that this structure may have some relationship to Kelly's "system" through the example given previously: runs entails energetic but not vice versa because running is an energetic activity. Thus energetic is superordinate to runs. In logical terms we would normally expect predicates applicable to different categories to have different names and note that the predicate energetic applied to an activity is different from the predicate energetic applied to a person. In everyday language, however, ellipsis of various sorts is common and such distinctions are dropped, or implicit. The rationale seems to be that someone who undertakes an energetic activity will themselves be termed energetic.

We can formalize this argument by considering two constructs m and n such that n is superordinate to m and such that LHPm is construed as being assigned to LHPn. If we now assume that ellipsis occurs in statements such that any element assigned to LHPm of the subordinate construct is also stated to be assigned to LHPn of the superordinate construct, then we have the entailment:

$$\text{LHPm} \rightarrow \text{LHPn} \qquad (16)$$

However, we do not have the converse entailment since it is possible for an element to be construed as assigned to LHPn without its being assigned to LHPm. This might happen, for example, through its being assigned to LHPo of an alternative subordinate construct o of construct n.

Thus we can see that the subordination/superordination hierarchical system defined by Kelly will show up as an entailment structure between the poles of constructs. However can an entailment itself always be construed as arising from superordination/subordination? Again a simple model of some natural language phenomena suggests that the answer is yes. KORZYBSKI (1933) has noted the wide ranging effects of the common phenomenon in nat-

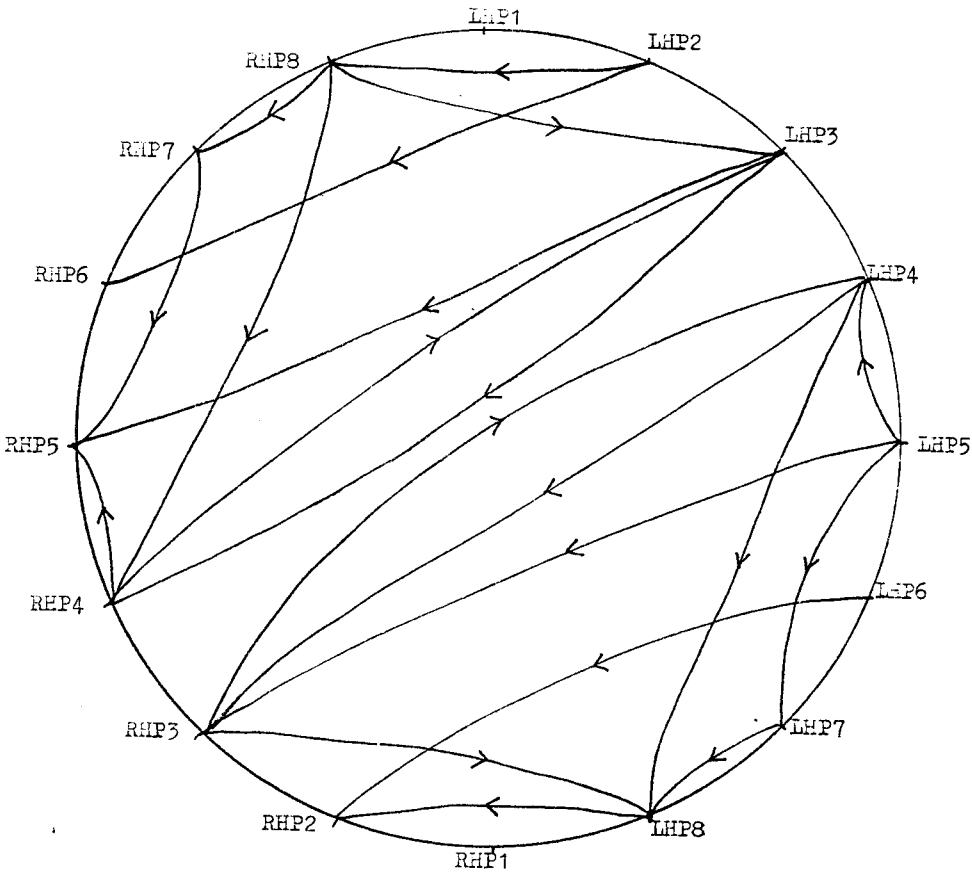


Fig. 11. Circle of Derivations Representation of ENTAIL Analysis of Jane's Constructs

ural language whereby we treat class-names as if they were those of individuals. If we have an entailment of the form of (16) then we may express this as LHPm "leads to" LHPn, meaning that any element assigned to LHPm is also assigned to LHPn. We may then through ellipsis treat LHPm itself as representing the class of elements assigned to it and hence itself being construed as an 'element' assigned to LHPn. There then exists a relation between the constructs on Kelly's definition whereby m is subordinate to n and n is superordinate to m.

This relation between entailment and the subordination/superordination hierarchy raises many other questions: how does it relate to other approaches to eliciting the hierarchy such as HINKLE's (1965) "laddering" and GLANVILLE's (1980) "circle of derivations"; how can we speak of a hierarchy of constructs when the converse entailment applies to the right-hand poles; does it throw light on the criticisms of the whole concept of a superordination/subordination hierarchy?

Firstly, the question of the relationship between implicitly derived structures in human rationality and explicitly verbalized ones is complex. Laddering derives the construct hierarchy directly by asking "why" questions to go up it and "how" questions to go down it: Q: why do you run? A: to be energetic; Q: how are you energetic? A: through running. We can infer from the first that running entails being energetic, and from the second being energetic is entailed by running. Thus, from a logical point of view the indirect elicitation of implicit entailments through ENTAIL and the direct elicitation of the construct hierarchy through laddering should correspond. There are two reasons in practice why this may not occur: that laddering tends to bring in additional constructs in that it is not only a structural analysis but also a different form of elicitation; and, more fundamentally, that the logical correspondence does not necessarily imply a psychological one - people's verbal expression of the rationale behind their behaviour can be quite dissociated from their actual behaviour.

Figure 11 shows the "circle of derivations" corresponding to Figure 9. This is what would be obtained directly from Jane using

Glanville's technique if she agreed totally with the entailments derived by ENTAIL. It would be interesting in future studies to use both Hinkle's and Glanville's methods to obtain directly entailment structures and compare them with those from ENTAIL. Any irresolvable disagreements between the directly derived and the indirectly derived structures would be evidence of dissociation between verbal and actual behaviour. This dissociation can be very significant in attempts to extract from a person information about their skilled behaviour (BAINBRIDGE 1979). Examples occur in the literature on expert systems (MICHIE 1979) which throw light on the difference between modelling the actual behaviour of people and accepting their own verbal models. MICHALSKI and CHILAUSKY (1980) have reported some interesting comparative results on a system for acquiring knowledge from experts on plant disease diagnosis where rules induced from the decision-making behaviour of an expert were far superior to those which the expert actually stated he was following.

The possibility of such dissociation between verbal and actual rationality does not affect just the relation between construct structures derived through ENTAIL and those derived through laddering. It is a general phenomenon whereby psychological and logical models of human rationality differ. For example, we might have someone who agrees that LHPm entails LHPn and also that LHPn entails LHPm, but does not agree that LHPm and LHPn are equivalent. WASON and JOHNSON-LAIRD (1972) have demonstrated that such pathology in the reasoning process is common in many human cognitive activities. Clearly there are many ways of resolving such conflicts. We can go back to the definitions of the terms, agree them and then point out the discrepancy, either in the general case or relative to the particular data. Such a "socratic" approach through explanation and example seems a natural extension of the PEGASUS program described in section 2.3. In concluding the discussion here we note that it is dangerous to assume that even basic logical relationships and results derived from them will always be obvious, or even accepted without debate, by people using personal construct structure analysis programs.

Our second question is on the direction of the construct hierarchy. We have already noted that in the conventional elicitation of constructs the entailment between two left-hand poles is inherently associated with a reverse entailment between the corresponding right-hand poles. Thus from (16) we can infer:

$$RHPn \rightarrow RHPm \qquad (17)$$

This association is often a natural one but there seem to us no logical grounds why it should be a necessary one and in section 11 we discuss an extension of conventional grid methodology which avoids the direct derivability of (17) from (16). However, regardless of this, there will still be a tendency for the left-hand and right-hand poles of a construct to be at opposite ends of the order relation derived by ENTAIL. This might seem to imply that any particular construct may be at either end of the hierarchy according to which pole one considers, and this then conflicts with Kelly's definition of the hierarchy in terms of generality.

This problem can be resolved in major part by noting that the inverse relation exemplified by (16) and (17) causes the entailment graph for the poles to split into two subgraphs that are duals of one another. Figure 9 illustrates this for the particular example analysed. Either subgraph gives rise to the same construct hierarchy but with the arrows reversed. Whether the direction of the arrows indicates increasing subordination or increasing superordination is often obvious by inspection in looking at the relative generality of the two extremes. Thus the entailment hierarchy can be used to derive the structure of the subordination/superordination hierarchy but its direction needs to be determined by other considerations.

The example of Jane's grid used as an illustration is exceptionally simple and in general more complex and fragmentary structures may be found. For example, the isolation of construct 1 in Figure 9 illustrates that there is no necessity for all the

constructs to fit into the same hierarchy. Each of the dual subgraphs may fragment into subgraphs and then one may have several different systems of subordination/superordination. Also, as noted above, there is no reason for the entailments to be necessarily construed as a subordination/superordination hierarchy. This is only one way in which entailment can arise and provides one possible rationale for explaining the entailments.

It is probable that the discussion of this section and particularly these final notes throw light on our third question about criticism of the whole concept of a subordination/superordination hierarchy. Slater presents a number of arguments that lead him to the conclusion that "the theory that construct systems are hierarchical appears questionable to some extent" (SLATER 1977, p. 46). In particular he quotes Kelly to the effect that "The ordinal relationship between constructs may reverse itself from time to time" (KELLY 1955). If our discussion here relating entailment through linguistic ellipsis to subordination/superordination provides a model of the actual processes at work then they may be expected to be variable and subject to change. Constructs themselves are conventional and so are the entailments between them and hence so is the structure of the construct system. If our construct systems are used to guide our actions then as our goals change it may well be that the structure of our construct system itself changes. What is a 'cause' in one context may become an 'effect' in another.

Thus viewing the basic ordinal structure of constructs as being one of logical entailment between poles does enable one to subsume other such structures and provide a basis for understanding their operation and dynamics.

3.5. ASYMMETRIC ELEMENT ANALYSIS

The logical analysis of construct structures through the asymmetrical implication relation makes sense both formally and intuitively. Is there a comparable analysis for the elements? At first sight the answer may appear to be negative. One element 'entailing' another is not necessarily a natural concept, whereas one element being 'near' another in construct space is much more so. We can interpret such 'nearness' as <u>similitude</u> and have a natural interpretation of the two elements being similar. However, there are two factors which should be taken into account in analysing the element structure.

Firstly, if we look at the relation between elements in terms of a distance structure based on the vectors of values of elements on constructs then the <u>weighting</u> assigned to each construct dimension is very significant in determining the element clusters. This weighting determines the relative significance that we attach to dissimilarities between elements in relation to differing constructs. If we apply a uniform weighting then we are effectively assuming that each construct is equally important in determining the grouping of elements. This clearly depends on how the grid was elicited and the purpose of doing the grouping.

Secondly, if we look at an asymmetric implication relation between elements we are again making assumptions about constructs and their significance. The type of relation will be:

$$\forall \text{ LHP} \quad \text{LHP Em} \supset \text{LHP En} \tag{18}$$

which we can abbreviate conveniently to:

$$\text{Em} \rightarrow \text{En} \tag{19}$$

The quantification is now over the predicates so that what we are considering is not expressible in the first-order predicate calculus. The meaning of the expression is dependent on some as-

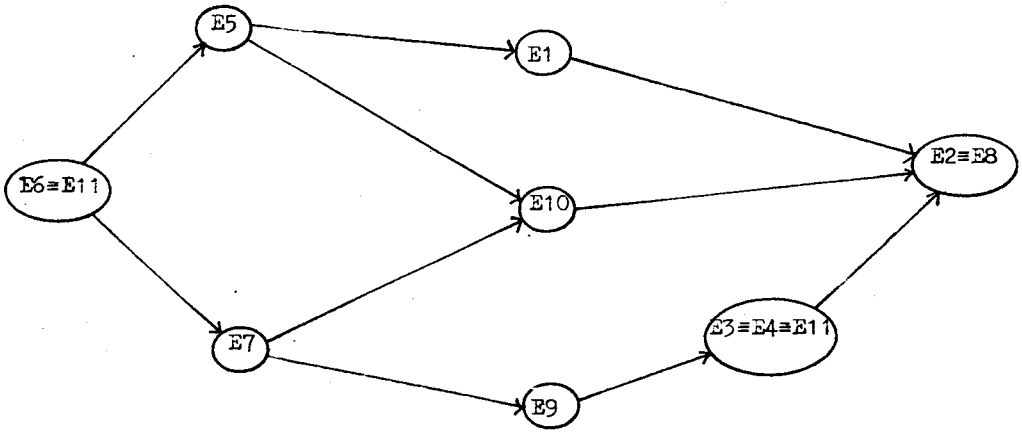


Fig. 12. Preference Analysis of Jane's Elements by ENTAIL

sumptions about the coherence of the class of predicates over which quantification occurs.

One possible source of coherence amongst constructs is that they have a preferred pole and we might then interpret the "for all" in (19) as "for all the preferred poles". The arrow in (19) then defines a preference order on elements since Em → En then means precisely that "En is construed as preferred on at least every construct where Em is preferred". If we use ENTAIL to analyse Jane's grid of Figure 1 for the preference relation between elements assuming for the purposes of this example that the preferred poles are the left-hand ones except for constructs 2 and 3, then we obtain the preference graph between elements shown in Figure 12. The close resemblence of this in terms of clustering between elements to the INGRID (Figure 6), QARMS (Figure 7) and FOCUS (Figure 8) analyses will be noted. However, Figure 12 also contains additional information since it gives a direction of preference.

Thus the logical approach to grid analysis also leads to a comparable element analysis to other approaches. It can also provide additional information about asymmetric relations between elements when an appropriate interpretation such as a preference relation over constructs exists.

3.6. EXTENDING ENTAILMENT TO RATING SCALES - FUZZY SEMANTICS

So far in this paper we have analysed grids with binary assignments of elements to poles using a classical logic with two truth values. In this section we show how the logical analysis extends to the multivalued logics (RESCHER 1969) with which one can analyse grids based on rating scales. The semantics of such rating scales presents a number of problems in its own right. Kelly's original binary assignments may be interpreted as the truth or falsity of predicates. Intermediate points on a rating scale are not so readily, or uniquely, interpretable. For example, the 'neutral' point 3 on a 1 to 5 scale say may be inter-

preted as "this element lies half way between the poles", or as "this element should be assigned to both poles", or as "neither pole is applicable to this element", or "sometimes this element comes under one pole and sometimes another", or "I am not sure what pole to put this under", or "I do not wish to construe this element in this way", and so on. In logical terms we are attempting to use a single truth value to encompass many different modalities (WHITE 1975).

The extension of binary distinctions to multi-valued ones may be treated at a fundamental level. We have already noted in section 2 the close relation of Kelly's constructive alternativism to Spencer Brown's "calculus of distinctions". VARELA (1979) has shown how Brown's calculus may be extended to the multi-valued case. Within a basic bipolar distinction may be interpolated others through logical operations that correspond to expressions that generate paradoxes of self-reference in classical logic. VARELA (1975) shows that an essentially three-valued logical calculus arises from the use of a single self-referential form in Brown's calculus of distinctions. GAINES (1976) shows how such "primitive paradoxes" may be iterated to give an indefinite number of distinctions between the poles of the distinction originally made, and hence how the truth value of an arbitrary proposition may be approximated to any accuracy on a continuous scale through a Dedekind section.

This move from a binary basis for making distinctions to a multivalued one raises problems of a semantic nature even at a fundamental level, particularly those of interpreting intermediate "truth-values" (HAACK 1979). However, the need for rating scales in practice, and an appropriate underlying theory, does seem an essential one in terms of the human construct systems and their logic. In the physical sciences the expected and preferred source system in which to represent data is quantitative. We use a source system of physical quantities and their precise measurement. However, the underlying constructs of physics have been derived and refined over a very long period and are themselves of a peculiar, and perhaps unique, nature. The existence

of continuous and limitless scales for physical variables of length, time, mass, charge and so on, is an important phenomenon that marks out the constructs involved as being different from those in many other sciences.

The existence of refined measuring schemes for some constructs should not blind us to their close relationships to other constructs for which no such physical measurement exists, for example, the concepts of 'tallness' and 'beauty' (GAINES 1976). The concept, the perception, of 'tallness' exists in a more primitive sense than does the measurement of 'height'. We are able to generate and follow arguments involving 'tallness' without having any concept of inches, centimetres, or any other metric scales. Whilst a 'scientific' analysis might conclude that there is a wide and ill-defined range of physical phenomena that combine in an extremely complex fashion to produce the subjective impression of 'beauty', in everyday reasoning it is as primitive a term as 'tallness'. We certainly do not distinguish between them in arguments such as:

> He likes girls that are tall and beautiful.
> Mary is not very tall but very beautiful.
> He will probably like Mary.

Such considerations led ZADEH (1965) to develop a theory of fuzzy sets that closely paralleled that of classical set theory but allowed for 'shades of grey' in set membership. He extended the definition of the characteristic function of a set to include not just the binary values 0 and 1 but also the continuous interval between them. In classical set theory the characteristic function of a subset maps the elements of the universal set into 1 if they belong to the subset and into 0 if they do not. Zadeh allowed the elements to take the values in between also and called them degrees of membership to the subset. He showed that it was possible to extend the normal set-theoretic operations such as union, intersection and complementation, in a simple and natural way to fuzzy sets with continuous characteristic functions.

Since Zadeh's original study there has been a rapid growth in the literature on fuzzy sets and their application to system theory, control engineering, psychological modelling, linguistics, and so on (GAINES & KOHOUT 1977). The related logical calculus derived from fuzzy set theory in the same way that the classical predicate calculus may be related to conventional set theory is of particular interest for this paper and has been presented as a system for fuzzy reasoning. This logic has been found to be one already studied by the Polish logician Lukasiewicz (RESCHER 1969) and of particular importance since WHITE (1979) has shown recently that it avoids paradoxes such as that of Russell's "barber" (HUGHES & BRECHT 1976) which arise from the unrestricted use of the axiom of comprehension in naive set theory. Since its inception fuzzy set theory has been used to model human verbal reasoning and concept processing. GOGUEN (1974) takes a formal axiomatic approach to the notion of a 'concept' in natural and artificial languages and shows within a very general category-theoretic framework that one obtains generalized fuzzy sets.

These considerations led SHAW and GAINES (1979, 1980) to propose a fuzzy set semantics for personal constructs that could deal with the analysis of entailment in repertory grids using rating scales. Here the fuzzy sets and logic have been left deliberately until this late section so that they do not confuse the basic discussion of systems of entailment and their derivation from grid data. Suppose in the discussion of section 3.2 one now assumes that the predicates LHP and RHP are not just true or false, but also have the possibility of intermediate degrees of membership to being true (with false interpreted as a degree of membership of 0 to being true). Then the rest of the discussion of that section follows virtually without change but one now has a model of entailment in grids whose values are not binary. The implication and entailment operations are now those of Lukasievicz multivalued logic, and entailment holding between two poles is now not just true or false but can also take intermediate values.

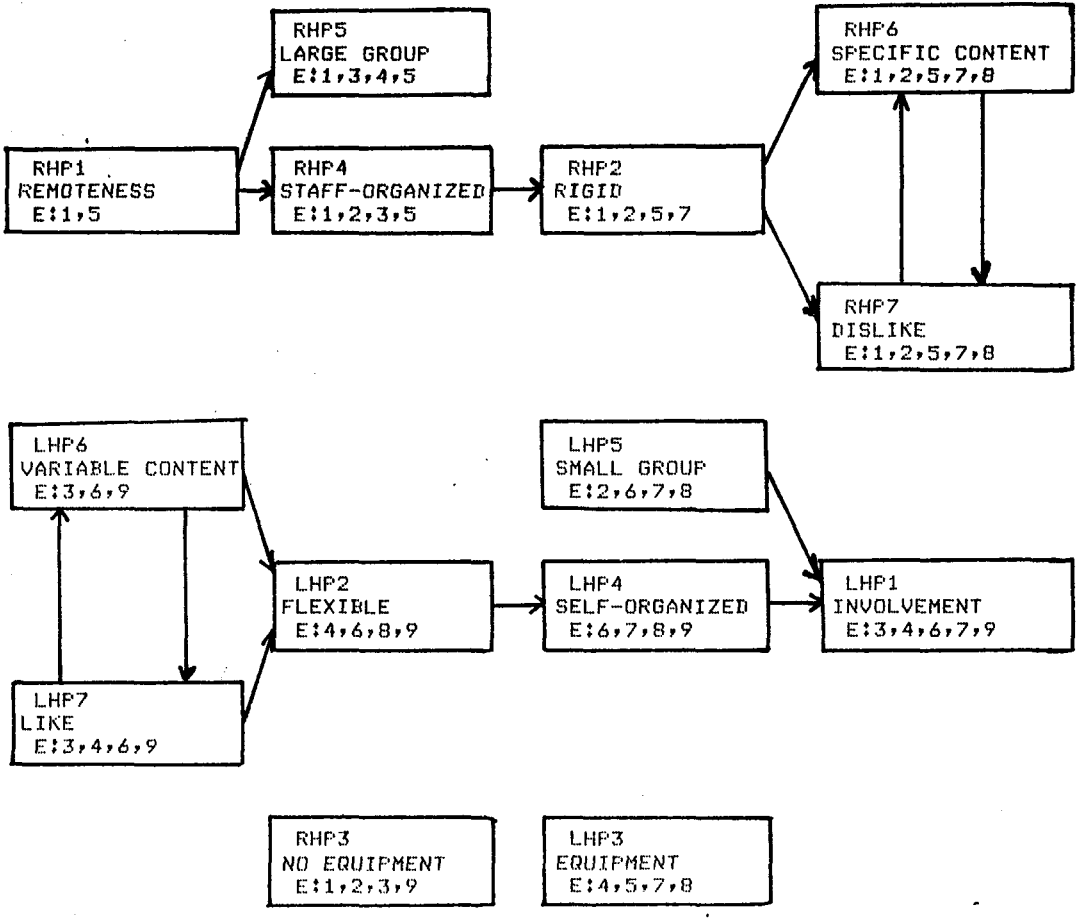


Fig. 13. Fuzzy Entailment Analysis of Data of Figure 2(f) by Entail

The program ENTAIL described in section 3.3 has been written to take into account such multivalued data (as have INGRID, QARMS and FOCUS). The discussions of sections 3.2, 3.3 and 3.5 also generalize immediately to multivalued data and logics. The logic system now provides a measure of the 'strength' of an entailment. Since Lukasiewicz logic defaults back to the standard propositional and predicate calculi when intermediate values are not used (GAINES 1978) it is actually more convenient to develop the whole of the theory of construct structures and analysis described here directly in terms of fuzzy logic and this would seem appropriate for future studies.

Figure 13 shows the ENTAIL analysis of the grid of Figure 2(e) which is based on a 1-5 rating scale. The clusters produced are again very close to those of the FOCUS analysis in Figure 2(e) and again the asymmetric analysis provides additional information. Figure 13 was derived from a table of entailments true at the 4 level on the rating scale. An entailment true at the 5 level (on a 1-5 scale) is absolutely true; that is, if LHPm entails LHPn at the 5 level, then the rating of any element on LHPn is greater or equal to that on LHPm. An entailment true at the 4 level (on a 1-5 scale) is almost true; that is, if LHPm entails LHPn at the 4 level, then the rating of any element on LHPn is at most 1 less than its rating on LHPm. Because the entailment is only approximate the elements under each pole (assigned to the pole if they are rated 4 or 5 on it) do not have the simple inclusion structure of Figure 10; the topological structure is now a fuzzy one.

One important feature of Zadeh's work has been its emphasis on the linguistic nature of human reasoning and the use of fuzzy set theory to model the use of hedges such as very and rather in human reasoning. This is similar to the interpretation of the points on a rating scale in terms of such hedges as "very", "slightly" and "quite" used in semantic differential techniques (OSGOOD, SUCI & TANNENBAUM 1957). Thus there are natural verbal interpretations of the rating scale when values are input and these may also be applied to the equivalent values resulting

from the ENTAIL analysis. One may say that there is a "quite strong" or a "very strong" entailment from one pole to another. Our requirement that the terminology and concepts of the analysis be those of the data thus continue to be satisfied in the extension to multivalued logics.

3.7. NEGATION, OPPOSITES AND RELEVANCE

A number of times here we have noted that the role of the two poles of a construct as opposites has not been adequately treated. In our logical analysis the left-hand pole and the right-hand pole have been treated as distinct predicates of equal status. We have noted (section 3.2) that the conventional elicitation of constructs lends to an inverse relation between the poles such that the predicate corresponding to one pole behaves as the logical negation of that corresponding to the other. This should perturb us since it appears to lead to precisely those defects of formal logic that Kelly warns against:

> "Now conventional logic would say that black and white should be treated as separate concepts. Moreover, it would say that the opposite of black can only be stated as not black, and the opposite of white can only be stated as not white. Thus the person whose field we mentioned would have shoes which would be just as much not white as the time of day, and he would write on paper which would be just as not black as the distance to his office." (Kelly 1955, p. 106)

Part of the problem that Kelly is discussing here is one of relevance. "Not white" is a predicate relevant to shoes but not to the time of day. The standard predicate calculus fails to distinguish between "not" and "not relevant". We noted in section 1 that it is only in recent years that logics accounting for 'relevance' in a very formal sense have been established (ANDERSON and BELNAP 1975). However, what even such logics do not encompass and Kelly brings out is the psychological role of the concept of opposite which has no logical counterpart - it is related to negation but not identical to it.

This introduction of the importance of modelling the role of opposites in human thinking is not peculiar to Kelly but is a

continuing theme in philosophy from early times. The Pythagoreans used a table of opposites in analysing entities with ten constructs such as "limited---unlimited" and "good---evil". MAO TSETUNG (1937) in his essay "On Contradiction" emphasizes the essential interdependence of opposites:

> "No contradictory aspect can exist in isolation. Without its opposite aspect, each loses the condition for its existence...Without life, there would be no death; without death there would be no life. Without above there would be no below; without below there would be no above...It is so with all opposites; in given conditions, on the one hand they are opposed to each other, and on the other hand they are interconnected, interpenetrating, interpermeating and interdependent" (Mao 1937, p. 61)

Mao also brings in the notion of relevance in defining opposites and uses the notions of contradiction yet identity amongst opposites in his exposition of an epistemology which closely mirrors Kelly's constructive alternativism.

This line of reasoning can be traced back through LENIN (1914) to Hegel whose basic logic of thesis and antithesis leading to a synthesis is founded on what seems to be the most careful distinction between opposite and negation in the philosophical literature. Hegel distinguishes between negation as an absolute difference and opposition as an essential difference, and BOGOMOLOV (1977) singles this out as the foundation of dialectical logic:

> "the investigation of the relation of two objects... begins with establishing the difference between them, expressed in the most general form, with their mutual negation (A and ~A). To put it differently the second object acts initially as the simple negation of the first and is naturally expressed in logic by its indefinite negation...Describing this kind of development of the concept, Hegel saw in it the transition from absolute difference to essential difference (variety), and from this to opposition (antithesis), as one of the stages of the general path from identity through difference to contradiction." (Bogomolov 1977, p. 137)

Thus we may see that Hegel's dialectics is crucially dependent on the transition from the concept of general negation to that of opposition. An opposite is some basis for there being negation, some reason for it, and it is the underlying construct

to which this opposition is relevant that Hegel regards as the "synthesis" of the opposition between thesis and antithesis. Thus there is a close relationship between the epistemology put forward by Kelly and that put forward by previous philosophers concerned with dialectics. However, neither Pythagoreans nor Hegelians justify in logical terms their assertion that opposites are fundamental to reasoning. Kelly does not himself do so except by quotations like that at the beginning of this section which point out by example the difference between the negation of a construct and an opposite to it. Indeed one may argue from the presentation so far of a classical logical analysis of the repertory grid that in its original form it has already lost the possibility of coping with either relevance or the distinction between negation and opposition.

If we start with essentially bipolar constructs such that an element must be assigned to one, and only one, pole then we cannot treat relevance within a uniform framework. Kelly has to introduce it separately in terms of constructs having a "range of convenience". However, by considering an element to have quite distinct assignments to the two poles of a construct, i.e. to a construct and its "opposite" we can also capture the concept of relevance. A construct is irrelevant to an element if the element is assigned to neither of its poles (or, in the context of fuzzy logic, if its degree of membership to both poles is zero). Thus, in terms of Kelly's example at the beginning of this section the construct "white---black" is irrelevant to the time of day because it is both not white and not black. Those who extended his bipolar notion to allow for multipoint rating scales also failed to allow for relevance when they made the scales a 1-dimensional interpolation between the two poles of a construct. However, the approach taken here is readily extended to the multipoint case by allowing separate ratings on the two poles of the construct. It is clearly debatable still whether this explication of relevance captures all its psychological connotations. We would suggest only that it captures some key ones.

What we have proposed is a very simple extension of Kelly's repertory grid methodology that gives us a logic capable of

dealing with relevance and Kelly's notion of a "range of convenience". The mechanism used is crucially dependent on every predicate having an "opposite" so that one can distinguish between the predicate being not true for an element (element assigned to opposite predicate) and its being not relevant for an element (element assigned to neither predicate nor opposite). This demonstrates the importance of the concept of an "opposite" emphasized by so many different philosophers and gives a formal model for the utility of opposites. In previous papers we have analysed the semantics of opposite predicates and developed various logical constraints upon them (SHAW & GAINES 1979, 1980). However, in the present context of repertory grid analysis an opposite predicate is just whatever the person from whom the grid is elicited chooses it to be. The ENTAIL analysis will cope with assignments to the two poles of a construct that are completely unconstrained in their mutual relationships.

The introduction of relevance in this way overcomes some of the objections raised against semantic differential techniques. For example, in his review of The Measurement of Meaning (OSGOOD, SUCI & TANNENBAUM 1957), Weinreich notes:

> "If warm is put at the midpoint of the hot-cold scale, this is probably a valid SEMANTIC judgement on an appropriate scale. Similarly, foreigner may occupy an AFFECTIVE midpoint on a good-bad scale. But the midpoint location of, say, boulder on the sweet-sour scale is not due to either semantic or affective judgements, but the inappropriateness of the scale to the concept. An investigation to be productive should include these considerations as axioms; subjects should be free to select and reject scales." (WEINREICH 1958, footnote 16)

Essentially our not relevant assignment provides just such a possibility of rejection.

The possibility of making separate assignments to the two poles of a construct and of analysing such extended forms of the repertory grid seems significant for other problems with grids already noted in the literature. SLATER (1977, p. 46) points out that missing data creates major problems for distance-based grid analysis, and yet it is a common problem. Kelly states:

> " The assumption which is specific to a grid form of the test is that all the figures fall within the range of

> convenience of the constructs...This may not be a good assumption in all cases; it may be that the client has left a void at a certain intersect simply because the construct does not seem to apply one way or the other." (KELLY 1955, p. 271)

LANDFIELD (1976, p. 97) gives an example of a grid elicited from a patient which goes beyond this and allows the two additional values "N" for neither pole applicable and "?" for either pole applicable. In terms of our discussion above his "N" corresponds to an assignment of false to both poles and his "?" corresponds to an assignment of true to both poles. Thus the grid he elicits is readily analysed by ENTAIL. Obviously when ENTAIL analyses a particular entailment between a pair of poles under these circumstances it is relative to the elements actually construed in relation to those poles. However, it is possible to provide an analysis which does draw as much as possible out of the data given and does not crucially depend on all elements being assigned to one pole of every construct.

We shall note in the next section that the logic being used by ENTAIL in dealing with Landfield's data is essentially that proposed by BELNAP (1976) for use in database systems.

3.8. SUMMARY

This section has put forward a fundamental logical analysis of semantic space expressed as constructs regarded as predicates applying to elements including other constructs. The operational form of this analysis in a computer program has been shown to give a richer structure to semantic space which includes and subsumes that of a variety of distance-based analyses. The hierarchical structure of the space, the fuzzy semantics underlying it and the particular roles of relevance and opposites in semantic space are included simply and naturally in the analysis. In the next, and final, major section of this paper we examine the normative influences of social communities on the personal semantic spaces defined by construct systems.

4. INTERACTION OF PERSONAL AND COMMUNAL SEMANTIC SPACE

So far in this paper we have presented foundational and operational models of personal semantic space with some practical examples of how interactive computer programs may be used to explore the space. Although we touched briefly on the problem of exploring the inter-relations between the semantic spaces of different individuals in section 2.4, the theoretical sections of this paper have concentrated on personal semantics. One of the key problems of a conventionalist epistemology such as that proposed by Kelly and developed in this paper is to explain the way in which people do come to have common constructs and are able to communicate with one another. If we take Kelly's viewpoint to be epitomized by the concept of man as a personal scientist, then we also have to explain how a commune of science can arise in which individuals can reach agreement on a common view of the world or on co-operative programmes of action. In this section we outline a model of the formation of communes of science which integrates Kelly's conventionalist epistemology with the normative epistemology of POPPER's (1968) concept of world 3s.

4.1. SURRENDER AND CATCH

At the end of section 2.5 we referred to the work of WOLFF (1976) on the fundamental nature of role playing in epistemology. His book about "experience and enquiry today" has much in it that is relevant to the discussion of this section. In particular his basic model of "surrender" to a situation so that one may "catch" the essence of it - of becoming through surrender someone different in order to catch hold of something different - gives a valuable insight into the nature of the processes involved in adopting construct systems in order to achieve knowledge and action. Wolff's book is fragmentary, discursive, literary, even poetic in places, and this is in itself also significant - we

shall not catch the essence of any theory of knowledge from within its own framework of abstraction.

Wolff's model of surrender is most relevant to the current context in terms of his discussion of the nature of the "mathematician":

> "From the standpoint of the world of everyday life, the mathematician, as we often put it, lives in the 'world of mathematics', dealing with 'nonreal' elements, notably numbers, whose relation to 'real' things, to 'reality', is not part of his concern. Analogously for the logician. What makes our subject-object approach to this attitude misleading is the fact that the subject, the student of mathematics or logic - his or her individuality, including motives and attitudes - is irrelevant for our understanding; the only thing that counts is the pursuit, with its results and questions." (WOLFF 1976, pp. 162-163)

He makes the key point that not only does the real world, the object, disappear to be replaced by the world of mathematics, but also that in entering into this world the person doing mathematics, the subject, also disappears to be replaced by a new entity, the mathematician.

Wolff was not alone in these insights and there are two further models which develop and complement his. In terms of his "subject" PASK's (1975) concept of "P-individuals" as coherent psychological processes capable of engaging in conversations is a useful representation of the results of Wolff's "surrender", particularly when we note that several P-individuals may execute within a single processor. Thus 'the mathematician' may engage in conversation with 'the physicist' or 'the statesman' all of whom happen to use the same brain for their processing. In terms of Wolff's "object" POPPER's (1968) concept of a "world 3" of "statements in themselves" is a useful representation of that which we "catch", particularly when we note its distinct ontological status:

> "I regard the third world as being essentially the product of the human mind. It is we who create third-world objects. That these objects have their own inherent or autonomous laws which create unintended and unforseable consequences is only one instance (although a very interesting one) of a more general rule, the rule that all our actions have such consequences." (POPPER in Schilpp 1974, p. 148)

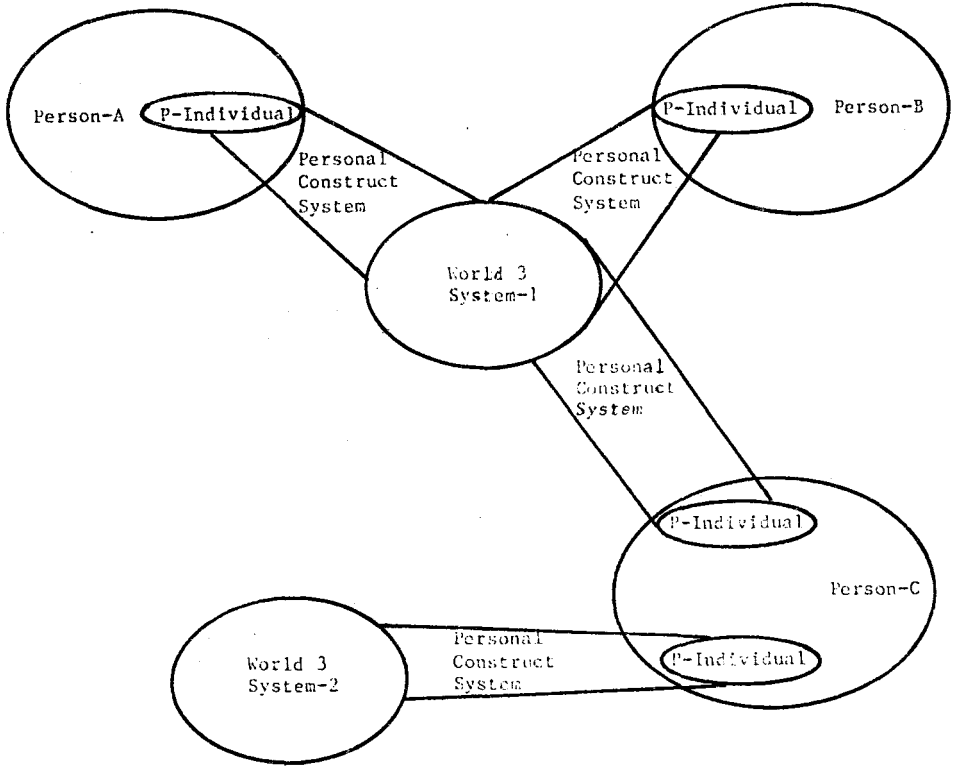


Fig. 14. P-Individuals Surrendering to Catch Essence of World 3s through Personal Construct Systems

SHAW (1980a) notes that each P-individual may have its own system of "personal constructs". We thus have a model of a P-individual within a particular person using a particular construct system by which to surrender himself to a particular world 3 in order to catch its essence.

This model of coherent intellectual processes, each with its own individuality, each using its own construct system to enter into an abstract world that has its own reality, provides a very cogent picture of the rationale behind, and the conflicts involved in, the interaction between our personal semantic spaces and our communal ones. It goes beyond any simple concepts of "role playing" by providing the basis for formal models of the mechanisms whereby P-individuals can enter into an abstract world. The "constructive alternativism" of Kelly provides a model for the extreme relativism of our conception of a world. However, its conventionalism needs counterbalancing by Popper's emphasis on the ontological independence of that world in itself.

Figure 14 illustrates diagrammatically the integrative model we have proposed. Person-A, Person-B and Person-C are each interacting with the same World 3 System-1 through personal construct systems that correspond to P-Individuals within each person. We have shown the construct systems of B and C overlapping to indicate that they have shared constructs. It is reasonable to suppose that they could establish linguistic communication between the two of them in terms of these shared constructs. Person-A, however, is shown as sharing no constructs with B and C even though he is interacting with the same World 3. This may seem strange but it is possible for someone to interact with essentially the same structure on a very different basis from someone else. For example consider two mathematicians, one of whom solves a geometry problem by direct geometric techniques and the other by algebraic techniques based on a co-ordinate system. It is the same problem they are tackling, and it is this which defined the world 3, but their views of it and the techniques they use are so different that they have virtually no

common vocabulary for describing their different solution methods, how they interact with the world 3. In more formal terms we would describe them as being analogous to one another through the minimal analogy of the truth category formed by the world 3; otherwise there is no analogy between their methods and construct systems (GAINES & SHAW 1980). This very clearly illustrates the ontological independence of a world 3 structure.

Figure 14 also shows a second P-Individual in Person-C interacting with a second World 3 System-2. The diagram may be extended to show situations where different P-individuals interact, either through sharing partially common construct systems, or by interacting with the same world 3, or both. It may also be used to show situations where different world 3s themselves overlap. It is when the world 3s have some basis of autonomously interacting back to the people that we seem to distinguish them as real worlds. Note that the so-called physical world is not the only one with this kind of power; certain ideas and ideals seem to have a charisma of their own which gives them the strength of categorical imperatives, ideologies, religious truths, etc. This is reminiscent of Henry ADAMS' (1961) comparison of the hall of dynamos at the Great Exposition in Paris where they "became a symbol of infinity... a moral force, much as the early Christians felt the cross", with the Cathedral at Chartres: "All the steam in the world could not, like the Virgin, build Chartres." GAINES (1979b) has suggested that the computer is a source of energy for traversing Popper's world 3 much as was the steam engine for his world 1. Programs such as PEGASUS described in section 2.3 show how such a computational 'engine' can operate in practice.

4.2. LOGICS AND CONFLICT RESOLUTION

Perhaps an obvious question at this stage is as to the necessity of the 'schizophrenia' within the person operating a number of distinct P-individuals. Even if we accept that this happens we do not have to accept that it is desirable. Quite the con-

trary - is not the ideal the "whole man" who integrates together all facets of his existence and is able to avoid conflicts between his being as a 'humanitarian' with his being as a 'scientist'? The basic problem here is one of over-determination. It is an easy one to neglect because we seem to pay far more attention to decisions under uncertainty where we have insufficient determination than to decisions under conflict where we have too much determination. We avoid such conflicts by using construct structures associated with particular world-views that filter out all other aspects of the world. To enter in the world of the mathematician is not only to learn to use new constructs that form the key to that world but also to learn to put aside others that would impede entry to that world. This is the way in which the world 3s themselves interact with the person to mould his construct system to make it possible for him to move about within them. In this process constructs may be changed or removed but this may endanger the capability of the person to operate in other worlds. Hence P-individuals are formed within coherent clusters of constructs through which a person enters a particular world 3.

Clearly essential conflicts do not somehow magically disappear when a person establishes different P-individuals. The overall result is that the conflicts may be avoided within a particular psychological processing unit and isolated to appear at the conversational boundaries between them. Each P-individual can then have its own coherence not only in terms of self-consistency but also in terms of mutual consistency with others who subscribe to the same 'rules of the game'. For example to be a 'behavioural psychologist' is to view man in a certain way using a certain methodology and pattern of allowable inferences. Within the framwork of psychological science there is an epistemic drive to explore fully all the behaviours of men. As 'psychologists' there are experiments which we would wish to perform which involve behaviour towards our fellow men which would clearly be unacceptable to us as 'humanitarians' (or 'law-abiding citizens' or ...). How we resolve the conflict between the differing objectives of the P-individuals interested in such

situations is clearly a significant problem. It has always been a major one for those involved in the science and technology of human life such as clinicians, and books such as The Social Responsibility of the Scientist (BROWN 1971) point out that there are undesirable side effects from many scientific and technological activities otherwise regarded as of great value.

The ethical conflicts of medicine and other sciences are real and irresolvable. There are also lesser conflicts which are conveniently treated in the same way - it is conventional for us in scientific papers not to discuss the way in which our family holidays interacted with our experimental program (although it would often be helpful to know what parameters are those of experimental design and what of expediency!). If we do attempt to take the 'whole man' approach then our 'sciences' assume new dimensions - "$E = MC^2$." might become TRUE but UNREPORTED (due to dangerous consequences). Technically we can cope with the injection of ethical and other value schemata into science by using many-sorted modal logics to deal with the differing deontic modalities involved. So far we have not done this formally even for epistemic logics, e.g. to treat 'unknown' as a logical value of equal status to 'true' and 'false'. In practice, of course, much of the actual argument of scientific debate is based on informal epistemic logic, and in some disciplines such as elementary particle physics and cosmic physics epistemic constraints have come to dominate the interaction between theory and experiment - we are dealing with entities whose time and space scales are intrinsically too small, or too large, for definitive experiments to be feasible.

It is only in the last decade that formal methods for treating many-sorted epistemic and deontic logics have become available (SNYDER 1971). The computational complexity of these treatments is such that the M-individual that is a human brain cannot be expected to support P-individuals that can cope with them. It seems feasible that computers, or man-computer symbiotic systems, may be able to do so. Indeed this is a natural, and probably necessary, extension of current database systems (GAINES

1979a); and BELNAP (1976) has already suggested that epistemic logics using "TOLD TRUE" and "TOLD FALSE" have a significant role in databases. He also notes that we might wish to go further and tag TOLD TRUE as TOLD TRUE BY X - clearly this may be extended to deontic predicates also, such as FELT WRONG BY Y, ILLEGAL IN FRANCE, and so on. It is interesting to note that the logic being used by ENTAIL to deal with Landfield's four "truth values" in section 3.7 is precisely that proposed by Belnap. GAINES (1979a) shows that such a logical structure also avoids the possibility of paradoxes such as that of Russell's barber arising through the imposition of semantic constraints on a database, and suggests the extension of the logic to continuous values in order to avoid deeper paradoxes.

The development of automated database and inference systems with such capabilities does seem to be of potentially great importance and significance. However, it does not in any way avoid the basic problems already referenced. Any attempt to implement a consensual decision-maker within such a system is doomed to failure in general since it involves mapping many modalities down into a single set of necessity and impossibility. Such a mapping will give rise to contradictions just at those points where there are conflicts: A is NECESSARY because A is DESIRABLE and A is IMPOSSIBLE because A is ILLEGAL. The assertion of such a contradiction allows the derivation of any other statement - the decision-maker oscillates or hangs. If we derive consensual sub-systems from the database, for example over all mathematicians, then we obtain the "world 3" (of 'mathematics') within which move particular P-individuals (who are 'mathematicians').

4.3. SUMMARY

This section has sketched out theoretical foundations for integrating the conventionalist model of personal semantic space developed in previous sections and the normative model of knowledge structures in Popper's theory of "three worlds". It

gives the beginnings of an account of how "personal scientists" interact with the "real world" and with one another to form "communes of science". It also again emphasizes the role of the computer as a tool with which to explore the semantic spaces formed through the existence of world 3s, and hence to explore these worlds themselves.

5. CONCLUSIONS

We have provided a logical model of personal semantic space based on Kelly's personal construct theory and have used Popper's three worlds model to explain the normative forces on personal constructs. We have shown how our theories have been operationalized through programs such as PEGASUS and ENTAIL which provide tools for the empirical analysis and exploration of personal semantic space. Our logical model is based on Zadeh's fuzzy semantics and provides an explicatum for the hierarchical structure of semantic space and the notions of relevance and opposite. It also provides a model of the formation of Pask's P-individuals and Wolff's notions of surrender and catch.

In the introduction it was noted that this work is part of an ongoing research programme concerned with the development of man-computer symbiotic systems for the amplification of human intelligence, creativity and similar capabilities necessary to cope with complex worlds of knowledge, decision and action. We hope we have provided some indication of the direction of this programme, its current status and possibility of future success.

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