out examples to make sure one has not gone astray. Also, the mathematical notation was not as clear as it could have been if some additional explanations had been given. Much of the algebraic manipulation of the derivation is left to be worked out by the reader. Finally, the paper gives no hint as to why it was undertaken, except to say that no synthesis previously existed. If, by way of background, a brief description of the reason for synthesizing all possible DAC ladders were included in the paper, it would have made it more interesting and less academic.

However, these are minor points. The paper is instructive and of interest to those involved in the design of digital-to-analog converters or analog computers.

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F. STOCHASTIC COMPUTATION

R68-18 Random Pulse Machines—S. T. Ribeiro (*IEEE Trans. Electronic Computers*, vol. EC-16, pp. 261–276, June 1967).

In conventional digital computation, continuous quantities are quantized and represented by binary words in which the number of bits determines the precision of representation. In stochastic computation, as described in this paper, continuous quantities are represented as a sequence of one-bit binary words (i.e., as a pulse stream or sequence of logic levels) in which the probability of an ON logic level is a measure of the quantity. Since probability is a continuous variable in the range $0 \leq p \leq 1$, this removes the effects of quantization. However, a probability cannot be measured precisely, only estimated subject to random variance, and hence there is an effective random noise in the output of the computer.

Alternatively, the stochastic computer may be compared with other forms of computer in terms of efficiency. Representation of quantities as random variables enables very simple elements to be used to perform arithmetic operations. For example, a single EXCLUSIVE-OR gate acts as a multiplier for two signed numbers; far more complex and expensive hardware is required for four-quadrant multiplication in either conventional analog or digital computers. Similarly, simple digital elements may be used in the stochastic computer to perform all the various operations—addition, subtraction, integration, and so on—commonly associated with the analog computer.

The stochastic computer is thus very efficient in its utilization of hardware. On the other hand, the noise inherent in stochastic computation makes N^2 pulses necessary to define a quantity with a precision of 1 part in N. This compares unfavorably with other pulse-counting computers, such as the DDA, which need N pulses for the same precision, and even more unfavorably with the general-purpose digital computer, which requires $\log_2 N$ bits to obtain this precision. The stochastic computer is thus inefficient in its use of the potential information content of a data stream, and hence its operations are slow compared with those of other machines. It scores when large amounts of low-bandwidth (10 Hz) data have to be processed in parallel in a fairly complex fashion, for example, in pattern-recognition or multivariable adaptive control.

It is difficult to trace the origins of stochastic computing back very far, although one has the suspicion that the extremely simple concept of representing analog variables by the probability of occurrence of discrete events may well have a fairly ancient lineage. However, although the early Chinese had a grasp of number theory which gives them some right to claim prior disclosure on patents for residue-number machines, the concept of a quantitative representation of random events is itself sufficiently recent to preclude this for stochastic machines. On the other hand, the human brain seems to use stochastic computing as its mainstay (perhaps), and might be considered prior art!

Certainly the possible genesis of stochastic computing may be found in the use of high-frequency sawtooth "dither" to round off the corners in the output of diode function generators, and, more recently, to linearize polarity coincidence correlators, but none of this work has ventured beyond the immediate application and generated a complete computing system based on the representation of continuous variables as probabilities.

Suddenly this has been done, with papers presented at both the Spring and Fall Joint Computer Conferences this year, together with the excellent introduction to stochastic computing principles which is the subject of the present review.

The author first introduces the concept that a logical interaction between two independent binary sequences in a two-input, single-output gate is equivalent to an arithmetic operation on their generating probabilities, and tabulates this operation for the 16 possible Boolean functions of two variables. He then presents the problem of coding any analog variable as a probability in such a way that this effect may be used to perform arithmetic. Finally, the author considers time-dependent operations, such as integration, gives a brief analysis of the errors inherent in stochastic computation, and suggests some possible applications.

In its treatment of memoryless computing elements and stochastic representations of quantity, this paper is particularly comprehensive and much to be welcomed. Time-dependent operations are not covered in as great a depth. In particular, the stochastic integrator with feedback, which is usually the outward interface of the computer and fundamental to the solution of differential equations on the stochastic computer, is hardly mentioned. Because of this, the wide range of potential applications in control systems, both for identification and optimization, is not described.

However, these are minor criticisms of the first paper to appear in this TRANSACTIONS on a new type of computer which presents exciting problems and a fruitful area for research. The main obstacle to the practical application of the stochastic computer is, at present, the generation of the random variables required in a reliable and economical manner. It may well be that we should look to truly random physical processes, such as photon–photon interactions, to provide the hardware foundation for stochastic computing systems.

This is a relatively unexplored region, however, where much further effort by mathematicians, engineers, and physicists will be required before we can fully define the potential impact of stochastic techniques on future computing systems. Fortunately, there is sufficient commercial promise and intellectual novelty in the concept of stochastic computing to attract ever-increasing research effort in a wide range of locations.

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