Gaines, B. R. (1970). Stochastic automata and learning systems. In Proceedings SRC Seminar on Mathematics for Control, pp. 22-27. Department of Engineering, University of Cambridge, Cambridge.

DISCUSSION OF STOCHASTIC AUTOMATA AND LEARNING SYSTEMS

B. R. Gaines

1. Background

Linear systems theory has formed the main mathematical foundation for control engineering, and has given rise to a number of powerful techniques for coping with practical situations. Even outside the domains where it is strictly applicable some form of quasi-linear theory, involving the informal neglect of some of the implications of linear theory, has proved very useful.

However, there is an increasing need for mathematical systems techniques capable of dealing with situations where no overall assumption of linearity is tenable, and capable of providing the maximum power (in system analysis and synthesis) available under the less-restricted conditions. The non-linear theory is particularly required in the following four types of situation -

(i) Plant essentially non-linear - the operation of plant in deliberately non-steady-state conditions can be attractive commercially but generally leads to wide swings in operating conditions over comparatively short periods, the system appearing both nonlinear and time-variant.

(ii) Controller essentially nonlinear - although digital computers may be used as linear PID controllers this is generally an inefficient use of resources and some conditional-action or 'decision making' mode of control based on boundaries in plant conditions is more effective - this leads to a discrete-action system with non-uniform sampling periods.

(iii) Both controller and plant essentially nonlinear - here we move into the domain of so-called learning systems, pattern-classifiers, and so on, where no form of linearity is assumed in the plant and

implemented, perhaps, only in some sub-section of the controller.

(iv) The overall system may be essentially assumed linear but the mathematical development based on linear systems theory is cumbersome. This situation tends to be neglected in discussion but is of profound importance to the progress of control theory - there are many important system properties which hold both for linear systems and for certain classes of nonlinear systems - i.e. certain postulates of linear systems theory are not required. As is usual in mathematics, superfluous postulates may lead to a cumbersome theory by offering a host of routes to the same goal. This is particularly true in the field of differential-integral equations where a quaint notation hides the essential nature of the operator structures involved.

2. Algebraic system theory

The algebraicising of system theory is not new, dating back at least to Heaviside, but it is only in recent years that the necessary mathematical tools have been developed. The first book on algebraic semigroups is of 1960 vintage and it is only from 1967 onwards that the literature and the range of established results has shown substantial expansion. In the same way that group theory was developed in major part by, and for, the physicist, it is not unreasonable to predict the development of semigroup theory by and for the systems and control engineer.

In hindsight the late development of algebraic system theories is surprising considering the purely algebraic content of many important properties of the homeomorphism in topology and of the essential simplicity of the techniques in functional analysis using Banach algebras. However, it is easy to forget that Eilenerg and MacLanes' theory of natural equivalences postdates Banach's work by some twenty years, and that functional analysis itself has only recently become a common tool

in engineering mathematics.

Whatever the historical antecedents, and they are fascinating in their own right particularly in the interplay between pure mathematics and its applications, it seems probable that we have reached the point of swing on the pendulum where the postulation of very strong topologies in a system and the development of theorems based on the concomitant algebraic properties will be replaced by the postulation of more or less strong algebraic properties and the derivation of concomitant topologies only as required.

3. General Implications

One mundane, but nevertheless important, implication of algebraic developments is that sleepless nights spent contemplating the iniquity of not teaching the full rigour of Laplace transform methodology to engineering students need be no more - it is so intuitively obvious that the 'rigour' based on convergence of infinitesimal processes is irrelevant, if not positively misleading, that this in itself should have lead to a reappraisal of the postulate system involved. This reappraisal is now virtually complete, theoretically at least, and it is probable that the name Kalman will be associated with transform theory as closely as that of Laplace at present.

The more profound implications are far less obvious - it is clear that the pendulum is swinging but not exactly where its orbit will take it. Informally, one can see a tendency towards the examination of systems for intrinsic properties rather than their conformance to externally required properties, for example, confluence sets rather than 'stability' - this is consistent with the increasing interest in system identification. In practice it means the examination of a wider variety of possible system modes for their commercial outcome rather than concentration on the problems of enforcing a pre-determined mode of

operation.

In particular, the distinction between 'local', or initial-value, control techniques and 'global', or boundary-value, control methods will become more sharply focussed. Controlling overall system constraints to create a suitable environment for the application of particular control strategies is the whole basis for the practical application of linear systems theory, but the concepts involved have far wider application. The local global distinction is vital to the development of self-optimising and learning controllers where the convergence of the local control strategy is highly dependent on the environmental situation. In the application of learning systems, controller synthesis becomes a problem of creating a suitable environment for training the learning system.

4. Objectives of Systems Theories.

Generality in a theory is expensive if it does not allow specific properties of particular systems to be encompassed. Thus a systems theory based on algebraic semigroups is only a starting point - the useful structures are the bridges between the general and the particular. In linear systems theory one has an algebraic structure consisting of an Abelian group embedded in a field, that is a linear manifold. The axioms of a linear manifold may each be interpreted in physical terms - that the basic structure is a group implies that every element has an inverse, that is that the system is reversible - that the group is commutative implies that the order of occurrence of 'events' does not matter - the distributive laws relating the embedding of the group in the field imply superposition and the extension of local knowledge to global knowledge. Having established the relationship of the structure to the field, the latter then itself provides a number of very convenient topologies, including a variety of metric spaces.

This structure is so enormously rich that the mathematician's problem becomes one of finding results that actually require all the axioms. Results which do not can be proved by so many routes of equal merit that the probability of the one with which his name is associated lasting long is vanishly small. Conversely, the engineer's problem is that of finding systems, or creating systems, which obey most, if not all, of the axioms. The main reason why so many systems work according to theory, even though they clearly contravene some postulates of the theory is that many alternative axiom systems would lead to the same result.

Thus more general systems theories can be given the very definite objectives of -

(i) Defining what axiom sets are necessary and sufficient to the proof of fundamental (i.e. useful) results.

(ii) Establishing the constraints which may be placed on a system which does not satisfy a given postulate, i.e. does not possess a certain global property, to cause it to satisfy that postulate.

At present one can see a definite program moving towards satisfying the first objective - it is basic theoretical spade-work that will be done. However, the second 'objective' is far more open-ended, and it is not at all clear whether it is a meaningful goal, or rather the fusion of all problems remaining once the first objective has been attained.

5. Stability, Stochastic Automata and Learning Systems

Examples of techniques for attaining the second objective given in the last section are numerous - anyone who has been involved in trouble-shooting an amplifier will have realised that 90% of it is concerned with making the other 10% operate in an analysable mode - a fault in the major part and theory no longer applies to the rest. However, the most interesting of such techniques are those in which a comparatively weak

global influence is used to turn a difficult and complex situation into a simple and manageable one. For example, the injection of noise to stabilise an otherwise unstable system to enable a simple control algorithm to replace a complex one, or to improve the numerical accuracy of a simulation. Such examples of stochastic behaviour are often counter-intuitive but form the basis of important phenomena, particularly in living systems.

In the main presentation, a framework for the analysis of adaptive and learning systems in terms of generalised stability theory will be outlined. The nature of inducing convergence in a learning system (training it) treated as a control problem and a global stability problem will be contrasted, and a brief example will be given of the behaviour of a simple stochastic learning automaton illustrating the locally-controlled induction of convergence, and the global induction through the injection of noise.

From: J. F. COALES, OBE, CENG, F.R.S.
Professor of Engineering (Control)

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CONTROL ENGINEERING GROUP
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June 18, 1970

Dear Gaines,

Symposium on Mathematics for Control

The Science Research Council has asked me to arrange a Symposium on Mathematics for Control at Cambridge during the weekend September 25-28 this year. With the assistance of my colleagues and in conjunction with Professor Lighthill I have proposed the attached programme and this has been agreed by the Control Engineering Committee and the Mathematics Committee of the Science Research Council.

From this programme you will see that we propose to have a number of discussions on topics important to control engineers which we believe to be of interest to mathematicians. The provisional plan is that each session should be of 1½ or 2 hours duration and that the subject for discussion should be introduced by two people, where possible first by an engineer and then by a mathematician. It is intended that the introductory talks should not take up more than 45 minutes in all, leaving approximately an hour for discussion. It is hoped that the two Openers of each session will be able either to meet or to correspond in order to agree on how best to divide the time allowed for the opening contributions.

The purpose of this letter is to invite you to attend the Symposium and to be one of the Openers for the discussion on Stochastic Automata. Accommodation, including meals, will be provided for you in St.John's College for the duration of the Symposium and you will be reimbursed for your travelling expenses in the U.K.

If you are able to accept this invitation to open the discussion on Stochastic Automata I should be grateful if you could let me have a manuscript of your opening remarks by the 12th September; we will then arrange for it to be duplicated and distributed at the beginning of the Symposium.

Since time is getting short I should be glad if I could have your reply in the course of the next fortnight.

Yours sincerely,

John F. Coales

J. F. Coales

B.R.Gaines,Esq.,
Electrical Engineering Science,
University of Essex,
Wivenhoe Park,
Colchester,
Essex.

Enclosure.

Programme

Cambridge Seminar on Mathematics for Control

FRIDAY, SEPTEMBER 25, 1970

2.30 p.m. to 4.30 p.m. — CALCULUS OF VARIATIONS

Chairman: Professor M.J. Lighthill

Openers: Professor J.H. Westcott
Professor L. Markus

6.30 p.m. — Dinner in St. John's College

8.30 p.m. to 10.30 p.m. — STOCHASTIC AUTOMATA (Learning Systems)

Chairman: Professor G.A. Barnard

Openers: Mr. B.R. Gaines
Professor H.C.Longuet-Higgins

SATURDAY, SEPTEMBER 26, 1970

9.15 a.m. to 10.45 a.m. — CANONICAL FORMS OF REPRESENTATION OF DIFFERENTIAL EQUATIONS

Chairman: Professor R.J.A. Paul

Opener: Professor H.H. Rosenbrock

11.15 a.m. to 12.45 p.m. — STABILITY OF NON-LINEAR SYSTEMS

Chairman: Professor H.H. Rosenbrock

Openers: Professor M.J. Lighthill
Professor C. Storey

1.00 p.m. — Lunch in St. John's College

2.30 p.m. to 4.00 p.m. — Demonstration of the Cambridge Hybrid Computer - CASSANDRA

4.30 p.m. to 6.30 p.m. — FUNCTIONAL ANALYSIS (Integral equations, orthogonal and Volterra expansions)

Chairman: Professor R.H. Sargent

Openers: Professor William L. Root
Dr. J.F. Barrett

7.30 p.m. Seminar Dinner to be held at Clare Hall in Herschel Road

SUNDAY, SEPTEMBER 27, 1970

1.00 p.m. — Lunch in St. John's College

4.30 p.m. to 6.30 p.m. — DECISION THEORY

Chairman: Professor C. Longuet-Higgins

Openers: Dr. J. Willems
Professor P. Rivett

7.30 p.m. — Dinner in St. John's College

MONDAY, SEPTEMBER 28, 1970

9.15 a.m. to 10.45 a.m.	STATE AND PARAMETER ESTIMATION	
	Chairman:	Professor J.H. Westcott
	Openers:	Dr. D.Q. Mayne Professor Y.H. Ho
11.15 a.m. to 12.45 p.m.	PARTIAL DIFFERENTIAL APPROXIMATIONS	
	Chairman:	Professor C. Storey
	Opener:	Dr. J.D. Roberts
1.00 p.m.	Lunch in St. John's College	
2.00 p.m. to 4.00 p.m.	NUMERICAL OPTIMISATION	
	Chairman:	Professor J.F. Coales
	Openers:	Dr. G.F. Bryant Mr. M.J.D. Powell

Accommodation has been reserved for those who requested it in St. John's College for the nights of Friday 25th, Saturday 26th and Sunday 27th September and meals, including breakfast, will be available in the Great Hall. A dinner has been arranged for the Saturday evening in Clare Hall, a new postgraduate College situated in Herschel Road.

All meetings will be held in the University Engineering Department, Control Group's premises in Mill Lane, Cambridge.

Accommodation and meals will be provided for those attending the Seminar; also travel costs within the U.K. can be recovered during the meeting from Dr. H.K. Cameron.

University of Cambridge

Control Engineering Group

Seminar on Mathematics for Control

25-28 September, 1970

List of participants

*	1.	Professor F.M. Arscott	(University of Surrey)
*	2.	Professor G.A. Barnard	(University of Essex)
*	3.	Dr. S. Barnett	(University of Bradford)
	4.	Dr. J.F. Barrett	(University of Cambridge)
*	5.	Professor J.A. Bather	(University of Sussex)
*	6.	Dr. N.H. Bingham	(Westfield College, University of London)
*	7.	Mr. P.A.N. Briggs	(Warren Spring Laboratory)
*	8.	Professor T.J.M. Boyd	(University College of N.Wales, Bangor)
Mon.28 only	9.	Dr. G.F. Bryant	(Imperial College of Science & Technology)
*	10.	Dr. G.H. Burkhardt	(University of Birmingham)
**	11.	Mr. E.G.C. Burt	(Royal Aircraft Establishment)
	12.	Professor J.F. Coales	(University of Cambridge)
*	13.	Professor J.L.B. Cooper	(Chelsea College of Science & Technology)
*	14.	Professor W.D. Collins	(University of Strathclyde)
*	15.	Dr. M.J. Davies	(University College of N.Wales, Aberystwyth)
*	16.	Dr. D.V. Evans	(University of Bristol)
	17.	Dr. A.T. Fuller	(University of Cambridge)
*	18.	Mr. B.R. Gaines	(University of Essex)
*	19.	Mr. P. Gill	(N.P.L. Teddington)
*	20.	Dr. B. Girling	(City University)
*	21.	Dr. J.A. Glen	(University of St. Andrews)

*	22.	Professor F.A. Goldsworthy	(University of Leeds)
*	23.	Dr. W.A. Green	(University of Nottingham)
**	24.	Professor H.B. Griffiths	(University of Southampton)
*	25.	Professor Y.C. Ho	(Harvard University)
*	26.	Dr. E. Huntley	(University of Sheffield)
*	27.	Dr. T.A.S. Jackson	(University of Liverpool)
*	28.	Professor A. Jeffrey	(University of Newcastle upon Tyne)
*	29.	Dr. G.H. Lawden	(University of Sussex)
	30.	Professor M.J. Lighthill	(University of Cambridge)
	31.	Professor H.C. Longuet-Higgins	(University of Edinburgh)
*	32.	Professor A.G. Mackie	(University of Edinburgh)
Sat.26 only	33.	Professor G.D.S. Maclellan	(University of Leicester)
**	34.	Dr. F.T. Malins Smith	(University College of Swansea)
*	35.	Professor L. Markus	(University of Minnesota)
*	36.	Mr. J.E. Marshall	(Bath University of Technology)
*	37.	Dr. D.Q. Mayne	(Imperial College of Science & Technology)
**	38.	Mr. W. Murray	(N.P.L. Teddington)
25,26 only **	39.	Professor R.J.A. Paul	(University College of N.Wales Bangor)
*	40.	Dr. P.H. Phillipson	(University of Leicester)
rr.11 p.m. 27th *	41.	Mr. M.J.D. Powell	(U.K.A.E.A.)
Sat.26 only	42.	Professor G.E.H. Reuter	(Imperial College of Science & Technology)
	43.	Professor P. Rivett	(University of Sussex)
*	44.	Dr. A.P. Roberts	(Queen's University, Belfast)
	45.	Dr. J.D. Roberts	(University of Cambridge)
*	46.	Professor A.P. Robertson	(University of Keele)
	47.	Professor W.L. Root	(University of Michigan - visitor at University of Cambridge)
*	48.	Professor H.H. Rosenbrock	(U.M.I.S.T.)

*	49.	Mr. J.L. Russell	(GEC - Kidsgrove)
*	50.	Professor R.H. Sargent	(Imperial College of Science & Technology)
	51.	Professor K. Stewartson	(University College, London)
*	52.	Professor C. Storey	(Loughborough University of Technology)
*	53.	Dr. A. Sumner	(University of Reading)
*	54.	Dr. R.M. Terrill	(University of Liverpool)
*	55.	Dr. J. Tweed	(University of Glasgow)
*	56.	Professor G.N. Ward	(University of Sussex)
*	57.	Professor J.H. Westcott	(Imperial College of Science & Technology)
*	58.	Mr. L.A. Wigglesworth	(University College, London)
	59.	Dr. J. Willems	(Massachusetts Institute of Technology)
*	60.	Dr. E.R. Williams	(University of Manchester)
*	61.	Professor W.E. Williams	(University of Surrey)
*	62.	Mr. B.I. Wood	(British Steel Corporation)
	63.	Professor E.C. Zeeman	(University of Warwick)

* staying in St. John's College

** accommodation for two.