Instructional Science 3 (1974) 151–176
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TRAINING, STABILITY AND CONTROL

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ABSTRACT

This paper presents a system-theoretic approach to the analysis of the problem of training formally relating it to the *control* of an abstract dynamic system, the "adaption automaton" of the trainee. The utility of this formulation and the possibility of basing real training strategies upon it are discussed, and it is argued that further constraints upon the automaton are both necessary, and available, in so far as the theory corresponds to practical reality. The minimal constraints generate an extended theory in which training is related to the *stability* of the adaption automaton. More practical constraints lead to theoretical foundations for strategies of "feedback" or "adaptive" training. Corresponding to each set of constraints a "training theorem" is proved which demonstrates that the constraint is adequate to lead to a simple universal training strategy.

Although this paper is highly theoretical it is argued that the formal concepts introduced correspond to intuitive models of the phenomena of learning and training which are implicit in the design of many training systems. It is hoped that the formal analysis will throw new light on these implicit assumptions and help to clarify discussion of practical approaches to training, including the possibility of "computer-aided instruction" given on our present level of knowledge of human cognitive skills or individual students.

1. Introduction

In Gaines (1972a) a strategy is derived for using performance feedback to generate a training sequence for a perceptual-motor skill, and experiments are described with both humans and simulated learning machines to illustrate the practical utility of the technique and demonstrate its independence of the nature of the trainee. It is argued in that paper that the training strategy utilised and the results obtained are of more general application and derive from fundamental characteristics of the nature of learning and training.

In Gaines (1972b) the nature of adaptive or learning behaviour is analysed and an explicatum of the accepted stereotypes of adaptive

behaviour is derived which demonstrates that the concept of adaption is a very rich one giving rise to many possible and distinct modes of behaviour characterized as "adaptive". This is formalised through a set of axioma for the description and analysis of adaptive behaviour leading to the concept of the "adaption automaton" of a learning system.

In the present paper these two previous studies, one experimental and related to training and the other theoretical and related to learning, are brought together to provide a formal analysis of the problem of training. In particular the specific analysis of the tracking task system in Gaines (1972a) is related to a non-specific analysis of the class of such problems, and the abstract characteristics of this class which make feedback training possible are derived.

The objective of these formal studies is to throw new light on the fundamental system-theoretic concepts implicit in our assumption that a particular training strategy is appropriate to a particular type of situation. For example, the contrast between the "stimulus-response" instructional mode and the "learning environment" approach may be seen to be the difference between "control-derived" and "stability-derived" training strategies in the context of the adaption automaton of the trainee. In particular, arguments about the feasibility of "computer-aided instruction" given our present limited knowledge of individual cognitive psychology are thrown into a new light by an explicit account of the type of information required about the trainee before any form of instructional strategy is possible.

2. Adaption Automata

In Gaines (1972b) a detailed discussion of the nature of adaptive and learning behaviour, and of previous attempts to define it, led to an automata-theoretic formulation of the definitions of adaptive behaviour. This is briefly reviewed in the following sections before extension to include phenomena of instruction and training.

2.1. ADAPTIVE BEHAVIOUR

The detailed arguments of Gaines (1972b) lead to the definition of a basic element on which all further analysis of adaptive behaviour may be based. Formally this is an abstract entity defined only by its properties, however, it was given the name, "task", as a convenient mnemonic. We define the term in two stages, noting firstly that the behaviour of any system is defined extensively as the set of all possible sequences of

descriptions of that behaviour, and secondly that to define adaption or learning we must have a performance criterion against which to evaluate the behaviour:

Definition: A task is a segment of description of behaviour about which it is possible to say whether or not it is satisfactory.

The justification of this definition and its implications are discussed at length in Gaines (1972b), where it is noted that the arbitrariness of the proposed segmentation is an intrinsic and important feature. There is no absolute basis for the discussion of adaption or learning and certain decisions must be taken before the discussion can begin. These decisions clearly provide fertile grounds for misunderstanding and argument long before any concepts of teaching and training are introduced. The arbitrariness also indicates that the necessity to monitor the "progress of learning" is not in itself a simplistic, nor intrinsically meaningful, requirement.

It may be fruitful at this point to place some stress on the intended neutrality of the above definition of an element of adaptive behaviour since we shall move on very rapidly to an automata-theoretic formulation which, however, is a formal mathematical convenience not an assumption of any particular philosophy of psychology. The definition above involves concepts of description, evaluation, and change which seem intrinsic to our concepts of adaptation and learning (Kilmer and Arbib, 1973). The systems of description and evaluation, however, are deliberately left completely open – a major area of theoretical endeavour is clearly the inter-relationship of different systems of description. The phenomenological psychologist (MacLeod, 1964) will have a different system of description from the strict behaviourist (Malcolm, 1964) and the physicist will have a different system of description from either. In the present study we are concerned only with establishing an axiomatic theoretical basis for the rigorous development of theories of learning and instruction. The formal basis is neutral to a very wide spectrum of approaches to the study of psychology. This is consistent with the arguments of Putnam (1964), Macklin (1972), Rorty (1972), Sher (1973), and other philosophical discussions of the interrelationship between physical and psychological descriptions of behaviour where the overall conclusion is that causal explanations of human action are *not logically impossible* and are open to empirical investigation. An automaton is a minimal mathematical formulation of (at least partially) causal explanations. The neutrality of the theory to the system of description leaves open the "two-vocabularies" question (of movement and action – Sher 1973) amongst many others.

Empirical findings may be expected to be rather less neutral and the theorems of Section 4 of this paper may be viewed as a demonstration that certain properties of our overall description/observation/inference system are desirable because they lead to successful application of the basic instruction and training techniques which we normally use. This type of result is closely related to Kalman's (1962) analysis of a system into four components based on combinations of *observability* and *controllability* – in general we do not use the results to find out whether a system is observable or controllable, but, instead, we re-structure our relationship with it until it is. In conclusion we may note that including the possibility of physical modes of description allows a formal relationship between *learning* and *stability* to be established which is, at the very least, a useful analogy that allows us to offset the, perhaps infeasible, concept of *controlling* learning by introducing that of *stabilising* it.

2.2 FROM BEHAVIOUR TO STRUCTURE

The basic concept of a "task" may be used to formulate definitions of different modes of adaptive behaviour. In the present context it is more useful to present these as properties of a derived structure underlying the adaptive behaviour, the *adaption automaton* of the learning system.

The justification for moving freely from a behavioural description to a structural equivalent is nowadays theoretically well-founded. When Wiener (1949) proposed that systems should be regarded as *cybernetically equivalent* if their behaviours were in one-to-one correspondence (co-extensive), there were no rigorous theoretical foundations for the study of such equivalences and the "minimal-structure necessary to realise behaviour" relationship. Many structure/behaviour embedding theorems have been derived since then, and these have been subsumed in recent years under a general relationship of "adjunction" in category theory (Goguen 1973a, b; Arbib and Manes 1973). The rigorous formulation of structure and behaviour as adjoint categories enables us to move freely between the two in a well-defined, unambiguous manner.

Hence, we define an *adaption-automaton* as a state-restricted machine with tasks as inputs and performance as output whose behaviour is cybernetically equivalent to that of a given learning system. Such an automaton will always exist – we consider later the problem of actually deriving it in practice.

More formally, consider an automaton defined by the quintuple

$$A \equiv (T,P,S,\sigma,\pi)$$

where T is a set of tasks forming inputs to the automaton; $P=(p^-,p^+)$ is a

set of two possible outputs from the automaton, p^+ indicating satisfactory performance and p^- unsatisfactory performance; S is a set of states for the automaton, and σ and π are the next state and output functions, respectively. Such a machine will be called an adaption-automaton representing the behaviour of some learning system.

In general the adaption automaton need not be determinate and both the next state and output functions must be assumed to be set-valued. Hence, if the automaton is in state, $s \epsilon S$, is given the task, $t \epsilon T$, then its next state, $s' \epsilon S$, and its output, $p \epsilon P$, belong to the transition, and output sets, respectively:

$$s' \epsilon \sigma(s,t) \tag{2}$$

$$p \epsilon \pi (s,t). \tag{3}$$

Since it is the effect of sequences of tasks, especially those generated by the repetition of a single task, which are of interest, it is important to have a clear notation distinguishing between tasks, sets of tasks, sequences of tasks, and sets of sequences of tasks. A sequence consisting of the tasks t_1 followed by the task t_2 will be written $t_1 t_2$ (with the obvious extension to longer sequences), a sequence consisting of the task, t, repeated n times will be written t^n. A typical set of tasks will be represented by the letter T (T_1, T_2, etc.); a typical sequence of tasks will be represented by the letter, u (u_1 u_2, etc.). The set of sequences generated by the set of tasks, T, as free generators will be written $U(T)$ – this is, the set of all possible sequences of tasks which may be formed using members of T. The function, σ, has an obvious extension from tasks to task sequences – if $u = tu'$, then:

$$\sigma(s,u) = \{\sigma (s',u') : s' \epsilon \sigma(s,t)\} \tag{4}$$

Having established this structure, it is possible to give set-theoretic definitions of various modes of adaption. These are characterised by the sets of states of the automaton in which behaviour can arise satisfying certain constraints.

2.3 ADAPTION TO A SINGLE TASK

The fundamental situation with which a learning system is expected to cope is to be coupled to a fixed environment and learn to control it satisfactorily. The interaction is equivalent to the system performing a sequence to tasks consisting of the same task repeated indefinitely. Such repetitive task sequences play a special role in the theory of adaptive behaviour because they correspond to the characterisation of an adaptive system by its changing response to the same situation.

Definition: An interaction between controller and environment consisting of the repetition of a single task is *acceptable* if it is eventually always satisfactory.

Thus, in an acceptable interaction, the initial performance of the system does not matter, and for a number of repetitions of the task it may be satisfactory, unsatisfactory, or waver between the two. However, it must eventually become satisfactory and remain so – an acceptable interaction is one which reaches a *stable* condition of satisfactoriness. In this condition we may say that the learning system has become "adapted" to the task.

Definition: An interaction between controller and environment consisting of the repetition of a single task is *immediately acceptable* if it is always satisfactory – an immediately acceptable interaction is obviously acceptable. A system in such a condition that it would have an immediately acceptable interaction with a task is *adapted* to that task.

These definitions may be re-phrased in terms of the adaption-automaton:

$$\text{Let} \quad W(t) = \{s : \pi(s,t) = p^+\} \tag{5}$$

that is, $W(t)$ is the set of states in which the system will have a *satisfactory* interaction given the task t.

$$\text{Let} \quad P(t) = \{s \; \exists N : \forall n \geqslant N, \; \sigma(s,t^n) \subset W(t)\} \tag{6}$$

then $P(t)$ is the set of states from which the system has *acceptable* behaviour given the task, t.

$$\text{Let} \quad A(t) = \{s : \forall n, \sigma(s,t^n) \subset W(t)\} \tag{7}$$

that is, $A(T)$ is the set of states in which the system is adapted to the task t, because, given a sequence of task consisting of t repeated, the interaction is always satisfactory.
These definitions are best illustrated in a diagram showing the sets as regions in the state-space of the adaption automation. In Figure 1 the set of all states are shown as a rectangular region within which are delimited the regions, $W(t)$, $A(t)$ and $P(t)$. Note that $A(t)$ must lie in the intersection of $W(t)$ and $P(t)$. On the same diagram it is convenient to show the "trajectories" in state-space generated by repetition of the task t. In general trajectories may be indeterminate and fork arbitrarily. Those, such as that from X, commencing within $P(t)$ will always remain within $P(t)$ and eventually lie entirely within $A(t)$. Those, such that from Y, commencing

outside $P(t)$ may enter $W(t)$ (become satisfactory) but will have at least one fork that leads out of $W(t)$.

The trajectories generated by repetition of t generate a topology on the state space, (the topology whose open sets are the transitive closures of any sets under t, i.e. for a set of states $R \subset S$, $\{s: \exists r \epsilon R,\ n: s = \sigma(r, t^n)\}$ is open). The topology will generally not be so neat as that illustrated in Figure 1. Note that $A(t)$ is connected to the region of $P(t)$ outside $W(t)$ since any trajectory between these two regions which does not leave $W(t)$ must lie in $A(t)$.

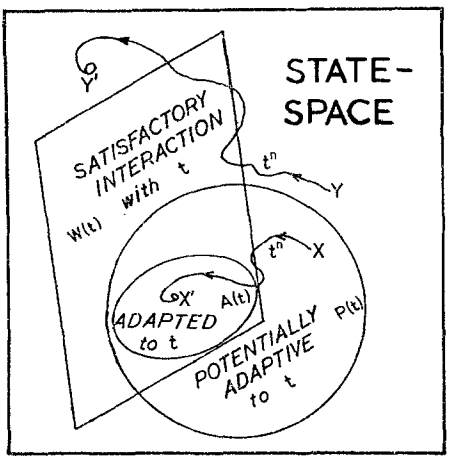

Fig. 1. Adaption to a single task.

2.4 ADAPTION TO MULTIPLE TASKS

The concepts of acceptability and adaptedness concern the performance of a single task, but adaptivity is generally advantageous because a particular task is incompletely specified or may change. The possible varieties of adaptive behaviour in a situation involving performance of any one of a number of tasks are many, but there are two modes of adaption of particular interest which may be defined.

Very often a learning system will be required to perform a single task which will not change, but whose characteristics cannot be specified in advance. It must be capable of having an acceptable interaction with any of a range of tasks, but need not necessarily be capable of adapting to a sequence of different tasks. This mode of behaviour is characterised in the following definition.

Definition: A system in such a condition that it will have an acceptable interaction with any one of a set of tasks is *potentially adaptive* to that set of tasks.

Let $P(T) = \{ s:\forall t\epsilon\ T, \exists N:\ \sigma(s,t^N) \subseteq A(t)\}$ (8)

that is, $P(T)$ is the set of states in which the system is potentially adaptive to the set of tasks, T, because, given any sequence of tasks consisting of a member of T repeated, the interaction is eventually always satisfactory.

This definition is compatible with that of (6) – essentially:

$$P(T) = \bigcap_{t\epsilon T} P(t) \tag{9}$$

The definition of $W(t)$ can be extended to multiple tasks in the same way, as can that of $A(t)$. However, note that the ultimate trajectory of a potentially adaptive system under a task $t\epsilon T$ lies within $A(t)$ not $A(T)$, indeed $A(T)$ may well be empty. Note also that the ultimate trajectory will lie within $P(t)$ but not necessarily within $P(T)$ – adaption to one task may destroy potential adaption to the others.

Thus potential adaption is a very weak criterion (that which is implied in statements like, "a shoe adapts itself to a foot") and if a learning system is to be able to cope on a continuing basis with a changing situation it must satisfy the stronger criterion of remaining potentially adaptive to the set as it adapts to any one task.

Definition: A system is *compatibly adaptive* to a set of tasks if, given any sequence of tasks from that set, it remains potentially adaptive to the set of tasks.

Let $C(T) = \{ s:\forall u\ \epsilon U(T),\ \sigma(s,u)\subset P(T)\}$ (10)

that is, $C(T)$ is the set of states in which the system is compatibly adaptive to the set of tasks, T, because, given any sequence of tasks consisting of members of T, its state remains in the sub-set which is potentially adaptive to the set of tasks, T.

These definitions are by no means exhaustive and Gaines (1972a, b) gives an analysis in greater depth. However, the concepts of the adaption automaton, trajectories in its state-space, and those of potential and compatible adaption, suffice for the analysis of training in the remainder of this paper.

3. Training, as a Control Problem

In this section the approach to learning behaviour developed in Section 2 is extended to provide a rigorous foundation for the analysis of training, and to enable different modes of training to be defined. In this way it is possible to regard training as a problem of control and stability in the state-space of the adaption-automaton of the trainee.

It has been shown that any learning sequence of behaviour may be represented as a trajectory in the state space of the adaption automaton. By considering the effects of one form of interaction, or "task sequence", upon the adaptivity of the learning system to particular "tasks", it is possible to formalise the concept of training and associated concepts such as, "negative and positive transfer".

Consider again Figure 1 which shows the regions of satisfactory interaction, $W(t)$, adaption, $A(t)$, and potential adaption, $P(t)$, within the state space of the adaption automaton for a particular learning system. If the objective of training is satisfactory performance of the task, t, then it is required that the state of the adaption automaton should finally lie within the region $A(t)$. However, it is adequate for training purposes that the state of the adaption automaton should lie within the region $P(t)$, since trajectories generated by the task, t, from within this region always eventually enter, and remain in, $A(t)$; that is, the learning system will itself adapt to the required task when the state of its adaption-automaton is initially within $P(t)$.

Thus the training control problem only becomes non-trivial when the initial state of the adaption-automaton is outside the region $P(t)$, and the objective of any training strategy must be to bring the state of the adaption-automaton within this region. When the initial state of a controller's adaption-automaton is outside the region of potential adaption to a task, then successful learning will not take place if it is given that task alone. Given some other sequence of tasks, however, the controller will adapt to them, and may, in so doing, become potentially adaptive to the original task – the sequence of tasks may be said to have *trained* it for the original task.

In Figure 2, for example, the point A is outside the region of potential adaption to the task, t and repetition of the task does not lead to stable satisfactory performance. The sequence of tasks, u, however, gives rise to a trajectory in the state--space which terminates at A_1, within the region of potential adaption – hence, from A, a sequence of tasks of the form, ut^n, causes the controller to become adapted to t, whereas a sequence of the form, t^n, does not. If the training sequence, u, consisted of another task, t', repeated many times, then it would be said that there had been *transfer of training* from t' to t.

The training sequence, u, will not necessarily be suitable for all initial conditions of the controller, and, for example, the trajectory induced by u from the point B in Figure 2 terminates at B_1 which is still outside the region of potential adaption. It is possible that the training sequence, u,

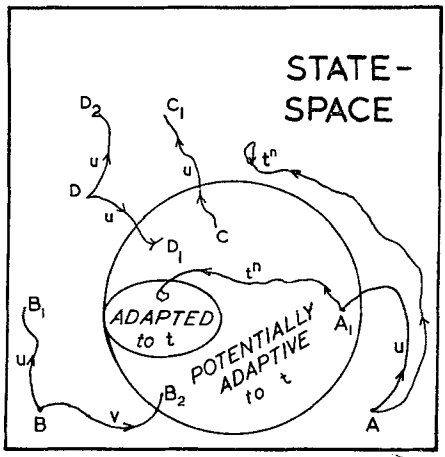

Fig. 2. Effects of open-loop training sequences.

may not only be ineffective in this way, but may also be detrimental in certain circumstances – for example, the point C is within the region of potential adaption, and yet the trajectory induced by u from C terminates at C_1 which is outside that region. In this case, if $u = (t')^n$, then it would be said that there had been *negative transfer* from t' to t.

Even when u is ineffective as a training sequence it may be possible to find an alternative sequence with the desired effect – for example, in Figure 2 the sequence, v, induces a trajectory from the point, B, which *does* terminate in the region of potential adaption. In the case of point C, it is clear that no training sequence is required, and the task, t, itself induces a trajectory within the region of potential adaption. Thus, even when the controller is not potentially adaptive to its required task, it may be possible to cause it to become so by giving it an appropriate training sequence. To choose between training sequences, however, and to decide whether one is required, it is necessary to have some information about the initial state of the adaption-automaton.

Hence, training may be analysed as the control problem: given an adaption-automaton structure, representing the possible adaptive behaviour of the trainee, generate a sequence of tasks to serve as an input to the automaton, such that its state terminates in a region where the controller is potentially adaptive to the required task. In general, the initial state of the automaton will be unknown, and information about the past behaviour of the trainee must be used in order to determine it. Since the adaption-automaton may be indeterminate in its state-transitions, an open-loop training sequence will not necessarily be adequate, and continuous feedback from observations of the behaviour and performance of the trainee may be necessary. In the following sections, different strategies for training are distinguished according to these considerations.

3.2 FIXED TRAINING

It is convenient to distinguish one strategy for "training" in which no attempt is made to solve the control problem described in the last section. In Section 2.3, it is stated that "the fundamental situation with which a controller is expected to cope is to be coupled to a fixed environment and learn to control it satisfactorily." This is, in a weak sense, a training situation in that the trainee is given an opportunity to learn, and the sequence of tasks corresponding to it, a repetition of a single task, will be called a *fixed training* sequence for the environment corresponding to that task.

The success of fixed training depends on the trainee being at least potentially adaptive to the task he has to perform, or, more strongly, compatibly adaptive to the set of tasks he may have to perform. The selection of tasks by the trainer involves no observation of the condition of the trainee's adaption-automaton, and he is given only those tasks which he is required to perform satisfactorily.

3.3. OPEN-LOOP TRAINING

In *open-loop training,* the trainer still does not observe the state of the adaption-automaton of the trainee, but prepares him for adaption to the main task by giving an initial sequence of auxiliary, or training, tasks for which the trainee is not necessarily required to be satisfactory. For example, the sequence u from point A of Figure 2, or the sequence v from point B, both induce trajectories which bring the state of the adaption-automaton within the region of potential adaption to the task, t. Fixed training on the task t alone is inadequate if the initial condition of the trainee corresponds to A or B, but the initial training sequences, u or v, respectively, enable adaption to take place, given further fixed training.

In open-loop training, however, there is no information available as to the state of the adaption-automaton, and if u were chosen to be the training sequence, then it would be given under all conditions.. From Figure 2 it is clear that u would not be an effective training sequence from B or C, and it becomes important to determine those regions of state-space in which u has the desired effect. In Figure 3 are delimited those states from which training with an initial sequence, u, causes the trainee to become adapted, or potentially adaptive to the task, t. If u were itself a single task, t', repeated, then the first region might be called one of "complete transfer" from t' to t, and the second region one of "partial transfer". It is interesting to note that neither the region where the controller is potentially adaptive, nor that where it is adapted, need be

contained in the region of partial transfer – the region of potential adaptivity – but no transfer from u, may be called one of "negative transfer". Clearly, the utility of u as a training sequence depends on maximising the region of partial transfer, and minimising that of negative transfer.

3.4. CONDITIONAL ADAPTION SETS

The behaviour of a controller relative to training by a given task sequence, and the various phenomena of transfer, may be defined by sets of the form shown in Figure 3, and set-theoretic definitions of trainability

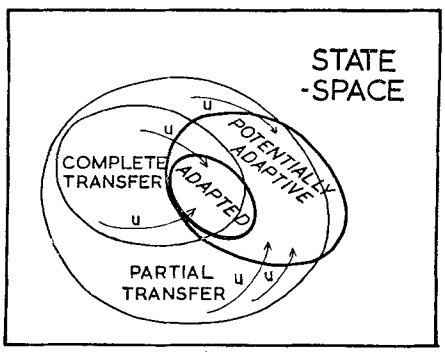

Fig. 3. Open-loop training spaces.

may be given similar to those for adaptivity. There is obviously a far greater variety of possible behaviour in training, since adaption is conditional upon previous learning in a training sequence which may itself have definable structure, and many phenomena are best analysed directly in terms of trajectories in state-space. A few conditional adaption sets are of sufficient general importance to be formally defined.

$$\text{Let } A(t{:}u) = [s{:}\ \sigma(s,u) \subset A(t)] \tag{11}$$

that is, $A(t{:}u)$ is the set of states in which the training sequence, u causes the controller to become adapted to the task, t. This is the set forming the region of "complete transfer" in Figure 3, and the trainee may be said to be *completely open-loop trainable* by the task sequence, u, for the task, t, when the state of his adaption-automaton is in $A(t{:}u)$.

$$\text{Let } P(T{:}u) = [s{:}\ \sigma(s,u) \subset P(T)] \tag{12}$$

that is, $P(T{:}u)$ is the set of states in which the training sequence, u, causes the controller to become potentially adaptive to the set of tasks, T. A trainee whose adaption-automaton is in one of these states may not adapt

to a task, $t \epsilon T$, when given the fixed training sequence, t^n, but will do so when given the open-loop training sequence, ut^n; a trainee in such a condition may be said to be *potentially open-loop trainable* by the task sequence, u, for the set of tasks, T.

A similar conditional-adaption set may be defined for *compatibly open-loop trainable:*

$$C(t:u) = [s: \sigma(s,u) \subset C(T)] \tag{13}$$

Apart from the greater variety of the conditional adaption sets themselves, there is also a far wider range of inclusion relations possible, and these will not be described in detail. The interesting inclusion relations are not those logically entailed (such as $A(t) \subset P(t) \cap W(t)$) but those which are a function of the learning system and tasks. For example, the region of negative transfer has already been defined as those states in which the controller is potentially adaptive to t, but not potentially open-loop trainable by u. For u to induce no negative transfer, we must have:

$$P(t:u) \supset P(t) \tag{14}$$

Another region of interest is that part of $A(t)$ which is outside $A(t:u)$ – in this region the controller is adapted to t, and this adaption is destroyed by giving it u. However, provided

$$P(t:u) \supset A(t) \tag{15}$$

the controller is able to re-adapt to t, and u has acted only as a transient distrubance to its performance. In this event we might say that the learning of the task, t, was *robust* to the influence of the sequence u.

Clearly consideration of the robustness of learning leads to an even more complex structure of adaption sets – $A(t)$ may be split into disjoint parts, $A_1(t)$, $A_2(t)$, with corresponding potential adaption sets, $P_1(t)$, $P_2(t)$, such that it shows robustness to u in $A_1(t)$ but not in $A_2(t)$ – and we may consider the problem of training a controller to, "adapt to t robustly with respect to u." Thus, the conditional adaption sets provide not only the means for evaluating various open-loop training sequences, but also a calculus for defining the effects of performing one task, or task sequence, when the controller is in various stages of adaption to another; the conditional adaption sets enable the *stability* of adaption to be evaluated.

3.5. FEEDBACK TRAINING

An open-loop training sequence would be chosen to make the conditional-adaption sets as large as possible, and clearly the optimum se-

quence would be such that at least $P(t{:}u)$ includes all possible initial states of the adaption-automaton. If this is not possible, some general restrictions, such as $P(t{:}u) \supset P(t)$, may be applied to ensure that training does not destroy adaptivity which is already present. However, it may not be possible to find a single training sequence which has all the desired properties, and hence it may be necessary to apply different training sequences dependent on the condition of the trainee.

So far, the training sequences have been taken as fixed sequences of tasks, but, since the adaption-automaton may be indeterminate in its behaviour, it is possible that an effective training sequence cannot be determined in advance even when the initial state of the automaton is known. For example, from D in Figure 2, the training sequence, u, might induce a trajectory which terminates either at D_1 within the region of potential adaption, or at D_2 which is outside the region; a further examination of the condition of the controller after having been given u is necessary to determine whether the training has been successful.

Even if the adaption-automaton is determinate, its initial state will generally be unknown, and observations of the controller's interaction with some environment will have to be made in order to determine the state of the automaton. The task sequence corresponding to this initial interaction clearly need not be a training sequence, and might rather be a *testing* sequence designed solely to obtain information about the controller prior to training. Such a testing sequence, or "probe", differs from a training sequence only in that it is designed for rapid observation, rather than to change the state of the adaption-automaton appreciably, and testing may be analysed within the same theoretical, and conceptual, framework as training.

Both when the adaption-automaton is indeterminate, and when it is to be determined by observation, the training sequence is contingent upon feedback from observations of the interaction between trainee and environment, and a system for implementing this feedback will be called a *feedback trainer.* For the feedback trainer, training has itself become a control problem: there is an "environment", physically the controller, conceptually its adaption-automaton, whose inputs are tasks and one of whose outputs is the satisfactoriness of the previous interaction; the control problem is to take the automaton from an initial state in which the controller (trainee) is not adapted to the task, to a final state in which it has become adapted to the task (or potentially, compatibly, or jointly, adapted to a *set* of tasks); the performance measure for this control problem may be based on the number of tasks given before the controller becomes adapted, or it may be a more complex cost function based on the cost of giving irrelevant tasks, and so on. In the following section the

"solution" of the feedback training control problem is analysed in more detail.

4. The Nature and Solution of the Training Control Problem

The derivation of open-loop training sequences and feedback training algorithms, given the structure of the adaption-automaton, is amenable to "solution" by some of the techniques of modern control theory, such as "dynamic programming" (Bellman, 1957). However, it is totally unrealistic to consider the application of such optimal control algorithms to training in any real situation since the adaption-automaton will never be known in sufficient detail. The derivation of structure as an adjoint to behaviour (Section 2.2) is a solution to the problem of complete induction and requires knowledge of all possible behaviours. In practice, however, due to the irreversibility of most learning systems it is not possible to collect descriptions of all possible behaviour and only partial information is available. Essentially the adaption-automaton will be only partially known, an approximate model of the adaptive behaviour.

Thus the training control problem is comparable with the majority of real control problems where there is only partial information about the plant and controllers are designed to perform reasonably well with a range of possible plants rather than optimally with a particular plant. In these circumstances feedback control is essential (because the state-transitions of the partially-known plant cannot be completely predicted in advance), and the prime initial problem is to ensure stability in control – that is, that the controller should bring the plant to (global stability), and maintain it in (local stability), a prescribed region of its state-space. In the following sections some weak conditions on the adaption-automaton are established which ensure the existence of *globally stable feedback* trainers.

4.1 FIRST TRAINING THEOREM

A necessary requirement for training to be possible is that there should be available training sequences to transfer the state of the adaption-automaton from any state it might reach during training into at least the potentially-adaptive region. Since an adaption-automaton is generally indeterminate it is also necessary to ensure that these sequences do actually cause the required transition, if not always at least "frequently" (in the mathematical sense). Given these postulates it is at least feasible to cause the learning system to adapt through training.
We can express these requirements as:

$$\forall s \in \Sigma, \exists v \in V : \pi(s,v) \in P(t) \text{ frequently} \qquad (16)$$

where t is the required task, V is a set of possible training sequences and Σ is the set of states which may be reached during training. This "reachability" requirement is not a strong one – if it did not apply there would exist at least one legitimate training sequence that would cause the learning system to become absolutely non-trainable. One can envisage practical situations where this could be so (trivially so, if the training sequence leads to the demise of the trainee), but it is not unreasonable to assume that the constraint of (16) is postulated in all practical training systems.

Whilst the constraint of (16) may be *necessary* for training to be possible at first sight it would appear to be unlikely that, given no other knowledge of the structure of the adaption automaton, it would be possible to devise a training strategy that was guaranteed to cause the trainee to become adapted to t. However, it turns out that (16) is also a *sufficient* condition for a *universal feedback training* strategy to be devised.

This training strategy has its own peculiarities in that the universal trainer which implements it cannot be a deterministic finite state automaton but must be either primitive recursive (infinite state) or finite state stochastic (non-determinate). This peculiarity does not result from the indeterminacy of the adaption automaton but is an example of general results derived by Gold (1971) for the recursive automaton and Gaines (1971) for the stochastic automaton. Their results were obtained within a framework of control and stability theory – in the present context they may be re-phrased to state that: "for any finite state feedback training system there exist trainees whose adaption automata satisfy the constraint of (16) but which are not trainable by the system; there also exists a primitive recursive automaton (Gold, 1971) and a two-state stochastic automaton (Gaines, 1971) which are *universal trainers* for any systems satisfying (16)."

The primitive recursive trainer is not of direct interest since its structure is necessarily complex. However, it is of interest to note that here we have an equivalence between automata in which a two-state stochastic device and an infinite-state deterministic device are of equal power and both more powerful than any finite state strategy. There are practical implications in this which are discussed in Section 5.

It is not difficult to construct a universal trainer and prove the result stated above. A suitable (necessarily weak) training strategy would be to give the required task, t, only once the state of the adaption-automaton is within the potentially adaptive region, $P(t)$, and to chose a training sequence at random if the state is outside $P(t)$. The random choice ensures that there is a finite probability of entering $P(t)$ and, since there is zero probability of leaving $P(t)$, entails convergence of the state to within $A(t)$.

Unfortunately, whether the state of the automaton is within $P(t)$ is not known since its performance may be unsatisfactory within $P(t)$ and it may be satisfactory outside it (within $W(t)-P(t)$).

A slightly more complex strategy can overcome this problem even though only performance feedback is available. We give the task t until the performance is unsatisfactory, and then select either t or one of the possible training sequences at random, then give the task t again until the performance is unsatisfactory, and so on. Clearly if t is given outside $P(t)$ but within $W(t)$ then, by the definitions of these sets (Section 2.3), the performance must eventually become unsatisfactory and some training sequence will be given with a finite probability of causing the state to enter the region $P(t)$. Equally if the state is within $P(t)$ but outside $W(t)$ there is a finite probability of t being given long enough for the state to enter $A(t)$. Convergence under this "training strategy" is clearly a weak result, but the postulates are so weak that no stronger result can be expected – the weakest possible conditions that will ensure the existence of a convergent feedback training algorithm, and hence a globally stable feedback trainer, are an important starting point from which to discuss stronger results based on stronger postulates.

For purposes of the theorems on training, the adaption-automaton will be taken to be finite-state. This is unnecessarily restrictive but makes the proof more transparent and meaningful in practical terms.

Theorem 1

Given a finite-state adaption-automaton and associated sets as defined in Section 2.3, a specified task $t \in T$, and a set of task sequences, $V \subset U(T)$, satisfying the conditions specified below, then there is a feedback training strategy (based on the performance feedback of equation (3) only) such that the probability of the output of the adaption-automaton being satisfactory tends to unity.

Conditions an Task Sequences

If Σ is the set of states generated by the following constraints on the adaption automaton:

(a) initial state belongs to some subset $S_0 \subset S$.

(b) the sequence of tasks input to the automaton is constructed of segments of the form:

(i) initial segment is t

(ii) if the output of the automaton was p^+ at the end of the previous segment then next segment is t

(iii) otherwise next segment is vt, where v is any task sequence such that $v \in V \cup t$

then:

$$\forall s \in \Sigma, \exists v \in V: \pi(s,v) \in P(t) \text{ frequently}$$

where "frequently" implies that the sequence of events in which the state s occurs and the input sequence v is given either terminates after a finite number of occurrences, or after any occurence of the pair (s,v) there always exists some future occurrence of the condition $\pi(s,v) \in P(t)$. This ensures that training is always possible taking into account the possible indeterminacy of the adaption automaton.

Proof

The proof is constructive – consider the training strategy defined by the use of the task segments described in condition (b), subject to the constraint that when a sequence is selected from $V \cup t$ the selection is probabilistic with uniform probability of selecting any possible sequence. It is sufficient to show that the state set, $A(t)$, is the only trapping set and that there is a finite probability of reaching a state within it at any point in the state trajectory.

$A(t)$ is trapping since by equation (7) for any state $s \in A(\tau)$, $\sigma(s,t) \in W(t)$ so that from equation (5) $\pi(s,t)=p^+$ and hence from condition (b) (ii) the next task will be t and from equation (7) the state will remain within $A(t)$.

No other state sets outside $A(t)$ can be trapping since for states outside $W(t)$ there is a finite probability that a sequence $v \in V$ will be used for input that will take the state within $P(t)$ and that a sequence containing only t will be used thereafter taking the state into $A(t)$. Whereas for states within $W(t) - A(t)$, a sequence consisting only of t cannot be maintained since the state must eventually leave $W(t)$ (by definition of $A(t)$) and there is then a finite probability of not selecting t.

4.2 SECOND TRAINING THEOREM

The first training theorem is clearly a weak result. However, it is worth noting that the training sequence had to be carefully constructed to avoid the problems caused by $W(t)$ and $P(t)$ not coinciding. It is also of interest that an essential random element was necessarily introduced into the training strategy to allow for the lack of knowledge of the "correct" training sequence for a given state. The training strategy is simple and does not involve identification of the state of the adaption-automaton or detailed knowledge of its structure.

Stronger results on training can only result from stronger constraints, that is, greater knowledge of the structure of the adaption automaton.

There are clearly infinitely many possible sets of constraints to ensure the existence of convergent training strategies with additional desirable properties such as rapid convergence. Only one further case will be considered since it corresponds to the type of training problem considered in Gaines (1972a). The assumption will be made that a *sequence* of tasks is known such that performance of one task leads to improved performance of the next task in the sequence. The training strategy is then to give the tasks in either forward or reverse sequence according to the performance.

Theorem 2

Given a finite-state adaption-automaton and associated sets as defined in Section 2.3, a specified task $t \in T$, and a finite sequence of tasks

$$t_0 , t_1 , t_2 , t_3 , \ldots , t_N$$

satisfying the following conditions:

(i) $t_N = t$
(ii) $\forall n : 0 < n \leqslant N , A(t_{n-1}) \subset P(t_n)$
(iii) $P(t_0) \supset \Sigma \cap S$

where Σ is the state semigroup generated by applying any sequence of t_0 through t_N from an initial set of states $S_0 \subset S$, then there is a feedback training strategy (based on the performance feedback of equation (3) only) such that the probability of the output of the adaption-automaton being satisfactory tends to unity.

Proof

The proof is again constructive – consider the strategy in which

(a) the initial task given is t_0
(b) if the previous task was t_n and the output of the automaton was p^+, the next task is chosen from t_n and t_{n+1} (t_N if t_{n+1} does not exist) with finite probability of chosing each.
(c) if the previous task was t_n and the output of the automaton was p^- the next task is chosen from t_n and t_{n-1} (t_0 if t_{n-1} does not exist) with finite probability of chosing each.

As shown in the proof of the first training theorem, the condition in which the task t is being given when the state of the automaton is within $A(t)$ is trapping. since the output will always be p^+ and by condition (b) above no change will be made in the task given.

Suppose that the task t_{n-1} is being given and the state of the automaton is within $P(t_{n-1})$, then there is a finite probability that t_n will continue to be given until the state comes within $A(t_{n-1})$ and that the

task will then be changed to t_n. Since, by condition (ii) of the theorem, $A(t_{n-1}) \subset P(t_n)$, there is thus a finite probability of changing from the condition in which t_{n-1} is being given within $P(t_{n-1})$ to the state in which t_n is being given within $P(t_{n-1})$ to the state in which t_n is being given within $P(t_n)$. Hence there is a finite probability of reaching the condition in which $t_N = t$ is being given when the state is within $A(t)$.

Suppose that t_n is being given and the state of the automaton is outside $P(t_n)$, then, by the definition of $A(t_n)$, there is a finite probability of t_n continuing to be given until the state is outside $W(t_n)$ and hence the output of the automaton is p^-. Hence by condition (c) above, there is a finite probability of the task being changed to t_{n-1}. Then either the state will be within $P(t_{n-1})$ in which case the preceeding paragraph applies, or if not the argument of the present paragraph applies. Thus eventually a condition must arise in which the state of the automaton is within the potentially adaptive region for the task being given, or the task must come to be t_0 but by condition (iii), $P(t_0) \supset \Sigma \cap S$, and the previous paragraph applies.

Thus the only trapping condition is t being given within $A(t)$ with output always satisfactory, and there is a finite probability of reaching this from any other state.

4.3 ANALYSIS OF FEEDBACK TRAINING

An immediate corollary to Theorem 2 is to consider a semi-lattice (Clifford and Preston, 1961, p.24) of tasks, L, ordered by a relationship, $<$, such that t is an upper bound, condition (ii) of Theorem 2 is satisfied by any pair of directly connected tasks in the lattice, and there is a set of tasks, L_0, each of which satisfies condition (iii). That is if:

(i) $\forall \lambda \in L, \lambda < t$

(ii) $\forall \lambda, \lambda' \in L: \lambda < \lambda'$, but there is no $\lambda^* \in L$: $\lambda < \lambda^* < \lambda'$, $A(\lambda) \subset P(\lambda')$

(iii) $\forall \lambda \in L_0 \subset L; P(\lambda) \supset S$

then it is clear that any directly connected chain of tasks starting within L_0 and terminating at t satisfies the conditions of Theorem 2. Hence the result of Theorem 2 can be generalised to strategies based on probabilistic movement along connected chains in a semi-lattice of tasks, with movement towards t when the output of the automaton is satisfactory, and movement towards L_0 when it is not.

This corollary gives a theoretical foundation for a variety of practical feedback strategies including that described in Gaines (1972a). It provides

a generalised justification of the feedback trainer for perceptual-motor skills making it plausible for the results to be extended to a wider variety of training situations. Condition (ii) of the previous paragraph gives one explication of what we mean by one task being "more difficult" than others – i.e. that the two tasks are related in such a way that the region of potential adaption for the easier is more extensive than for the more difficult and adaption to the easier gives positive transfer to the more difficult. The actual training strategy of Theorem 2 may be described as, "increase the difficulty of the task when the trainee is doing well and decrease it when he is doing badly," or alternatively "keep the difficulty of the task *constant for the trainee.*" Such strategies seem to be implicit in most of our actual approaches to teaching and training, and have been successfully automated not only for teaching perceptual-motor skills but also for teaching arithmetic (Wexler, 1970). The informal statements of the strategy have their dangers as evinced by Holding's (1962) demonstration of the experimental inconclusiveness of the question as to whether transfer is better from "easier" to "more difficult" tasks or vice versa. Such questions require a formal analysis to determine their operational interpretation (and its ambiguity) before they are amenable to resolution by experiment.

The analysis of the specific training strategy adopted in Gaines (1972a) which most closely resembles the abstract approach of Theorem 2 is that in control-theoretical terms given in Gaines (1968). It is noted that for a family of servomechanisms of increasing performance the stability boundaries in the natural-frequency/damping-ratio plane (the task plane) of the tracking task are nested within one another (and hence generate a lattice ordering task by natural-frequency/damping-ratio). It is hypothesized that learning only takes place when the trainee/plant interaction is stable. Since the effect of learning is to move the stability boundary outwards to encompass new areas of the tasks plane, the requirements of the corollary to Theorem 2 are satisfied.

Even the analysis of the dynamics of feedback trainer given in Gaines (1972a) can be given a more general interpretation. One may picture the task given as tending to travel upwards through the lattice as learning progresses and the automaton's output becomes satisfactory for tasks of increasing difficulty, and downwards through the lattice if the automaton's output becomes unsatisfactory and it is suspected that the task is too difficult. If the path length from entering $P(t)$ to entering $A(t)$ is long, then a high probability to changing the task will tend to take the task up to too high a level on the one hand, and down to a level for which the automaton is already adapted on the other. Hence the level of difficulty will tend to oscillate with increasing rapidity as the probability of changes

goes towards one. Equally, however, if the probability of change is very low, the transition to a more difficult task will be made later than is necessary but the automaton will be well adapted to the current task and hence within the region of potential adaption for the next – there will be little fluctuation of the task but steady progress towards τ .

It is possible to equate the probability of changing the task with the "loop-gain" of the feedback training by equating p^+ to +1 units of performance, p^- to -1 units of performance, calling the difficulty of t_m $\delta = n/N$, and the probability of changing the task, p. Then we have from (a) and (b) of Theorem 2:

expected change in $\delta = p$ times current performance.

Hence, the argument of the last paragraph may be interpreted as showing that too high a loop-gain will lead to slow learning through oscillating task difficult, whereas too low a level will lead to slow but steady learning and increase of task difficulty. This is the type of dynamic behaviour which would be expected of a simple servo system, and the result underpins the demonstration in Gaines (1972a section 3.2) that an actual feedback trainer, although highly nonlinear, can be reasonably analysed as a simple servo system.

5. From Theory to Practice

This paper presents an attempt to analyse the problems of teaching, instruction and training at an abstract, system-theoretical level. The abstraction is by no means an objective in its own right but serves two practical purposes:

(a) To achieve generality – or, at least, to investigate to what extent it is achievable.
(b) To remove pre-conceptions and clarify fundamental aspects of the problem of training a learning system.

It is possible to argue that in the area of learning and training generality is itself a myth. However, much of the debate on the feasibility of automated teaching systems has been at a high level of generality, and it is an open and important problem to determine what, if anything, can be said. The results of Section 4 do indicate that remarkable and meaningful conclusions can be drawn based on very weak and general assumptions. Section 4.3 relates these conclusions back to a specific training system that has already been investigated both theoretically and experimentally. In this section the discussion will be widened to include more general implications.

The approach taken may be summarised as:

(i) Critically analyse the types of system which are the subject of instruction and training, i.e. "adaptive" or "learning" systems (Gaines, 1972b).

(ii) Give purely behavioural explicata for the concepts underlying these terms (Section 2.1).

(iii) Develop a calculus for the analysis of modes of adaptive and learning behaviour in terms of the explicata (Sections 2.3 and 2.4).

(iv) Derive a mathematical structure accounting for this behaviour, the minimal adjoint, or cybernetically equivalent, adaption-automaton (Section 2.2).

(v) Express the analysis of adaptive behaviour in terms of the adaption-automaton so that the behaviour itself may be described as a trajectory in the state-space of the adaption-automaton (Section 2.3).

(vi) Analyse training as a problem of control in the state-space of the adaption-automaton – this introduces "fixed", "open-loop" and "feedback" training (Section 3).

(vii) Note that the control problem cannot be solved in terms of an optimal law relating the state of the adaption automaton to the task since the structure of the adaption automaton cannot in practice be determined (Section 4).

(viii) Investigate the possibility of inducing adaption by *stabilising* an adaption automaton whose structure is unknown but which satisfies certain very weak (and necessary) constraints (Section 4.1).

(ix) Note that there are *universal trainers* whose structure is either that of a stochastic automaton or that of a primitive recursive automaton (Section 4.1).

(x) Investigate the possibility of inducing adaption by a performance-feedback strategy based on a set of tasks ordered in difficulty (Section 4.2).

(xi) Relate the abstract analysis of feedback training to the specific analysis of a particular trainer (Section 4.3).

The steps in the logic which have important implications for automated instructional systems are those of (vi), (vii) and (viii). Whilst training is quite clearly a "control-theoretical problem" involving actions by the trainer dependent upon feedback from the trainee, it does not follow that the training system satisfies the conventional stereotype of a "controller" taking account of each output from the plant by means of pre-determined, optimal response. As Pask (1960, 1965) has strongly emphasized in his analyses of training as a control problem, it is best to

think of the training system *stabilising* the trainee rather than controlling him.

Stabilisation is the appropriate control strategy to adopt under conditions of uncertainty. Most practical control theory is concerned not with optimal control based on an exact knowledge of the plant, but with the stability of fairly universal controllers (the classical 3-term controller) connected to a *range* of differing plants. In many programmed texts and in much early computer-based-learning material one may see the adoption of the modern control strategy of optimal control in which an exact action is prescribed for each plant condition. In practice the development of optimum control strategies has thrown much light on the fundamentals of control theory and introduced rigour in its foundations, but it has had little impact in most practical situations where the classical approach is still most useful.

In the study of instruction and training these considerations apply even more forcibly. The classical servo-mechanisms based on linear systems theory which have served control engineers so well have, unfortunately, very limited utility in the context of human behaviour. The nonlinear, stimulus-response, approach of optimum control theory appears to be more relevant. However, it is not linearity in itself but rather the ease of *global analysis* of linear systems which is important. The global analysis of nonlinear systems is very much more difficult, and the appropriate tools for it are algebraic and automata-theoretic (Arbib, 1965, 1966). What we need are foundations for the development of training systems based on global properties of algebraic systems, such as stability.

The possibility of solving the training control problem with the simple strategy given in Theorem 1 does not arise in the general control of an automaton. It is specific to systems with broad interconnected regions of equilibrium – exactly those properties with which Ashby (1960) has characterised learning structures. These may be seen to arise through what Pask (1967) epitomises as "Man as a system that needs to learn." The training systems of Theorems 1 and 2 do not teach but instead they *generate conditions under which learning can take place.*

It is worth emphasising that whilst the feedback training strategy of Theorem 2 does not depend on, or utilise, detailed knowledge of the trainee, it does lead to an individualised training sequence that is generated by *interaction between trainee and trainer.* This is a fundamental property of systems incorporating negative feedback – Theorem 1 demonstrates just how little information is required about the structure of the trainee for it to be demonstrable that a strategy based on negative feedback will be effective.

The discussion preceeding Theorem 1 on relationship between

stochastic and primitive recursive trainers also has practical implications. Faced with the problem of generating a requisite variety of training situations and exemplars, we find that a deterministic system needs to be very complex (potentially infinite state if repetition is to be avoided) whereas a random system to achieve the same objective can be very simple. Conversational interactive programs such as ELIZA (Weizenbaum, 1966) owe must of their realism to controlled random selection, and a deterministic system rapidly becomes obviously mechanical unless it maintains and uses a growing corpus of complex information. Generative systems based on controlled random selection require a detailed knowledge of the subject matter under discussion rather than of the discussant.

In Gaines (1972a) it is argued that knowledge of the subject matter can generally be extended to knowledge of the epistemological problems faced by any learning system grappling with that subject matter. It is this level of knowledge that is required to devise feedback training strategies based on Theorem 2. Thus it is feasible to construct complete automated instructional systems based only on knowledge of the subject matter and the assumption that the trainee is a learning system reacting to that subject matter.

It is obvious, and may be proved in the context of Theorem 1 and the related discussion, that detailed knowledge of the structure of the adaption-automaton of a trainee may be used to speed up training. None of the arguments of this paper suggest that the pursuit of such knowledge is not worthwhile. Increasing knowledge of human cognitive processes undoubtedly gives potential for increasingly effective and efficient instructional systems. However detailed knowledge of individual cognitive processes is not a pre-requisite for successful teaching, and indeed the majority of current instruction, teaching and training proceeds without such knowledge. The results of this paper give some indication of how this is possible and what form of knowledge is required for the development of automated instructional systems.

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