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A NOVEL SPECIAL PURPOSE COMPUTER (U)

by

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ABSTRACT

A type of computer is described for the real time evaluation of integrals of the form $I = \int ydx$, where x and y are functions of time. It is believed to be novel in its use of quasi-random processes, operating on pulse trains, to achieve the desired result. The method may be extended to cases where y is a function of several variables dependent on time. Accuracies comparable with analog methods are obtainable without the drift problems usually associated with analog methods.

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1.0 INTRODUCTION

Many computing applications require the real time evaluation of integrals of the form $I = \int y\,dx$ where x and y are functions of time.

This is done by many methods, such as ordinary digital means, by analog methods such as ball and disc integrators or by electronic analogs in which the integral is expressed in the form $I = \int y.z\,dt$,

and where $z = \frac{dx}{dt}$

This paper describes another form of computer to perform this integration. It is simple in concept and has some of the advantages of both analog and digital methods. In addition to the simple integration it can also do computation, namely addition, multiplication, and division, inside the integration sign. In other words it can be used to evaluate integrals of the form $I = \int (a + b \cdot c)/y\,dx$ for example, without using any special purpose components. The system can be built completely with solid state electronic components and hence can be small, rugged and reliable. It is believed to be novel in its use of statistical rather than deterministic means to achieve the desired result.

The computer is not a general purpose device but should have wide application in areas such as navigation computers and process control. Its use in these applications is illustrated in this report and the Appendix describes a simple navigation computer which was built to demonstrate the method.

2.0 METHOD

The integral $I = \int_{X_1}^{X_2} y\,dx$ may be approximated by a summation process

$$\text{i.e. } I = \sum_{j=1}^{N} y_j \cdot \Delta x_j \qquad (1)$$

$$\text{where } \sum_{j=1}^{N} \Delta x_j = X_2 - X_1$$

and y_j is the value of y taken somewhere within the j^{th} increment of x.

If all values of Δx_j are equal,

$$\text{i.e. } \Delta x_j = \Delta x = (X_2 - X_1)/N$$

the integral reduces to $I = \Delta x \sum_{j=1}^{N} y_j$. (2)

The summation in eq (2) can be approximated by statistical methods. In a binary random process, if there are N trials and the probability that an event occurs at the j^{th} trial is p_j, then the expected number of events will be $\sum_{j=1}^{N} p_j$. This summation resembles the summation in eq (2). The actual number of events which occur will of course be a random number, but its expected or mean value will be the desired value, and as shown later its deviation from this expected value can be kept within reasonable bounds.

A means of implementing this procedure is shown in fig. 1.

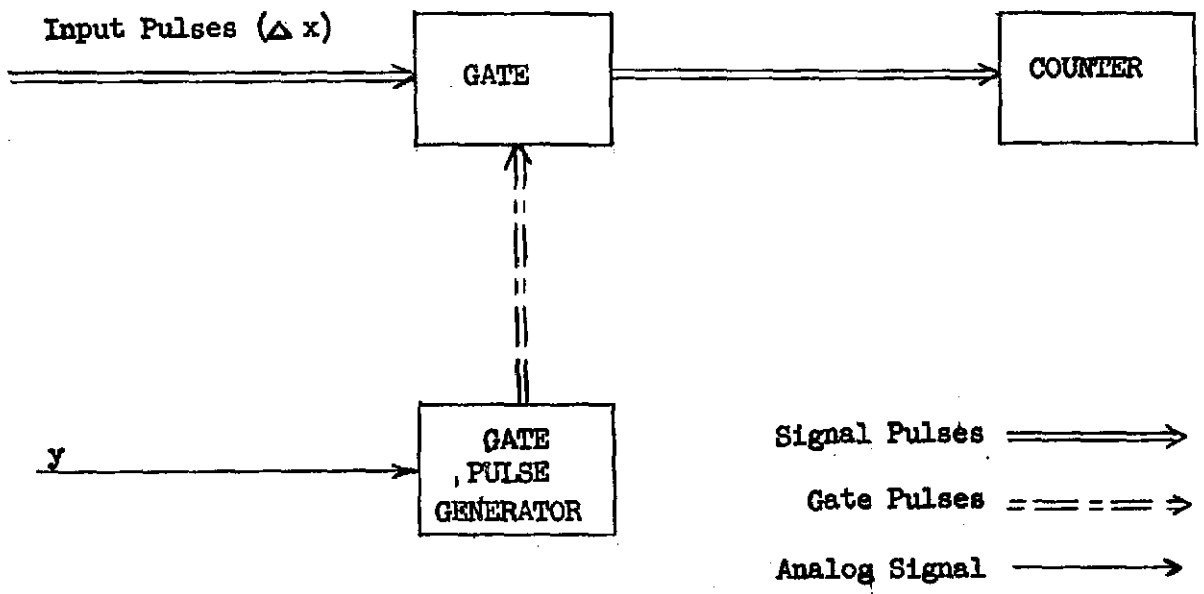


Fig. 1

Each increment of the variable x is denoted by a pulse applied to the input of the gate. The gate operates as an "and" gate in that an input pulse is passed through to the counter only if a gate pulse is present when the input pulse arrives. The gate pulses are generated at random with respect to the input pulses, but in such a manner that the probability, p, of the gate being open at a given time is proportional to the variable y.

i.e. $p = Ky$, where K is a constant.

(This is possible only if y is always positive, or always negative, since p must be within the interval:

$$0 \leq p \leq 1$$

We will restrict the present discussion to this case, and cover the general case later.)

If the j^{th} input pulse denotes the j^{th} increment of x then the probability of this pulse getting through the gate will be $p_j = K.y_j$. For N input pulses the mean or expected value of the number, M, of pulses getting through the gate is

$$E(M) = \sum_{j=1}^{N} p_j = K \sum_{j=1}^{N} y_j .$$

The value of the integral is therefore

$$I = \frac{\Delta x \cdot E(M)}{K} , \qquad (3)$$

The number M accumulated in the counter will not, in general, be equal to E(M), but will be a random number whose mean is E(M) and whose variance is

$$\sigma_M^2 = \sum_{j=1}^{N} (p_j - p_j^2) .$$

σ_M^2 is bounded since it is zero for

$$p_j = 0 \text{ or } p_j = 1 ,$$

and because $\frac{\partial \sigma_m^2}{\partial p_r} = 1 - 2 p_r$

σ_m^2 will have a maximum value when all $p_r = 1/2$.

$$\text{i.e. } \sigma_m^2(max) = \sum_{j=1}^{N} (1/2 - 1/4) = \frac{N}{4} ;$$

and the maximum value of the standard deviation is

$$\sigma_m(max) = \frac{\sqrt{N}}{2} .$$

If then we substitute M for its expected value in (3) we will get a value for the integral

$$I' = \frac{\Delta x \cdot M}{K} \quad . \qquad (4)$$

I' will be a random number having a nearly normal distribution whose expected value is I, and whose maximum standard deviation will be

$$\sigma_I \text{ (Max)} = \frac{\Delta x}{K} \frac{\sqrt{N}}{2} \quad .$$

Since $\Delta x = (x_2 - x_1)/N$, where $(x_2 - x_1)$ is the interval of x over which the integral is taken, and N is the number of increments, then

$$\sigma_I \text{ (Max)} = \frac{1}{2K} \sqrt{\Delta x \cdot (x_2 - x_1)} \quad . \qquad (5)$$

Thus if the summation approximation is valid the system will compute the value of the integral as given by equation (4) with a random error whose standard deviation will not exceed the value given by equation (5).

The random errors can be reduced by reducing the size of the increment. This will also improve the accuracy of the summation process. If it is not feasible to reduce the increment size below a certain level then the random errors can still be further reduced by causing several pulses to be generated at each increment. If K pulses are so generated then the random errors will be reduced by $\sqrt{K}$. This will not, of course, improve the accuracy of the summation approximation.

The foregoing assumed that y could not change sign. If y can be positive or negative then two gates are necessary. The system is then as shown in fig. 2.

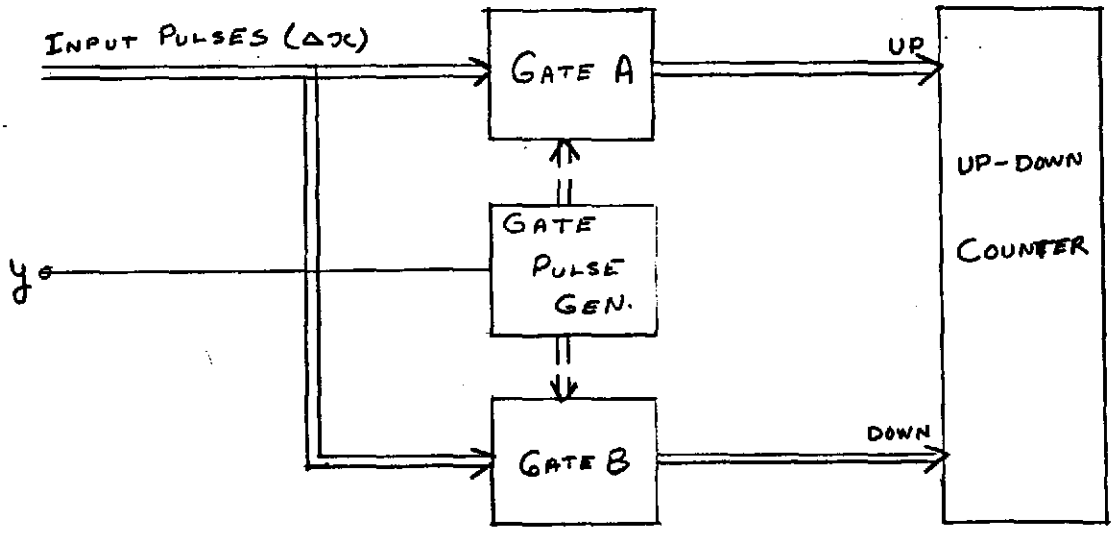


Fig. 2

Gate A operates when y is positive and its probability p_A of being open is given by:

$$p_A = K.y \text{ for } y \geq 0$$

$$p_A = 0 \text{ for } y < 0$$

Gate B operates when y is negative and its probability of being open is given by:

$$p_B = -K.y \text{ for } y \leq 0$$

$$p_B = 0 \text{ for } y > 0$$

An output from gate A increments the counter by 1, an output from gate B decrements the counter by 1. The expected value of the net count M is therefore $E(M) = \sum_{j=1}^{N} (P_{Aj} - P_{Bj})$. Since outputs from the two gates are mutually exclusive then $P_{Aj} - P_{Bj} = K \cdot y_j$

$$E(M) = K \sum_{j=1}^{N} y_j \quad \text{as before,}$$

also $\sigma_M^2(\max)$ can be shown to be $\frac{N}{2}$ as before.

3.0 A SIMPLE NAVIGATIONAL COMPUTER

The usefulness of this method is best illustrated by an example. In a dead-reckoning computer the distance, s_A, travelled along a reference direction is given by $s_A = \int \cos\theta \, ds$, where s is the distance travelled on a plane surface and θ is the angle between the direction of motion and the reference axis in the plane. The distance S_R travelled at right angles to the reference axis is given by

$$S_R = \int \sin\theta \, ds.$$

If we consider only the first integral,

$S_R = \int \cos\theta \, ds$, this can be implemented as shown in the diagram in fig.2, where the input $y = \cos\theta$ and the increment $\Delta x = \Delta s$.

An input pulse is generated at each increment of distance travelled. This pulse could be generated, for example, at each turn of the speedometer cable of a land vehicle. Cos θ is an analog voltage developed from some form of resolver on the reference direction indicator. The gate pulse generator developes pulses to operate gates A and B so that the probability P_A of gate A being open at any instant is given by:

$$P_A = \cos\theta \text{ for } \cos\theta \geq 0$$

$$P_A = 0 \text{ for } \cos\theta < 0$$

similarly

$$P_B = -\cos\theta \text{ for } \cos\theta \leq 0$$

$$P_B = 0 \text{ for } \cos\theta > 0$$

Each pulse from gate A represents one increment of distance up the reference axis, and increments the counter by 1. Each pulse from gate B represents one increment down the reference axis and decrements the counter by 1. The number held in the counter represents the initial setting plus the net distance travelled up the reference axis.

An identical circuit will compute the distance at right angles to the reference axis if sin θ is substituted for cos θ at the analog input. The two counters then in effect continuously display two co-ordinates indicating position on the plane surface.

A circuit diagram for such a navigational computer is described in the Appendix.

4.0 ERROR ANALYSIS

The errors introduced by non-ideal behaviour of components are not relevant to this discussion. They are determined by the actual circuit design and can usually be reduced to levels consistent with the accuracy of input transducers.

Two other types of error are introduced. The first is in the summation approximation to the integral. In general the upper bound placed on the value of the incremental distance is determined by the manner in which the other variable, in this case cos θ, varies with distance. The situation is best summed up as this: the approximation will be good if the increment of distance is too short to allow the vehicle any appreciable angle maneuver within that increment.

The other type is the random error discussed previously. The maximum standard deviation of s_A will be $\sigma_{s_A(max)} = \sqrt{\Delta S \cdot S}$, where S is the total distance travelled, and ΔS the increment used.

Hence the maximum value of the variance of the error varies as the square root of the total distance travelled on the plane surface and as the square root of the increment used.

As an example, if the vehicle travels a total distance of 10,000 feet and a distance increment of one foot is used, then the maximum value of the standard deviation of the computing error will be 50 feet. The actual standard deviation will of course depend on the path travelled and can be anywhere from zero to this maximum value. Since the absolute error increases as the root of the distance travelled, the relative error decreases as the root of the distance travelled. This is to be expected since a

random process becomes more predictable as the number of events increases.

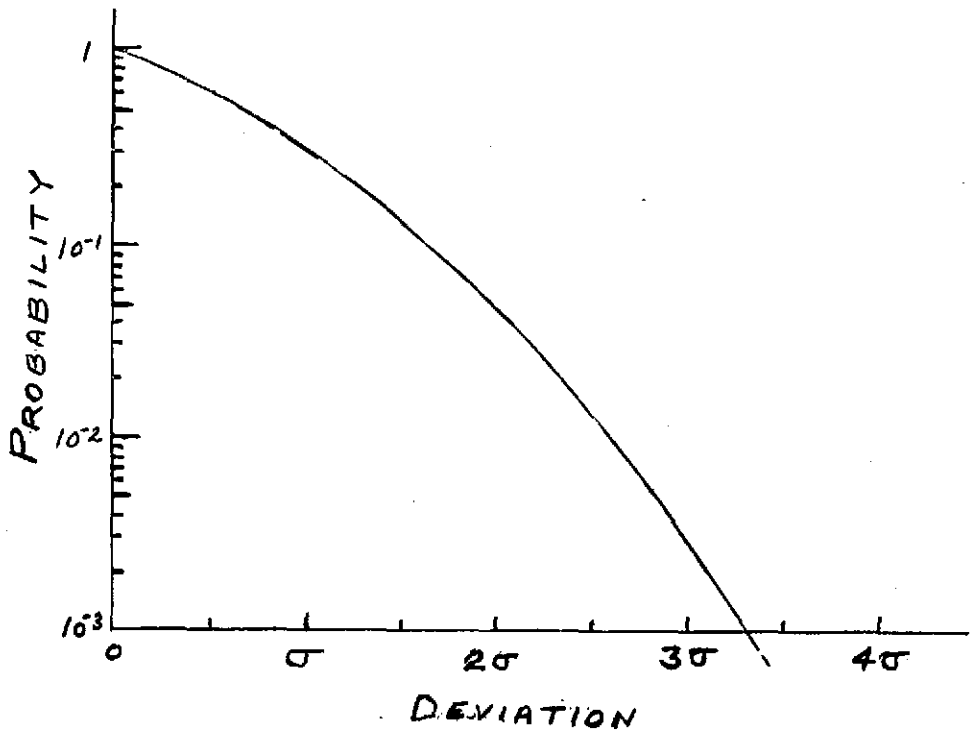


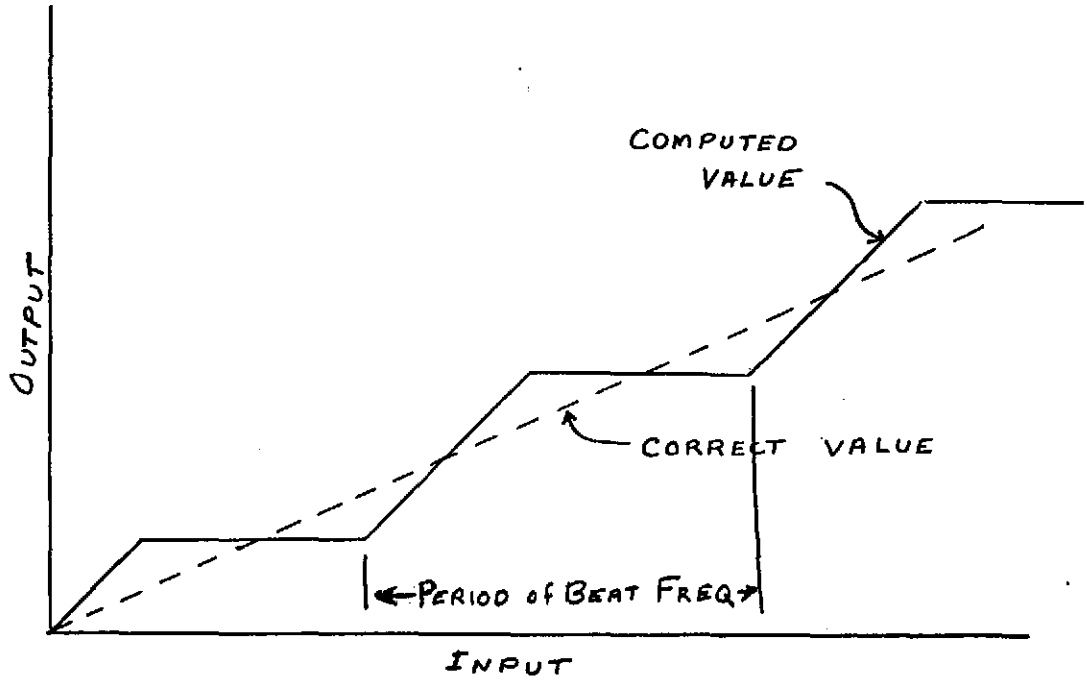
Fig. 3

The curve of fig. 3 gives the probability that the deviation of a normal distribution will exceed a given value. This shows that in the example given, where σ_I (max) = 50, the probability is one-half that the error will exceed 38 feet. However in only one run in a thousand will it exceed 170 feet. This is under the worst condition where $\cos\theta = \pm 0.5$. For $\cos\theta = 0$ or ± 1 the random error will be zero.

5.0 THE GENERATION OF RANDOM GATES

The foregoing has assumed that the operation of the gate is random with respect to input pulse operation. This does not necessarily imply that the gate must be random with respect to time. In fact in a lot of applications there will be sufficient variation in the frequency of the input pulses to give random operation of the gate even though the gate itself operates at fixed frequency. If both gates and input pulses do arrive at a fixed rate, and provided there is no simple harmonic relation between the two frequencies, they will give a beat frequency. It can be

shown that, if averaged over a complete cycle of the lowest beat frequency of the pulse and gate frequencies, the performance of the computer will be correct. The output for a fixed gate width will be roughly as shown in fig. 4.



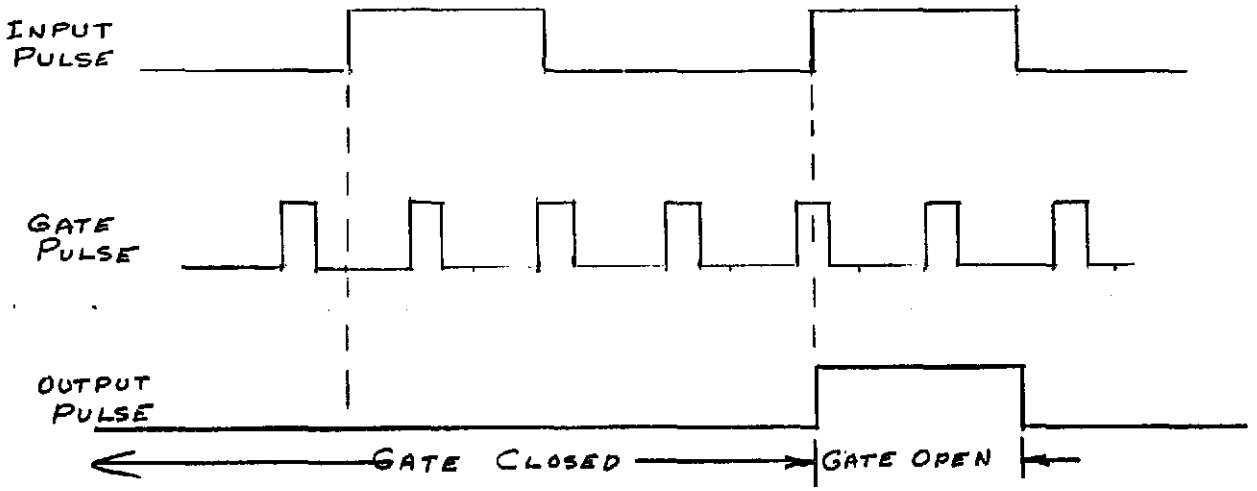
Output for constant gate width and fixed gate and pulse frequencies.

Fig. 4

Where possible the gate should be operated at a much higher frequency than the input pulses. In this case the lowest beat frequencies will occur between the fundamental of the gate frequency and a high harmonic of the pulse frequency. The input pulse rate would then have to be extremely stable to produce a sustained stable beat frequency, and this would be very unlikely in most applications. If it were possible that such periods of stable input pulse frequencies could occur, then the input pulses could be time-displaced in a random fashion over a time period greater than one period of the gate pulse. This would satisfy the conditions for random gate occurences.

Since the gate pulses can occur at a fixed frequency their generation is quite simple. All that is required is that the pulse width, expressed as a fraction of the total period, be equal to the probability required.

The gating itself must be done carefully. Any circuit delays which tend to add to the effective gating period will of course cause consistent errors. In the navigational computer described in the Appendix the gating is effectively done on the leading edge of the input pulse. If the gate pulse is "on" when the leading edge of the input pulse arrives the input pulse is passed through unchanged. If the gate pulse is "off" at the arrival of the input pulse the gate remains off for the duration of the input pulse. This operation is illustrated in the diagram in fig. 5.



Operation of Pulse Gate

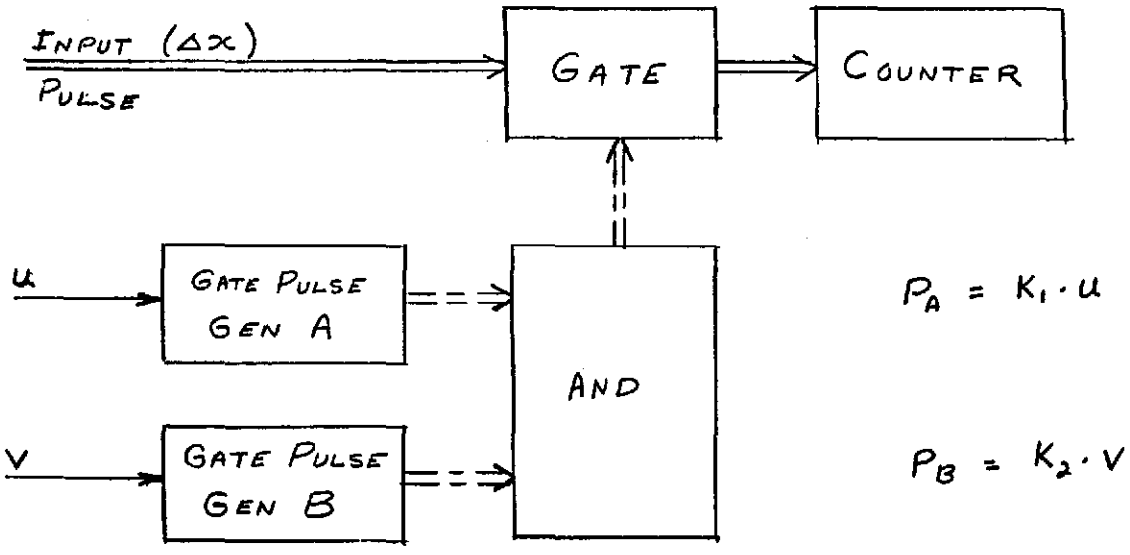
Fig. 5

6.0 ADDITIONAL COMPUTING

Expressing the value of a variable by the probability of occurence of a pulse, as was done in the computing system described, opens the possibility for additional computing of these variables by using the combining properties of random events.

6.1 Multiplication

Two variables can be multiplied by combining their respective gate pulses in an "and" gate. This is illustrated in fig. 6.



$$I = \int u \cdot v \, dx \approx \frac{\Delta x}{K_1 \cdot K_2} \sum_{j=1}^{N} u_j \cdot v_j$$

Fig. 6

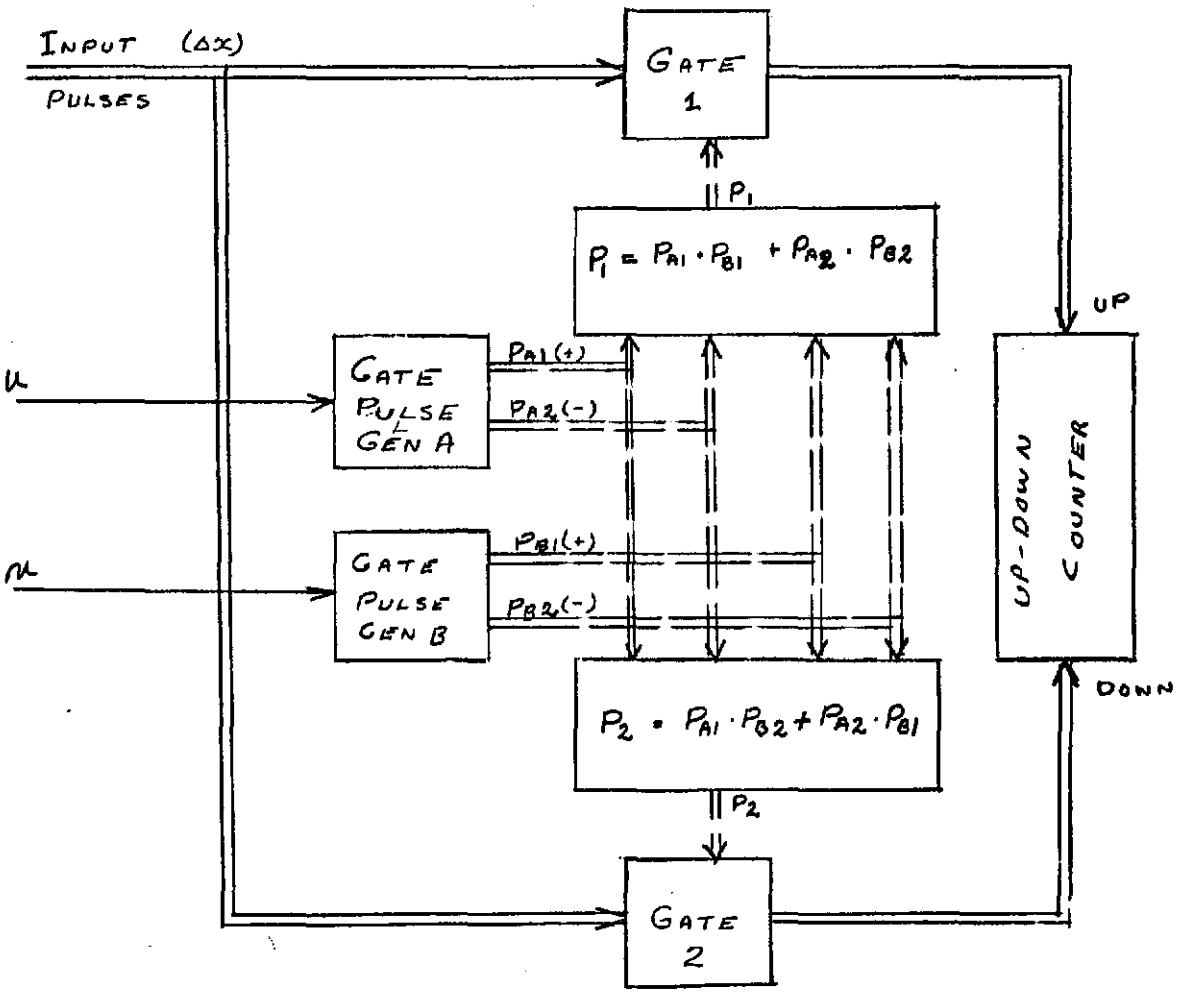
If P_A is the probability that an "on" voltage will be present from gate pulse generator A, and P_B is the similar probability for B, and if these events are statistically independent, then the voltage from the "and" gate will have a probability of $P_A \cdot P_B$ of being "on." Then, if $P_A = K_1.u$, and $P_B = K_2.v$, the expected value of M in fig. 6 is

$E(M) = \sum_{j=1}^{N} P_{A_j} \cdot P_{B_j}$ hence the system can be used to compute $I = \int u \cdot v \, dx$

Fig. 7 shows the same system when u and v can both have positive and negative values.

This method of multiplication assumes statistical independence of the gate pulses representing the two variables. Here again if the two gate pulses are at fixed frequencies the method can be shown to be correct if averaged over the beat frequency of the two gates. Also the value of the product is only of importance at the arrival of an input pulse, which is effectively a random occurence. Hence the requirement for random generation is satisfied if the average value of the output of the AND gate is correct, and if any cyclic variation in the value of the output is not correlated with the arrival of input pulses.

If this condition does not obtain then one or both of the gate pulses can be made to occur at a random frequency, although this will complicate the pulse generation.



System for forming $I = \int u \cdot v \, dx$ when u and v can both have positive and negative values.

Fig. 7

6.2 Addition

If A and B are mutually exclusive events with probabilities of occurence of P_A and P_B, then the probability of occurence of A or B is $P_A + P_B$. Therefore two variables can be added by combining their gate pulses in an "OR" gate, provided the two pulses are mutually exclusive. Fig. 8 shows a means of implementing

$$I = \int (a \cdot u + b \cdot v)\, dx$$

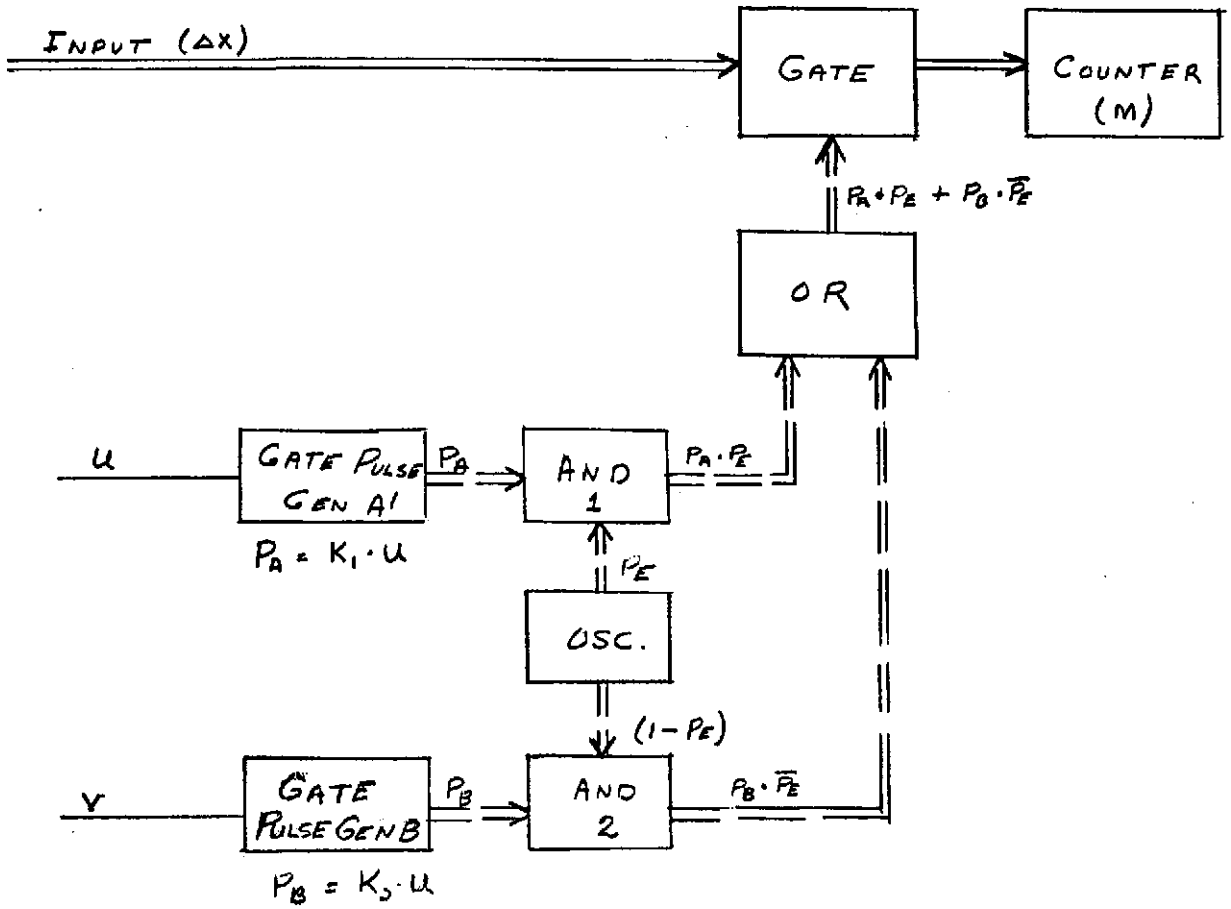


Fig. 8

The two AND gates 1 and 2 are used to make the pulses from A and B mutually exclusive. The gates are driven by the oscillator so that when 1 is "on" 2 is "off" and vice versa. AND 1 is "on" for a fraction P_E of time and AND 2 for a fraction $(1-P_E)$ of time. The outputs of these two gates are then mutually exclusive.

The expected value of M is now

$$E(M) = \sum_{j=1}^{N} \left(P_{Aj} \cdot P_{Ej} + P_{Bj} (1 - P_{Ej}) \right)$$

$$= K_1 \cdot P_E \sum_{j=1}^{N} u_j + K_2 (1-P_E) \sum_{j=1}^{N} v_j$$

By choosing appropriate values for K_1, K_2 and P_E one can then form the integral $I = \int (a \cdot u + b \cdot v)\, dx$

If u and v are analog voltages it is simpler to add them in that form. However if u or v is the product of two other variables then analog addition is not possible and the above method can be used, in conjunction with the multiplication method described in para. 6.1.

6.3 Division

Division can be obtained by placing the gate of fig. 1 in a feedback loop as shown in fig. 9

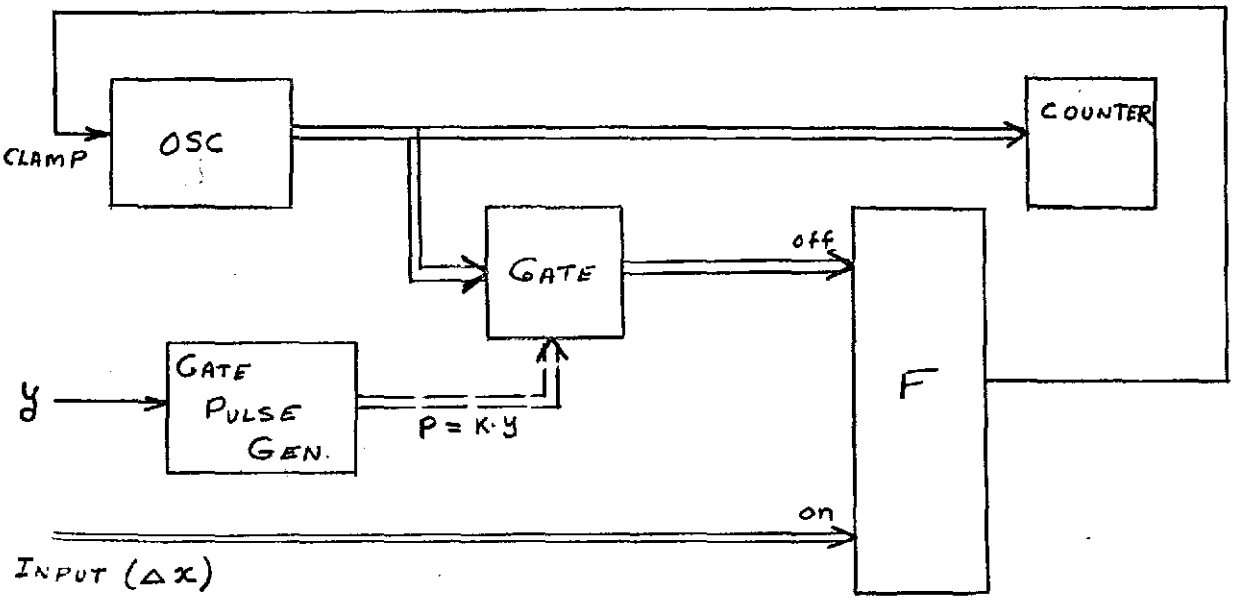


Fig. 9

F is a flip-flop which is turned on by an input pulse and turned off by a pulse from the gate. F controls the oscillator so that output pulses are generated when F is on. These output pulses are also applied to the gate input.

The operation is then as follows. When an input pulse arrives it turns F on. This starts the oscillator which generates output pulses until one of these pulses gets through the gate and turns F off. F remains off until the next input pulse. Thus the number of output pulses will depend inversely on the probability p of a pulse getting through the gate.

In common with all methods of division this system will have large errors when the divisor, in this case p, is small. To assess the errors of the system it is necessary to assume a lower limit to the value p. i.e. $p \geq p_{min}$.

The description of operation assumed that the flip-flop F is always turned off before the next input pulse arrives. If it is not then a truncation error occurs. Let R be the maximum number of output pulses that can be generated between successive input pulses. Then for $S < R$ the probability $p(S)$ that S pulses will be generated is the probability that the S^{th} pulse gets through the gate and that all preceding pulses were blocked. Since p is the probability that the gate is open,

$$p(S) = p\,(1-p)^{S-1} \quad \text{for} \quad 1 \leq S < R$$

The probability that R pulses are generated is the probability that none of the previous R-1 pulses get through the gate.

$$\text{i.e.} \quad p(R) = (1-p)^{R-1}$$

Therefore the expected value of S is

$$E(S) = \sum_{S=1}^{R-1} Sp(1-p)^{S-1} + R\,(1-p)^{R-1}$$

$$\therefore E(S) = \frac{1}{p} - \frac{(1-p)^R}{p}$$

For N input pulses the expected value of M, the number of output pulses, is

$$E(M) = \sum_{j=1}^{N} E(S_j) = \sum_{j=1}^{N} \left(1/p_j - \frac{(1-p_j)^{R_j}}{p_j} \right)$$

if $(1-p_j)^{R_j}$ is always small then $E(M) = \sum_{j=1}^{N} 1/p_j$,

and $$\int_{x_1}^{x_2} \frac{dx}{y} = \Delta x \sum_{j=1}^{N} \frac{1}{y_j} = K.\Delta x.\, E(M)$$

The error introduced in E(M) by a finite value of $(1-p_j)^{R_j}$ is

$$E = \sum_{j=1}^{N} \frac{(1-p_j)^{R_j}}{p_j} .$$

The relative error $E/E(M) = \sum_{j=1}^{N} \frac{(1-p_j)^{R_j}}{p_j} \Big/ \sum_{j=1}^{N} 1/p_j$

Since the derivative $\frac{\partial (E/E(M))}{\partial p_R}$ is always negative the maximum value of the relative error will occur when $p_j = p_{min}$ and $R_j = R_{min}$.

$$\therefore \left[E/E(M) \right]_{max} = (1-p_{min})^{R_{min}}$$

R_{min} is the maximum number of oscillator pulses which may be generated between successive input pulses, at the minimum input pulse separation. The minimum allowed oscillator frequency will therefore depend on the accuracy required, the maximum value of input pulse frequency, and the minimum expected value of p_j.

As an example, suppose that a maximum variation of 2:1 is expected in the value of y, and that the error must be kept below 0.1%. i.e. $(1-p_{min})^{R_{min}} \leq 10^{-3}$. By proper choice of K the value of p_j can be restricted to the range $0.5 \leq p_j \leq 1$. i.e. $p_{min} = 0.5$.

$$\therefore (0.5)^{R_{min}} \leq 10^{-3}$$

$$R_{min} \geq 10$$

This means that the oscillator frequency for this case must be at least ten times the maximum input pulse rate to ensure the required accuracy under all conditions.

In addition to the truncation error described above there will be a random error. The variance of M is given by

$$\sigma_m^2 = \sum_{j=1}^{N} \left(\frac{1}{p_j^2} - \frac{1}{p_j} \right)$$

This will be maximum when $p = p_{min}$

$$\text{i.e. } \sigma_m^2(max) = \frac{N(1-p_{min})}{p_{min}^2}$$

$$\sigma_{m(max)} = \frac{\sqrt{N(1-p_{min})}}{p_{min}} .$$

At this value of p the relative error will be $\frac{\sigma_m}{E(M)} = \sqrt{\frac{(1-p_{min})}{N}} .$

6.4 Terms in the numerator and denominator

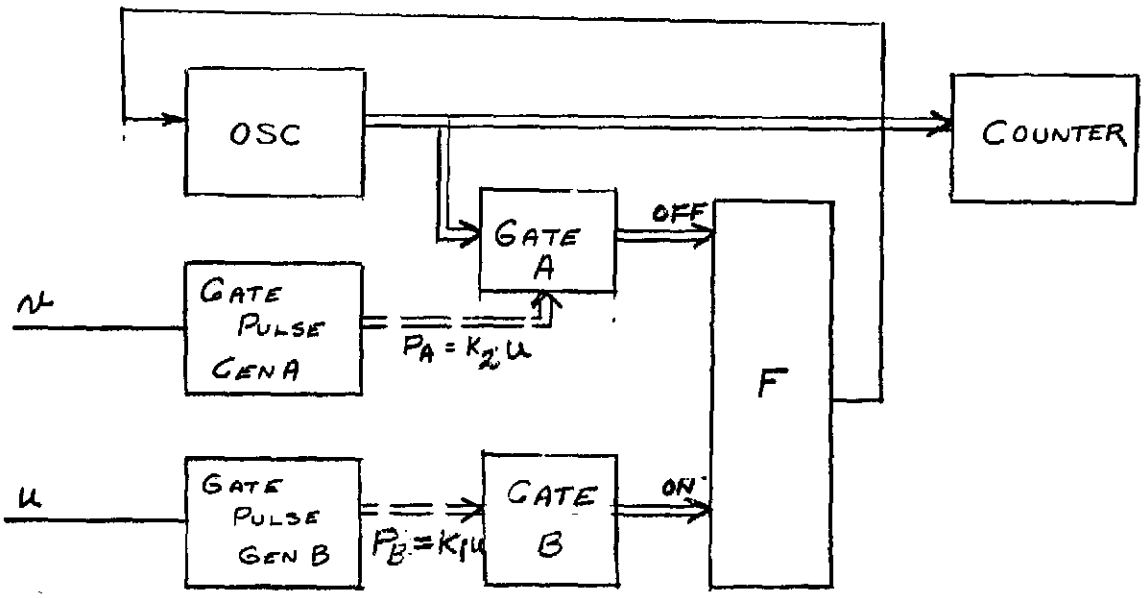


Fig. 10 Method of evaluating $\int \frac{u}{v}\,dx$

Fig. 10 shows a method for combining terms in both the numerator and denominator. It is easily shown that the expected value of the number of output pulses is given by

$$E(M) = \sum_{j=1}^{N} \frac{P_{Bj}}{P_{Aj}} = \frac{K_1}{K_2} \sum_{j=1}^{N} \frac{u_j}{v_j}$$

where N is the number of input pulses.

P_A and P_B can both be combinations of a number of functions, as described previously. Additional functional dependence can of course be built into the gate pulse generation circuitry. One is only limited here by the ingenuity of the circuit designer and the state of the art.

Hence the method can be extended to the general case where $I = \int y/z\,dx$, and y and z may be functions of other variables.

7.0 EXAMPLE

One further application is illustrated below.

If a gas is flowing in a pipe with a pressure p and an absolute temperate T, then mass of the gas flowing through the pipe is

$$m = K \int \frac{Pv}{T}\,dt$$

Where v is the velocity of the gas.

Since $v = \frac{dx}{dt}$, where x is the distance travelled, then

$$m = K\int \frac{P}{T}\, dx$$

If a pulse is generated at each turn of a propeller in the pipe, then each pulse represent an incremental distance, Δx, travelled by the gas. If these pulses are applied to the input of the network of fig. 9, and $p_B = K_1 p$ and $p_A = K_2 T$, then the number of output pulses is

$$M = \sum_{j=1}^{N} P_{Bj}/P_{Aj} = (K_1/K_2)\sum_{j=1}^{N} P_j/T_j$$

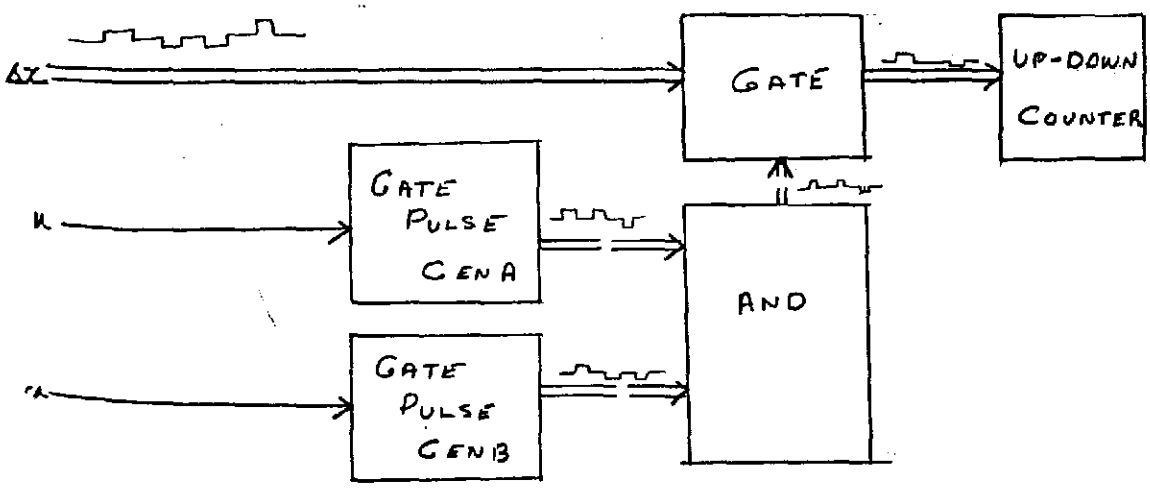
and hence is proportional to the mass of gas flowing past the measuring station.

8.0 GENERAL REMARKS

The systems outlined in block diagram form in this report have not been engineered for simplicity or economy of components. The idea was not developed with any application in mind and the systems have been chosen for ease of description. Nor has any mention been made of the case where the variable represented by the input pulses does not increase monotonically with time. This case can be handled by a parallel system for negative increments, or by reversing the action of the up-down counter during negative increments. There are several methods of dealing with changes of sign of any of the variables and the one chosen will depend on the particular application.

One method which appears worthy of mention is to represent positive and negative quantities by positive and negative pulses, respectively, at the same terminal. If the individual gates were then designed to operate with these bipolar inputs and to output with the correct signs, a considerable saving of components could probably be obtained and certainly the block diagrams would be considerably simplified.

Fig. 11 shows the system of fig. 7 redrawn using this bipolar system, and extended to include negative increments of the variable x.



System for evaluating $I = \int u \cdot n\, dx$. Where all pulse components are designed for bipolar inputs and outputs.

Fig. 11

9.0 CONCLUSION

A method of machine computation has been described which is simple in concept and easy to instrument. It can be built using solid state circuitry and can be smaller and more rugged than equivalent analog methods now in use. While not a general purpose device it has possible wide application within the Armed Services in simplifying present forms of navigation computers and in extending the use of these computers to smaller installations than is presently contemplated.

The problems of data acquisition and display still remain, of course, but new devices are becoming available. The Hall-effect magnetometer and fast stepping motors are examples of new devices which are compatable with the computer. These should enable one to build a complete dead reckoning navigation system which is compact and rugged with a low power drain, but which is accurate within the limits defined by methods of magnetic orientation.

The computer also has possible commercial applications, both in the field of navigation computers and process control and automation.

10.0 ACKNOWLEDGEMENTS

The author wishes to thank Mr. C.R. Iverson, Superintendent of Systems Wing, CARDE, for his encouragement to complete this work.

The demonstration unit described in the Appendix was built and tested by Mr. Marc Savard of CARDE.

APPENDIX

1.0 INTRODUCTION

This appendix gives the circuitry and the performance of a simple navigational computer using the methods outlined in the main report. The circuit is given in block diagram form in Fig. 1A. The output increments can be used to operate counters or a digital X-Y plotting table.

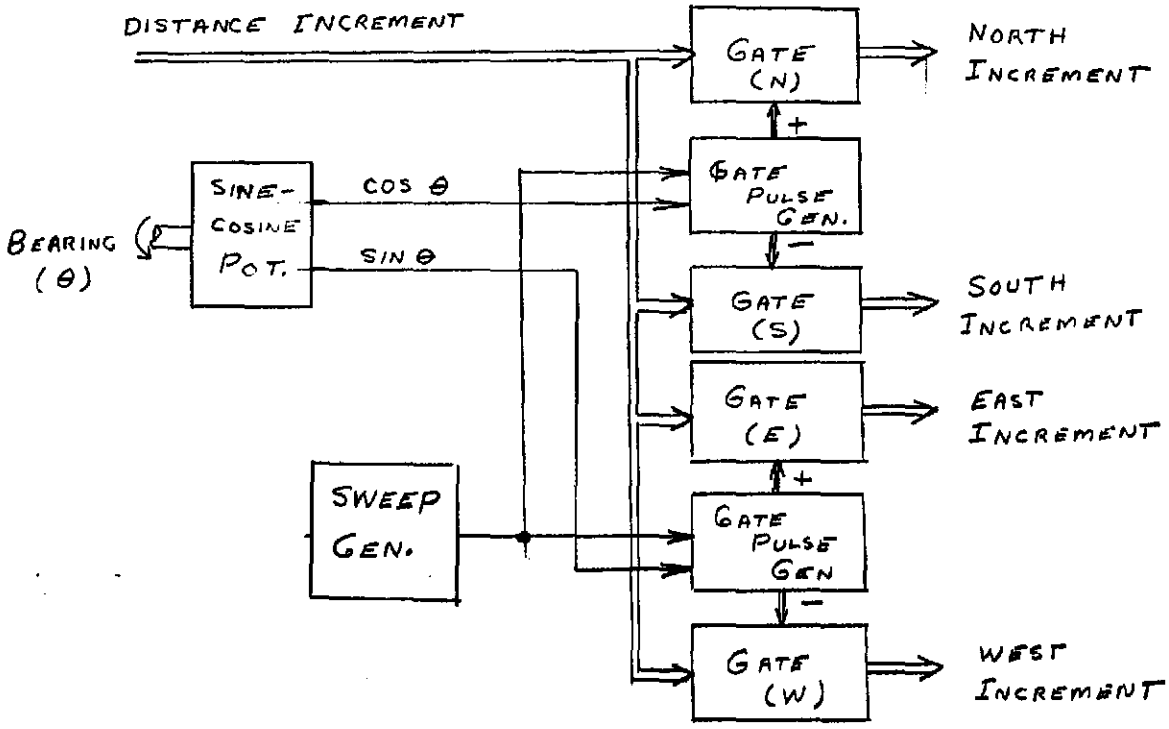


Fig. 1A

The gate pulse generators generate pulses whose dwell times are proportional to the magnitude of the input functions, sin θ or cos θ. The outputs are from the positive or the negative lines, depending on the sign of the input functions. The outputs are obtained by comparing the input voltages with a linear triangular sweep from the sweep generator. Fig. 2A shows typical waveforms for one such pulse generator.

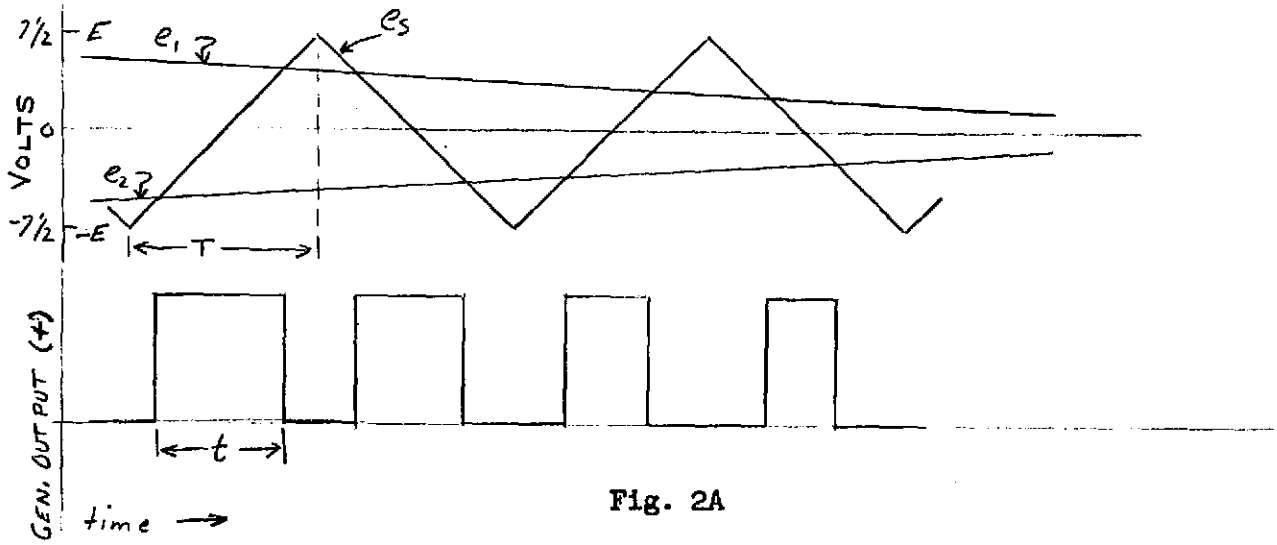


Fig. 2A

e_s is the triangular sweep voltage. e_1 and e_2 represent the input function, say sin θ, such that $e_1 - e_2 = 2E \sin \theta$ and $e_1 + e_2 \approx 0$. These voltages are derived from the sine-cosine potentiometer which has a double-ended output.

Pulses from the positive output are "on" when $e_1 \geq e_s \geq e_2$. For $e_1 > e_2$ they will be "on" for a total time $t = \frac{e_1 - e_2}{2E} \cdot T$ for each half cycle of the sweep voltage.

i.e. $\frac{t}{T} = \frac{e_1 - e_2}{2E} = \sin \theta$

For $e_1 < e_2$ there will be no pulses from the positive output.

Similarly when $e_1 < e_2$ the pulses from the negative line will be "on" for a proportion of time $\frac{t}{T} = -\sin \theta$. For $e_1 > e_2$ these pulses will not occur.

2.0 SWEEP GENERATOR

The sweep generator is shown in Fig. 3A. Transistors T_5 to T_{10} form a high gain amplifier, which with capacitive feedback acts as an integrator. The input to the integrator is from the flip-flop formed by transistors T_1 to T_4. With T_2 and T_3 "on" a positive voltage is applied to the integrator. This causes its output to fall at a fixed rate. When the integrator output reaches -7 1/2 volts (-E) T_{12} conducts and reverses the flip-flop, causing T_1 and T_4 to conduct. This applies a negative voltage to the integrator input, causing a positive going output. When the output reaches + 7 1/2 volts (+E) T_{11} conducts and again reverses the flip-flop. Thus a free-running linear triangular sweep voltage is generated whose excursions are limited to ± 7.5 volts. The repetition rate of the generator is approximately 350 cps.

3.0 PULSE GENERATOR

A pulse generator circuit is shown in Fig. 4A. The long-tailed pairs, T_1 and T_2, and T_3 and T_4 are voltage coincidence networks. T_5 and T_6 provide constant current at the emitters. T_7 is turned off if T_2 and T_3 are both off. This occurs when $e_s > e_2$ and $e_1 > e_s$, i.e. when $e_1 \geq e_s \geq e_2$. Thus the positive gate pulse is considered "on" when T_7 is "off". Similarly the negative gate pulse is "on" when T_8 is off.

4.0 GATE OPERATION

The gate uses direct-coupled rather than triggered logic. This method was used because it gives wider lattitude to input waveforms and rise times. If it were designed for a specific application where specifications of input waveforms were known some component saving could probably be achieved if triggered logic were used.

The gate operation is shown by the merged flow diagram[1] of Fig. 5A. An output is associated with states 5 and 6. P represents the input pulse and G the gate pulse. Y is a secondary variable. The flow diagram shows that an output can be generated only if the gate was on when the pulse came on. The output then remains on until the input pulse disappears. The excitation matrices for Y and the output are also given in Fig.5A, with the resulting Boolean expressions. Note that although the term G.$\bar{y}$ in the expression for Y is redundant as an algebraic term it is necessary in the circuit to ensure correct transition between states 5 and 3.

The circuit diagram of Fig. 6A shows the circuit to implement this flow diagram. The logic levels used are: ground potential represents binary 1, negative potential (below - 4 volts) represents binary 0. The gate pulse inputs can be connected directly to the output of the gate pulse generator. The input pulse and its complement are required at the logic levels described above.

5.0 RESULTS

The equipment was tested using the set-up shown in Fig. 7A. The input pulses were obtained from a pulse generator operating at about 50 pulses per second. The input voltages were obtained from a sine-cosine potentiometer connected between the ± 7 1/2 volt supplies. Its total winding impedance is approximately 35 KΩ and its stated accuracy was.25%.

The input pulses and the North and East output increments were counted. The tests were conducted in the NE quadrant only. For each setting of the input angle five runs were taken. At each run approximately 10,000 input pulses were applied, the resulting North and East increments were counted and from these the computed distance and bearing were computed and compared with the original. The results are tabulated below.

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1. S.H. CALDWELL, Switching Circuits and Logical Design, (New-York, John Wiley & Sons Inc, 1958).

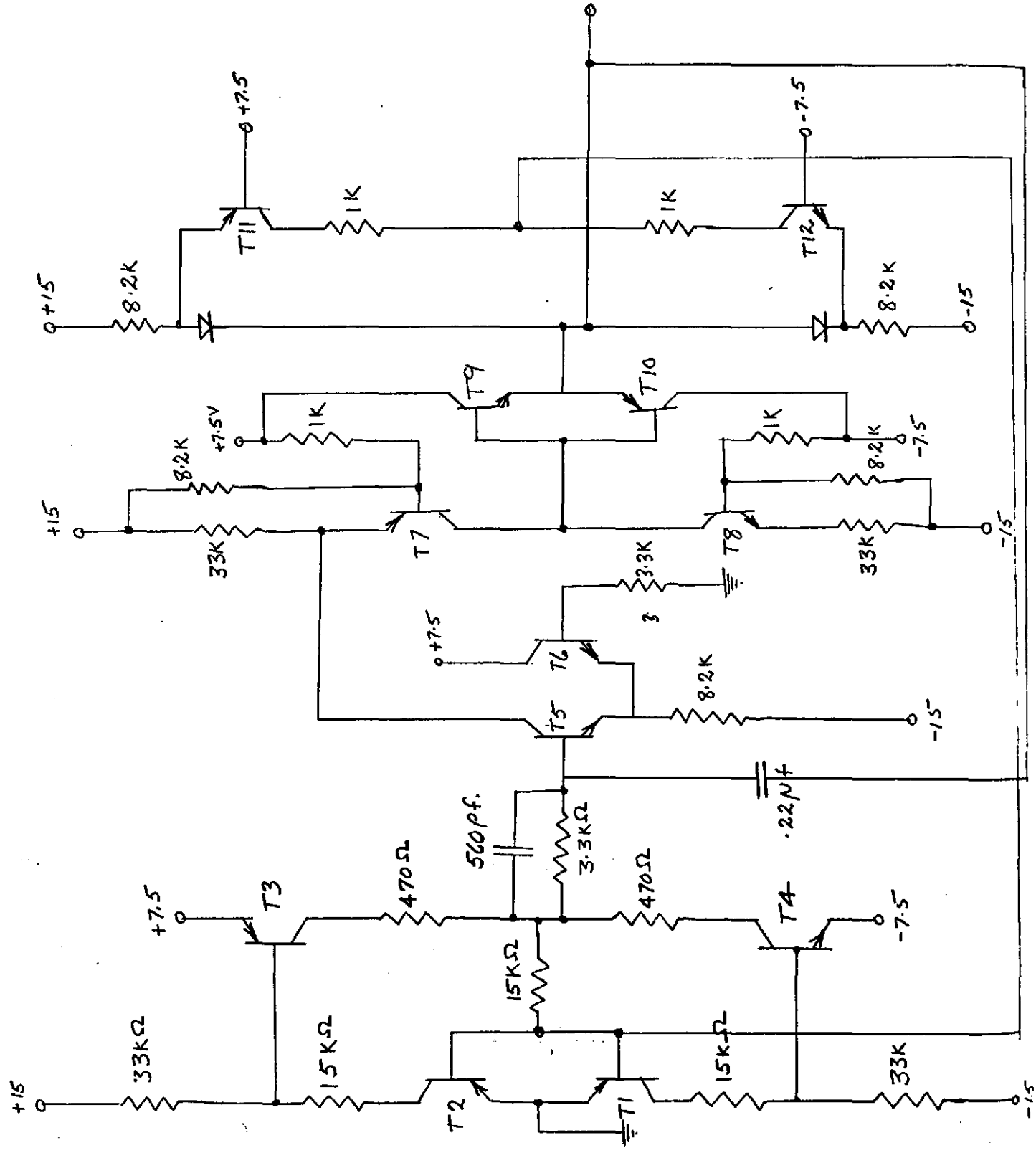
INPUT		OUTPUT			
θ(°)	S	Distance East	Distance North	Distance Error(%)	Calculated Bearing (°)
0	10065	345	9999	-.60	1.98
	10067	345	10013	-.48	1.97
	10057	342	10010	-.41	1.96
	10071	366	9999	-.65	2.10
	10063	322	10014	-.44	1.84
	Mean Distance Error = -.52%			Std. Deviation = .09%	
	Mean Bearing = 1.97°			Std. Deviation = .08°	
10	10052	2135	9863	.38	12.21
	10049	2138	9866	.45	12.23
	10067	2106	9874	.28	12.04
	10056	2128	9855	.25	12.18
	10052	2152	9854	.33	12.32
	Mean Distance Error = .34%			Std. Deviation = .07%	
	Mean Bearing = 12.20°			Std. Deviation = .09°	
20	10052	3803	9356	.46	22.12
	10056	3799	9353	.38	22.11
	10057	3826	9365	.58	22.22
	10055	3800	9349	.36	22.12
	10061	3790	9370	.45	22.02
	Mean Distance Error = .45%			Std. Deviation = .09%	
	Mean Bearing = 22.12°			Std. Deviation = .06°	
30	10055	5310	8585	.38	31.74
	10057	5308	8580	.31	31.74
	10057	5311	8613	.61	31.66
	10056	5288	8602	.40	31.58
	10058	5297	8584	.28	31.68
	Mean Distance Error = .40%			Std. Deviation = .12%	
	Mean Bearing = 31.68°			Std. Deviation = .06°	

INPUT		OUTPUT			
θ(°)	S	Distance East	Distance North	Distance Error(%)	Calculated Bearing (°)
45	10056	7363	6916	.45	46.79
	10064	7390	6897	.43	46.98
	10057	7359	6885	.20	46.91
	10061	7371	6910	.41	46.85
	10058	7351	6880	.09	46.90
	Mean Distance Error = .32%			Std. Deviation = .14%	
	Mean Bearing = 46.89°			Std. Deviation = .06°	
60	10059	8867	4832	.38	61.41
	10056	8893	4803	.50	61.63
	10058	8901	4810	.59	61.61
	10059	8900	4821	.62	61.56
	10063	8853	4814	.13	61.46
	Mean Distance Error = .44%			Std. Deviation = .18%	
	Mean Bearing = 61.53°			Std. Deviation = .08°	
80	10066	9938	1531	- .11	81.24
	10060	9941	1486	- .09	81.50
	10069	9935	1467	- .25	81.42
	10073	9955	1447	- .13	81.73
	10076	9950	1472	- .18	81.58
	Mean Distance Error = -.15%			Std. Deviation = .06%	
	Mean Bearing = 81.49°			Std. Deviation = .16°	

The angle measuring equipment used in the experiment was accurate to only about 1/2°, and contained an index error of approximately 2°. The computed angle does not, therefore, accurately represent the input angle. However at a given angle setting the standard deviation of the computed angle is generally less than 0.1°.

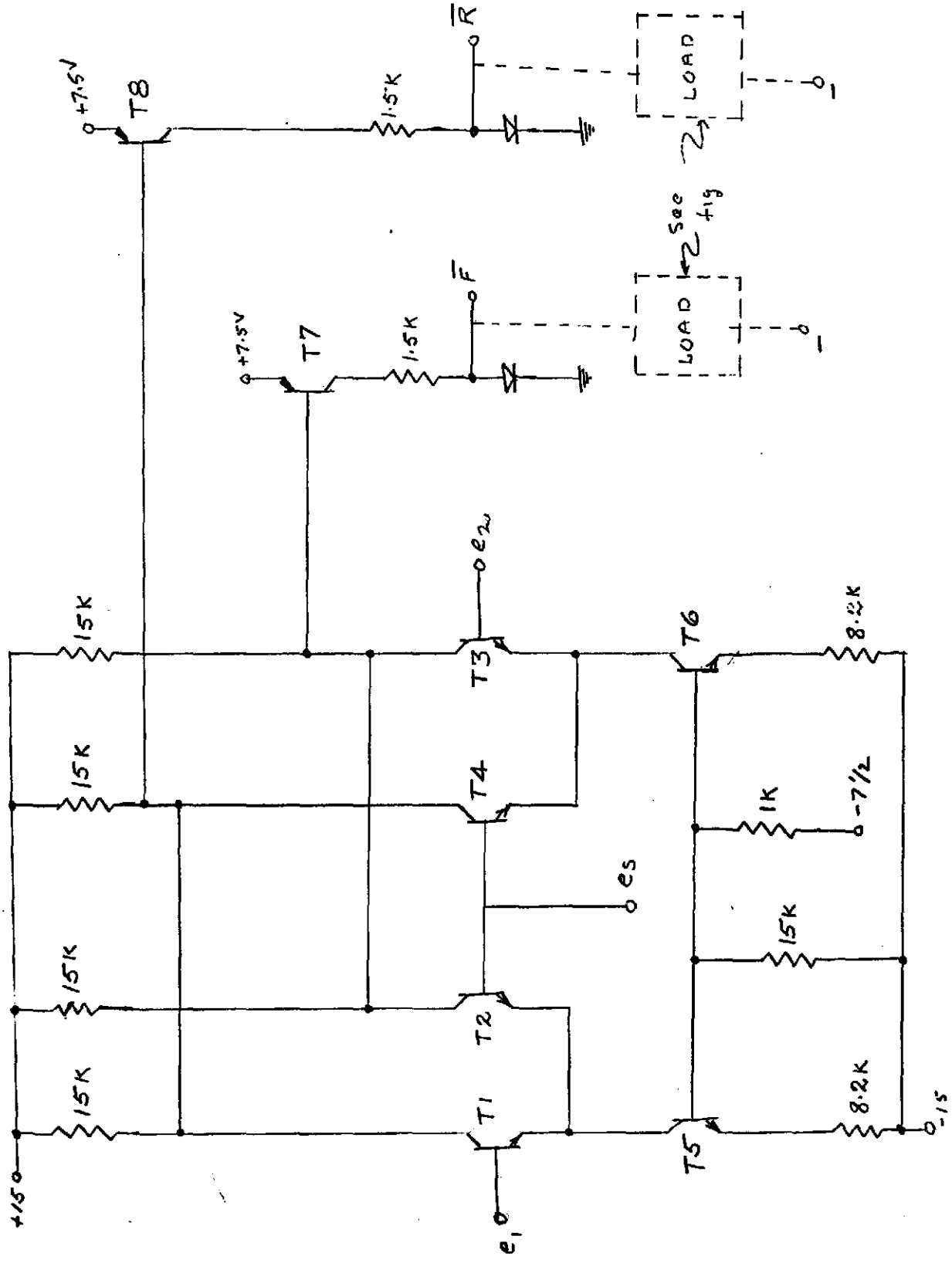
There is a consistent error in the computed distance ranging from -.5% to .5 % depending on the input angle. This is in part due to the sine-cosine potentiometer and in part to circuit error. The random errors are generally less than predicted because the input pulses are not random. Some random fluctuations are also present from circuit noise. The value has not been assessed.

The circuits shown have not been engineered for economy or for environmental conditions but they do show the feasibility of this method. The circuitry is much simpler than in a comparable digital method and has none of the long term drift problem of electronic analog methods.



LINEAR SWEEP GENERATOR

Fig. 3A



GATE PULSE GENERATOR

Fig. 4A

MERGED FLOW DIAGRAM

Y	PG			
—	00	10	11	01
0	(1)	(2)	(4)	3
1	1	(6)	(5)	(3)

Y- EXCITATION MATRIX

0	0	0	0	1
1	0	1	1	1

OUTPUT MATRIX

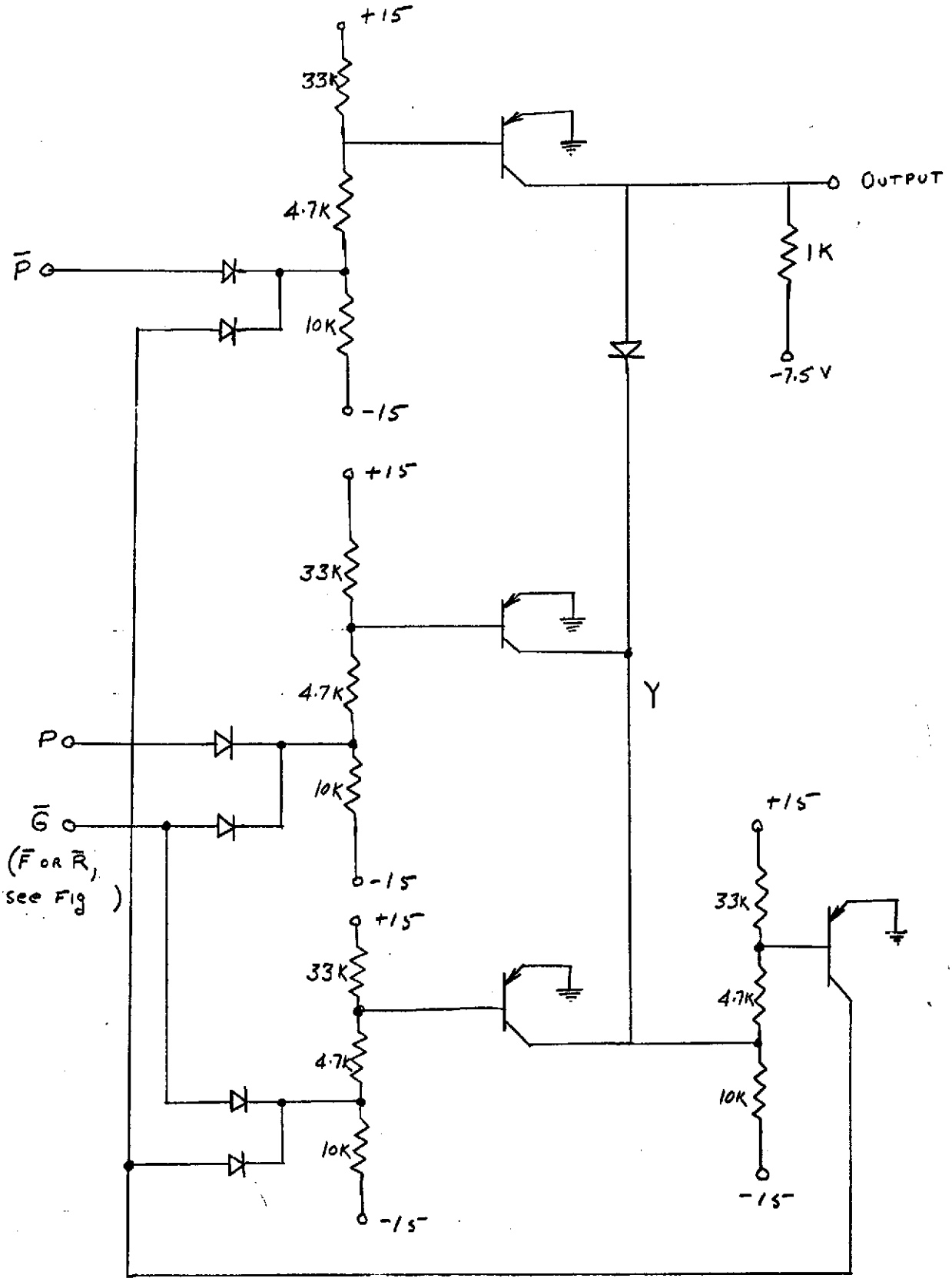
0	0	0	0	0
1	0	1	1	0

$$Y = P.y + G.y + \bar{P}.G$$

$$OUTPUT = P.y$$

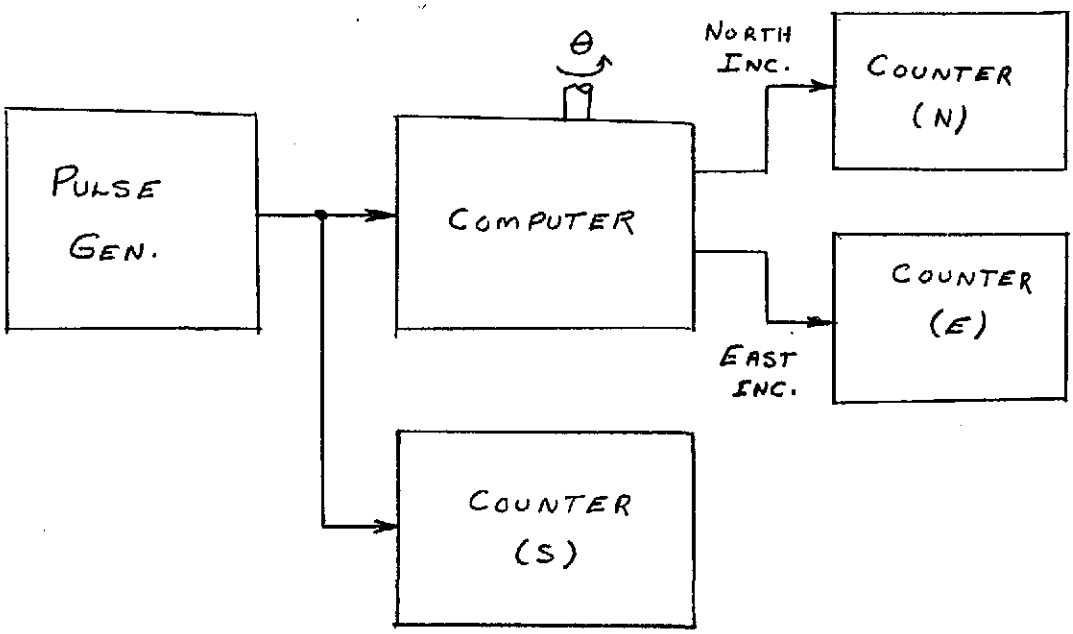
FLOW DIAGRAM AND EXCITATION MATRICES OF THE GATE

Fig. 5A



GATE CIRCUIT

Fig. 6A



TEST SET-UP

Fig. 7A

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